Cyclic constitutive law of HUJEUX for the Résumé

grounds:

The model of behavior known as of “Hujeux”, conceived at the MSSMat laboratory of the ECP [bib 5], is one of the cyclic elastoplastic models of soil mechanics (géomatériaux granular: sandy, normally consolidated or on-consolidated, serious clays…) more adapted for simulations of works geotechnics in earthquake. It moreover is exploited since many years, its parameter setting being thus well controlled.

This model multi-mechanisms (spherical - for a way of consolidation - and déviatoires) with variables mémoratrices are characterized by eight surfaces of load with hardening, defined for monotonous ways and cyclic ways. The mechanisms are defined by fixed plans, which induces an orthotropy of behavior of the ground. Inside these surfaces of reversibility, the material is elastic nonlinear. Hardening is governed by several variables and the flow rule normal is adopted for the mechanisms of consolidation, while the flow rule for the mechanisms déviatoires is nonassociated, according to the rule of dilatancy of Roscoe. Like other models of behavior of grounds, hardening is positive in phase pre-peak and negative in phase post-peak, which corresponds to the effect of dilatancy; these effects induce the behaviour of “liquefaction” of the ground. The plastic strain tensor results from the office plurality of the contributions of various active mechanisms. The voluminal plastic strain couples the mechanisms.

One describes the equations of the model, his parameter setting, then his numerical integration in an implicit diagram of Newton.

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1 theoretical Formulation

1.1 Behavior model of Hujeux

models It described here is the model known as of Hujeux. The model was developed by THE SCHOOL Centrale Paris (LMSSMat laboratory) in the Eighties in order to represent the rheology of the grounds for alternated loadings, for example in earthquake. This behavior model of soil mechanics is a model multi-mechanisms, characterized by twice four surfaces of load with hardening: three connected to mechanisms déviatoires and with a mechanism of spherical consolidation, defined for monotonous ways and cyclic ways. The mechanisms déviatoires are defined on three fixed plans, which induces an orthotropy of behavior of the ground. The spherical mechanisms reproduce the strong non-linearity of géomatériaux on a path of consolidation.

Each one of these planes, subscripted by k , is defined by the basic vectors $(\mathbf{e}_{ik}, \mathbf{e}_{jk})$, extracted the orthonormal base $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ of space with 3 dimensions. The indices are noted:

$$i_k = 1 + \mathrm{mod}(k, 3) \quad \text{and} \quad j_k = 1 + \mathrm{mod}(k+1, 3) \qquad \textbf{éq 1.1}$$

Remarque:

In situation 2D plane strains, the plane $k=3$ corresponds to the plan $(\mathbf{e}_1, \mathbf{e}_2)$ of the model .

The model comprises a nonlinear elasticity controlled by the Young modulus depend on the confining pressure.

A three-dimensional limiting criterion near to that of Mohr-Coulomb is considered to take into account the influence of the effective mean stress on the stiffness of the ground and the characteristics of fracture. The concept of critical condition is also integrated into this model to represent the coupling between the stresses déviatoires and the variations of volume and to formulate isotropic hardening related to the granular medium. A kinematic hardening is added to represent the behavior cyclic of the grounds and coupled with hardening of unit weight of the earth.

The model of Hujeux is expressed in effective stresses - defined as being the difference between the total stresses and the pressure of water in the case of water-logged soils - in the case of the hydraulic coupling: *i.e. one does not take into account the hydrostatic pressure of the fluid which can circulate in the pores, the aforementioned being computed in modelizations THM.*

1.1.1 Definition of the variables of state and statement of the free energy

1.1.1.1 Variables of state

Les variables being used to describe the state of the material point are the following ones:

- $\boldsymbol{\varepsilon}$: tensor strain
- $\boldsymbol{\varepsilon}^{\mathbf{p}}$: tensor plastic strain; one notes in particular $\varepsilon_v^p = \mathrm{tr}\,\varepsilon^p = \boldsymbol{\varepsilon}^{\mathbf{p}}.\mathbf{I}$ the voluminal plastic strain, $\mathbf{I}$ being the tensor identity
- r_k^m : "factors of mobilization" of the mechanisms déviatoires in monotonous way: it is a hardening in the plane pressure-shears
- r_k^c : "factors of mobilization" of the cyclic mechanisms déviatoires: it is an isotropic and kinematical mixed hardening in the plane pressure-shears.
- r_4^m : the factor of mobilization of the mechanism of consolidation in monotonous way: it is a hardening of the spherical mechanism of consolidation (scalar)

- r_4^c : the factor of mobilization of the mechanism of consolidation in cyclic way: it is an isotropic and kinematical mixed hardening (scalar),

accompanied by a certain number of discontinuous variables of history, described hereafter.

The various mechanisms of elastoplastic evolution utilize variables of hardening: factors of mobilization associated with each mechanism and plastic voluminal strain $\varepsilon_v^p = \mathrm{tr}\,\boldsymbol{\varepsilon}^{\mathbf{p}}$, commune with all the mechanisms and coupling them. The latter modifies surfaces of load even if they are not active, because of hardening operated on other activated surfaces of load: see them [§1.1.2 and 1.1.3].

One notes by $\boldsymbol{\sigma}$ the tensor of the stresses (effective), the confining pressure being: $p(\boldsymbol{\sigma}) = \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})$.

By convention in *Code_Aster*, the case of compression corresponds to the stresses (and the strains) negative. For each plastic mechanism in the plane k , one notes by:

$$\boldsymbol{\sigma}_{(k)} = \mathbf{P}_{(k)} \cdot \boldsymbol{\sigma} \cdot \mathbf{P}_{(k)} \quad \textbf{éq 1.1.1-1}$$

stresses in the plane k , oùdésigne $\mathbf{P}_{(k)}$ the tensor (symmetric) of projection as regards base $(\mathbf{e}_{i_k}, \mathbf{e}_{j_k})$, norm $\mathbf{e}_k$:

$$\mathbf{P}_{(k)} = \mathbf{e}_{i_k} \otimes \mathbf{e}_{j_k} + \mathbf{e}_{j_k} \otimes \mathbf{e}_{i_k} \quad \textbf{éq 1.1.1-2}$$

with $i_k = 1 + \mathrm{mod}(k, 3)$ and $j_k = 1 + \mathrm{mod}(k+1, 3)$, is:

$$\boldsymbol{\sigma}_{(k)} = \boldsymbol{\sigma}_{i_k i_k} . \mathbf{e}_{i_k} \otimes \mathbf{e}_{i_k} + \boldsymbol{\sigma}_{j_k j_k} . \mathbf{e}_{j_k} \otimes \mathbf{e}_{j_k} + \boldsymbol{\sigma}_{i_k j_k} . \mathbf{e}_{i_k} \otimes_s \mathbf{e}_{j_k} \quad \textbf{éq 1.1.1-3}$$

then confining pressure in the plane k :

$$p_k(\boldsymbol{\sigma}) = \frac{1}{2}\mathrm{tr}(\boldsymbol{\sigma}_{(k)}) \quad \textbf{éq 1.1.1-4}$$

One defines also the tensor of the stresses déviatoires $\mathbf{S}_{(k)}$ in the plane k , using the tensor identity of order 2 in the plane: $\mathbf{I}_{(k)} = \delta_{i_k j_k} \mathbf{e}_{i_k} \otimes \mathbf{e}_{j_k}$:

$$\mathbf{S}_{(k)}(\boldsymbol{\sigma}) = \boldsymbol{\sigma}_{(k)} - p_k(\boldsymbol{\sigma}) \,.\, \mathbf{I}_{(k)} \quad \textbf{éq 1.1.1-5}$$

One has well $\mathrm{tr}\,\mathbf{S}_{(k)} = 0$. One notes finally the norm (von Mises) tensor of the stresses déviatoires $\mathbf{S}_{(k)}$:

$$q_k(\boldsymbol{\sigma}) = \left\| \mathbf{S}_{(k)}(\boldsymbol{\sigma}) \right\|_{VM}^{2D} = \sqrt{\frac{1}{2}\, \mathbf{S}_{(k)\alpha\beta} . \mathbf{S}_{(k)}^{\alpha\beta}} \quad \textbf{éq 1.1.1-6}$$

Par ailleurs, several variables mémoratrices of irreversible history preserve during certain phases of the evolution the values of the variables of state at the beginning of these phases, and govern the subsequent evolutions, recording the value of this variable at the beginning of the way where a cyclic mechanism engages:

- p_H : scalar variable discontinuous mémoratrice, recording the value of the confining pressure $p(\boldsymbol{\sigma})$
- ε_{vH}^p : scalar variable discontinuous mémoratrice, recording the value of the voluminal plastic strain
- p_{kH} : scalar variable discontinuous mémoratrice, recording the value of the “confining pressure of the plane k ” p_k

- $\mathbf{S}_{(k)H}$: tensorial variable discontinuous mémoratrice, recording the value of the deviators of the stresses $\mathbf{S}_{(k)H}(\boldsymbol{\sigma})$ in each plane k, whose statement is given in [éq 1.1.1-5]
- $\mathbf{S}^c_{(k)H}$: tensorial variable discontinuous mémoratrice, recording the value of the deviators of the stresses "modified" $\mathbf{S}^c_{(k)}\left(\boldsymbol{\sigma}, \varepsilon^p_v, r^c_k\right)$ in each deviative plane k, whose statement is given in [éq 1.1.2-12].

Note:

> *The formulation by orthogonal plans of the model of Hujeux does not make it possible to represent axisymmetric states correctly; cf [bib9] .*

1.1.1.2 FREE ENERGY

One notes the free energy by the sum of an elastic contribution and a contribution of hardening:

$$\mathcal{F}\left(\boldsymbol{\varepsilon}(\vec{u}), \boldsymbol{\varepsilon}^{\mathbf{p}}, r^K_k\right) = \mathcal{F}_{él}\left(\boldsymbol{\varepsilon}(\vec{u}), \boldsymbol{\varepsilon}^{\mathbf{p}}\right) + \mathcal{H}_{écr}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}_{\mathbf{v}}, r^K_k\right) \quad \textbf{éq 1.1.1-7}$$

for $K=m, c$; $k=1,\ldots,4$. Thermodynamic dissipation is obtained by difference starting from the density of power of strain, this in an isothermal process:

$$\mathcal{D}=\boldsymbol{\sigma}.\boldsymbol{\varepsilon}(\dot{\vec{u}}) - \dot{\mathcal{F}}\left(\boldsymbol{\varepsilon}(\vec{u}), \boldsymbol{\varepsilon}^p, r^K_k\right) = \boldsymbol{\sigma}.\dot{\boldsymbol{\varepsilon}}^p - \dot{\mathcal{H}}_{écr}\left(\varepsilon^p_v, r^K_k\right) \quad \textbf{éq 1.1.1-8}$$

Dans la mesure où one does not solve the thermal coupling, one cannot differentiate the density of power dissipated from rate of energy of hardening, corresponding in the term of locked energy. However $\boldsymbol{\sigma}\cdot\dot{\boldsymbol{\varepsilon}}^p$ is not necessarily positive, contrary to $\mathcal{D}$. As one will see it hereafter, the elastic share of the model of Hujeux is nonlinear: one cannot easily extract the statement from the potential $\mathcal{F}_{él}\left(\boldsymbol{\varepsilon}(\vec{u}), \boldsymbol{\varepsilon}^p\right)$.

As the constitutive law of Hujeux has a particular nonlinear elasticity, cf [§1.1.1.3], one cannot give the integrated statement of the free energy.

1.1.1.3 Nonlinear elastoplastic model of Hujeux

One admits that the Young's modulus of the ground depends on the confining pressure $p(\boldsymbol{\sigma})$ by a model power, in the field of compressions $p(\boldsymbol{\sigma})<0$. The nonlinear elastoplastic behavior model of Hujeux is written:

$$\boldsymbol{\sigma}=\frac{\partial}{\partial \boldsymbol{\varepsilon}} \boldsymbol{F}_{él}\left(\boldsymbol{\varepsilon}(\vec{u}), \boldsymbol{\varepsilon}^p\right) = \mathbf{C}(p) . \left(\boldsymbol{\varepsilon}(\vec{u}) - \boldsymbol{\varepsilon}^p\right) \quad \textbf{éq 1.1.1-9}$$

where the elasticity tensor $\mathbf{C}(p)$ is isotropic.

In practice, one cannot express the relation [éq 1.1.1.9] in analytical form, and one identifies it in incremental form (hypoelastic) the relation between the elastic voluminal strain and the confining pressure:

$$\dot{p}=\frac{K_0}{1-n}.\left|\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right|^n . \operatorname{tr}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p\right) \quad \textbf{éq 1.1.1-10}$$

where two parameters are defined: $n \in [\ 0, 1\ [$ and $P_{réf}$ (non-zero), as well as the initial moduli E_0 and ν_0 (or, according to the practice in soil mechanics, positive moduli of initial G_0 shears and K_0 compressibility measured with the confining pressure of reference $P_{réf}$) which define the initial elasticity tensor $\mathbf{C^0}$. The case $n=0$ corresponds to linear elasticity.
The components of the elasticity tensor of the nonlinear elastic model are thus written:

$$C_{ijkl}(p)=C^0_{ijkl}\cdot\left|\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right|^n=G_0\cdot\left(\delta_{ik}\delta_{j\ell}+\delta_{jk}\delta_{i\ell}+\frac{2\nu_0}{1-2\nu_0}\delta_{ij}\delta_{k\ell}\right)\cdot\left|\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right|^n \quad \text{éq 1.1.1-11}$$

One will note the coefficient of compressibility thereafter:

$$K(p)=K_0\cdot\left|\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right|^n \quad \text{éq 1.1.1-12}$$

One notes that the elastic model is not differentiable if the confining pressure is null (and if $n\neq 0$). In practice, this model leads to a greater rigidity of the ground as one goes down in-depth.

The criteria (or surfaces of plastic load) utilize the function of critical pressure, which characterizes the resistance of the ground, depend on the index of the vacuums in the material (hardening of density):

$$P_c\left(\varepsilon_v^p\right)=P_{c0}\cdot e^{-\beta\varepsilon_v^p} \quad \text{éq 1.1.1-13}$$

where two parameters material intervene: critical pressure of initial reference P_{c0} (negative) and β the plastic compressibility of the material (positive). The critical pressure grows in absolute value when the material underwent a negative voluminal plastic strain (compression).

1.1.2 Elastoplastic mechanisms déviatoires

One considers three mechanisms déviatoires: one in each slip surface k (of norm $\mathbf{e}_k$; moreover one identifies a valid behavior during the ways of local monotonous load, and another behavior, controlled by a variable mémoratrice of history, on the ways of cyclic loading (as of the discharge starting from the state reached at the end of a monotonous way).

1.1.2.1 Déviatoires deviatoric criteria in

monotonic loading Les mechanisms in the plane $\left(\mathbf{e}_{ik},\mathbf{e}_{jk}\right)$ for a monotonous way are governed by the criterion:

$$\mathrm{f}_\mathrm{k}^\mathrm{m}\left(\boldsymbol{\sigma},\varepsilon_v^p,r_k^m\right)=q_k(\boldsymbol{\sigma})+p_k(\boldsymbol{\sigma})\cdot\mathrm{F}\left(p_k(\boldsymbol{\sigma}),\varepsilon_v^p\right)\cdot\left(r_k^m+r_{éla}^d\right)\leq 0 \quad \text{éq 1.1.2-1}$$

with:

$p_k(\boldsymbol{\sigma})$ and $q_k(\boldsymbol{\sigma})$ defined in [éq1.1.1], [éq1.1.1] and [éq7];

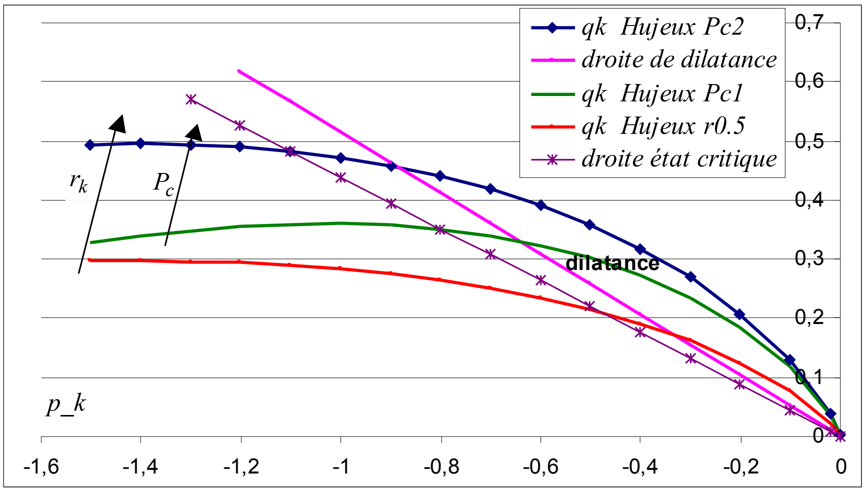
and the function which characterizes the resistance of the ground function of the voluminal plastic strain:

$$\mathrm{F}\left(p_k(\boldsymbol{\sigma}),\varepsilon_v^p\right)=M\cdot\left(1-b_h\ln\left|\frac{p_k(\boldsymbol{\sigma})}{P_c\left(\varepsilon_v^p\right)}\right|\right)\Rightarrow\frac{\partial F}{\partial p_k}=-\frac{b_h M}{p_k};\ \frac{\partial F}{\partial\varepsilon_v^p}=-b_h\beta M \quad \text{éq 1.1.2-2}$$

where three other parameters material intervene: $M=\sin\phi_{pp}$ (ϕ_{pp} is the internal friction angle), the plus coefficient b_h and the parameter $r_{éla}^d\in]0,1[$ which characterizes the size of the threshold in an initial state); the critical pressure $P_c(\varepsilon_v^p)$ is defined by [éq1.1.1].

The criterion [éq1.1.2] is strongly inspired by the model Camwood-Clay [bib8] original. Nevertheless, the statement was modified by Hujeux [bib4], because this model underestimated the strains déviatoires for on-consolidated clays. To correct this weakness, it modified the surface of load by introducing a dependence according to the strains déviatoires without modifying the voluminal flow rule of dilatancy of Roscoe [bib7], cf [§1.1.2]. Compared to Camwood-Clay, models it of Hujeux allows to better modelize the cycles.

[Fig. 1.1] visualizes the effect of the critical pressure $P_c(\varepsilon_v^p)$ and the factor of mobilization r_k^m on the threshold positioned compared to the line of critical condition (angle ϕ_{pp}) and with the straight line of dilatancy (angle ψ , cf [éq1.1.2]).



Appear 1.1-a: Surfaces of load déviatoires (in the plane of the stresses $p_k(\boldsymbol{\sigma})$, $q_k(\boldsymbol{\sigma})$).

Note:

One can reformulate the monotonous criterion [éq1.1.2] by standardizing the equivalent stress $q_k(\boldsymbol{\sigma})$ by $p_k(\boldsymbol{\sigma}).\mathrm{F}\left(p_k(\boldsymbol{\sigma}),\varepsilon_v^p\right)$, in such a way that one can geometrically interpret the criterion by a circle of radius $r_k^m+r_{éla}^d$: $\tilde{\mathrm{f}}_k^m\left(\boldsymbol{\sigma},\varepsilon_v^p,r_k^m\right)=\tilde{q}_k\left(\boldsymbol{\sigma},\varepsilon_v^p\right)-\left(r_k^m+r_{éla}^d\right)\le 0$ *, cf [fig. 1.1-b].*

The criterion [éq 1.1.2-1] requires the presence of an initial state of consolidation $p_k(\boldsymbol{\sigma})\neq 0$ in the material so that the threshold is not null.

Note:

One names liquefaction of the ground the situation which occurs when the effective pressure of consolidation of the ground is close to zero. This state can intervene after a cyclic loading for example for loose sands, but also for a monotonic loading, on a loose sand in not drained conditions.

1.1.2.2 Models of flow and hardening in monotonic loading for the mechanisms déviatoires

the first part the plastic strainrates for the mechanism déviatoire breaks up into a purely deviatoric share, respecting an associated flow model (normal flow), and a spherical share, nonassociated, respecting the principles of the model of dilatancy of Roscoe [bib7]. It is about a “standard material not generalized”. The flow model is written as follows:

$$\left(\dot{\boldsymbol{\varepsilon}}^p\right)_{(k)}^m=\dot{\lambda}_k^m.\boldsymbol{\Psi}_{(k)}^m=\dot{\lambda}_k^m.\left(\frac{\mathbf{S}_{(k)}}{2\mathrm{q}_k}-\frac{\zeta_0.\zeta\left(r_k^m+r_{éla}^d\right)}{2}.\left(\sin\psi+\frac{q_k}{p_k}\right).\mathbf{I}_{(k)}\right) \quad \text{éq 1.1.2-3}$$

the tensor $\boldsymbol{\Psi}_{(k)}^m$ indicates the flow direction. The angle of dilatancy ψ and ζ_0 are parameters material. The function $\zeta(r)$ makes it possible to control the effect of the variation of volume during an yielding on a mechanism déviatoire (flow which is then *not standard*). Up to a certain level of shears, the variation of volume is null; beyond it intervenes. The function $\zeta(r)$ has as a statement:

$$\zeta(r)=\begin{cases} 0 & si\ r\le r_{hys} \quad domaine\ pseudo-élastique \\ \left(\dfrac{r-r_{hys}}{r_{mob}-r_{hys}}\right)^{x_m} & si\ r_{hys}<r\le r_{mob} \quad domaine\ hystérétique \\ 1 & si\ r>r_{mob} \quad domaine\ plastique \end{cases} \quad \textbf{éq 1.1.2-4}$$

where r_{hys}, r_{mob}, x_m are new parameters of the mechanisms déviatoires.

One names *characteristic state* the situation corresponding to evolutions where $\dot{\varepsilon}_v^p=0$ (null evolution of *the contractance*). In the model Cam Clay, cf 5). this null evolution of *the contractance*, followed by a phase of *dilatancy* where $\dot{\varepsilon}_v^p=\dot{\boldsymbol{\varepsilon}}^p.\mathbf{I}>0$, the stop of the positive hardening of the material produces, but it is not the case of the model of Hujeux, for which negative hardening it resistance decreases, cf [éq1.1.2] - occurs later on.

The law of evolution followed by the intern variable of hardening r_k^m (monotonous factor of mobilization) is governed by the same plastic multipliers $\dot{\lambda}_k^m$ as on $(\dot{\boldsymbol{\varepsilon}}^p)_{(k)}^m$:

$$\dot{r}_k^m=\dot{\lambda}_k^m.\rho_k^m=\dot{\lambda}_k^m\frac{\left(1-r_k^m-r_{éla}^d\right)^2}{a_c+\zeta\left(r_k^m+r_{éla}^d\right).\left(a_m-a_c\right)} \quad \textbf{éq 1.1.2-5}$$

where a_m , a_c are new parameters (strictly positive) of the mechanisms déviatoires. One must always have $\dot{r}_k^m\ge 0$; moreover, [éq1.1.2] imposes that $r_k^m+r_{éla}^d\le 1$. Hardening is positive in phase pre-peak and negative in phase post-peak.

The plastic multipliers $\dot{\lambda}_k^m$, which must be positive, are obtained by resolution of the equation of “complementarity” of Kühn-Tücker, jointly with the condition of “coherence”:

$$\dot{\lambda}_k^m.\mathrm{f}_k^m\left(\sigma,\varepsilon_v^p,r_k^m\right)=0 \quad \text{etéq} \quad \dot{\mathrm{f}}_k^m\left(\boldsymbol{\sigma},\varepsilon_v^p,r_k^m\right)=0=\mathrm{f}_{k,\boldsymbol{\sigma}}^m.\dot{\boldsymbol{\sigma}}+\mathrm{f}_{k,\varepsilon_v^p}^m.\dot{\varepsilon}_v^p+\mathrm{f}_{k,r_k^m}^m.\dot{r}_k^m \quad \textbf{1.1.2-6}$$

from where, while combining with [éq 1.1.1], if only this monotonous mechanism k is activated:

$$\dot{\lambda}_k^m=-\frac{\mathrm{f}_{k,\boldsymbol{\sigma}}^m.\dot{\boldsymbol{\sigma}}}{\mathrm{f}_{k,\varepsilon_v^p}^m.\boldsymbol{\Psi}_{(k)}^m.\mathbf{I}+f_{k,r_k^m}^m.\rho_k^m}=\frac{\langle\mathrm{f}_{k,\sigma}^m.\mathbf{C}.\dot{\boldsymbol{\varepsilon}}\rangle_+}{\mathrm{f}_{k,\boldsymbol{\sigma}}^m.\mathbf{C}.\boldsymbol{\Psi}_{(k)}^m-\mathrm{f}_{k,\varepsilon_v^p}^m.\boldsymbol{\Psi}_{(k)}^m.\mathbf{I}-\mathrm{f}_{k,r_k^m}^m.\rho_k^m} \quad \textbf{éq 1.1.2-7}$$

and in the general case, it is necessary to take account of the contribution of all the active mechanisms on yielding $\dot{\boldsymbol{\varepsilon}}^p$, cf [éq1.1.5]:

$$\dot{\lambda}_k^m=-\frac{\mathrm{f}_{k,\boldsymbol{\sigma}}^m.\dot{\boldsymbol{\sigma}}+\mathrm{f}_{k,\varepsilon_v^p}^m.\left(\sum_{(K,t)\neq(m,k)}\dot{\lambda}_t^K\boldsymbol{\Psi}_{(t)}^K.\mathbf{I}\right)}{\mathrm{f}_{k,\varepsilon_v^p}^m.\boldsymbol{\Psi}_{(k)}^m.\mathbf{I}+\mathrm{f}_{k,r_k^m}^m.\rho_k^m}=\frac{\left\langle\mathrm{f}_{k,\boldsymbol{\sigma}}^m.\mathbf{C}.\dot{\boldsymbol{\varepsilon}}+\left(\sum_{(K,t)\neq(m,k)}\dot{\lambda}_t^K\boldsymbol{\Psi}_{(t)}^K\right).\left(\mathrm{f}_{k,\sigma}^m.\mathbf{C}-\mathrm{f}_{k,\varepsilon_v^p}^m.\mathbf{I}\right)\right\rangle_+}{\mathrm{f}_{k,\boldsymbol{\sigma}}^m.\mathbf{C}.\boldsymbol{\Psi}_{(k)}^m-\mathrm{f}_{k,\varepsilon_v^p}^m.\boldsymbol{\Psi}_{(k)}^m.\mathbf{I}-\mathrm{f}_{k,r_k^m}^m.\rho_k^m} \quad \textbf{éq 1.1.2-8}$$

here appearing Les various terms are computed in [éq 7], [éq 7], [éq 7]. The statement [éq1.1.2] contributes to the computation of the increase in stresses $\dot{\sigma}$ from where the tangent operator is fired, cf [§2].

1.1.2.3 Déviatoires criteria in cyclic loading

Quand the cyclic mechanisms intervene then the monotonous mechanisms "are solidified", i.e. the associated intern variables remain constant. A cyclic mechanism deviatoric "wires" engages when a discharge occurs on a monotonous mechanism "father" previously activated, and that one violates the criterion of this cyclic mechanism on the actual position at time t . The condition is written as follows:

$$f^{m}_{k,\sigma}\left(\boldsymbol{\sigma}(t),\varepsilon^{p}_{v}(t),r^{m}_{k}(t)\right).\mathbf{C}\left(\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right).\dot{\boldsymbol{\varepsilon}}(t)<0 \quad \text{etéq} \quad f^{c}_{k}\left(\boldsymbol{\sigma}(t),\varepsilon^{p}_{v}(t),r^{c}_{k}(t)\right)>0 \qquad \textbf{1.1.2-9a}$$

Cela can also occur between two cyclic mechanisms, "father" and "wires", when a change of management intervenes, cf [fig. 1.1]:

$$f^{c1}_{k,\sigma}\left(\boldsymbol{\sigma}(t),\varepsilon^{p}_{v}(t),r^{c1}_{k}(t)\right).\mathbf{C}\left(\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right).\dot{\boldsymbol{\varepsilon}}(t)<0 \quad \text{etéq} \quad f^{c2}_{k}\left(\boldsymbol{\sigma}(t),\varepsilon^{p}_{v}(t),r^{c2}_{k}(t)\right)>0 \qquad \textbf{1.1.2-9b}$$

a criterion of relative proximity is however introduced in order to leave active to the mechanism "father", to the detriment of the mechanism "wires".

In the event of microcomputer-discharge on a way of stress, followed by an increase in load, the mechanism "wires" is deactivated with the profit of the mechanism "father", in order to rather take again the value of the hardening modulus of this last than to traverse an elastic way of slope, which would be physically contestable, which is treated with the level of numerical integration, cf [§2.2.3.1], not d) vii. 1.a.

It can as occur as a monotonous mechanism "succeeds" a cyclic mechanism, when its surface of load reaches that of the monotonous mechanism, cf [fig. 1.1].

The precise description of the sequence of the monotonous and cyclic mechanisms with the record of the variables discrete mémoratrices and the possible restorations of the variables of hardening is a component of the model of Hujeux: it is presented in detail to [§2.2.3.1].

The surface of load of each cyclic mechanism is a general formulation common to the three déviatoires planes considered. The cyclic criteria deviatoric are written:

$$f^{c}_{k}\left(\boldsymbol{\sigma},\varepsilon^{p}_{v},r^{c}_{k}\right)=q^{c}_{k}\left(\boldsymbol{\sigma},\varepsilon^{p}_{v},r^{c}_{k}\right)+p_{k}(\boldsymbol{\sigma}).\,F\left(p_{k}(\boldsymbol{\sigma}),\varepsilon^{p}_{v}\right).\left(r^{c}_{k}+r^{dc}_{éla}\right)\le 0 \qquad \textbf{éq 1.1.2-10}$$

oùest $q^{c}_{k}\left(\boldsymbol{\sigma},\varepsilon^{p}_{v},r^{c}_{k}\right)$ an alternative for the cyclic mechanisms of $q_{k}(\boldsymbol{\sigma})$:

$$q^{c}_{k}\left(\boldsymbol{\sigma},\varepsilon^{p}_{v},r^{c}_{k}\right)=\left\|\mathbf{S}^{c}_{(k)}\left(\boldsymbol{\sigma},\varepsilon^{p}_{v},r^{c}_{k}\right)\right\|^{2D}_{VM} \qquad \textbf{éq 1.1.2-11}$$

with:

$$\mathbf{S}^{c}_{(k)}\left(\boldsymbol{\sigma},\varepsilon^{p}_{v},r^{c}_{k}\right)=\mathbf{S}_{(k)}(\boldsymbol{\sigma})-p_{k}(\boldsymbol{\sigma}).F\left(p_{k}(\boldsymbol{\sigma}),\varepsilon^{p}_{v}\right).\left(\mathbf{X}^{H}_{(k)}+\frac{\mathbf{S}^{c}_{(k)H}}{\left\|\mathbf{S}^{c}_{(k)H}\right\|^{2D}_{VM}}.\left(r^{c}_{k}+r^{dc}_{éla}\right)\right) \qquad \textbf{éq 1.1.2 the -12}$$

function $F\left(p_{k}(\boldsymbol{\sigma}),\varepsilon^{p}_{v}\right)$ is same as that defined in [éq 1.1.2]. The parameter $r^{dc}_{éla}\in]0,1[$ often has the same value as $r^{d}_{éla}$ for the déviatoire criterion in monotonous way, cf [§1.1.2].

The tensor $\mathbf{X}^{H}_{(k)}$ is a function of the discontinuous variables mémoratrices [§1.1.1], in the plan of the mechanism considered, necessary to the description of the history of each mechanism déviatoires, defined initially starting from the monotonous mechanism; it makes it possible to describe the kinematical behaviour of hardening. The indices H refer to the variables mémoratrices modified with each transition from mechanism with change of meaning of the stresses. Its statement is:

$$\mathbf{X}_{(k)}^{H}=\frac{\mathbf{S}_{(k)H}}{p_{kH}\,.\mathrm{F}\left(p_{kH},\varepsilon_{vH}^{p}\right)}$$ **éq 1.1.2 -13**

Remarque:

Like $\mathbf{S}_{(k)}^{c}\cdot\mathbf{I}=0$ and also $\mathbf{S}_{(k)}^{c}.\mathbf{I}_{(k)}=0$ *, it is well a tensor deviatoric.*

One can reformulate the cyclic criterion [éq1.1.2] *by standardizing the equivalent stress* q_{k}^{c} *by* $p_{k}(\boldsymbol{\sigma})\,.\mathrm{F}\left(p_{k}(\boldsymbol{\sigma}),\varepsilon_{v}^{p}\right)$ *, in such a way that one can geometrically interpret the criterion by a circle of radius* $r_{k}^{c}+r_{ela}^{d}$ *:*

$$\tilde{\mathrm{f}}_{\mathrm{k}}^{\mathrm{c}}\left(\boldsymbol{\sigma},\mathbf{X}_{(k)},\varepsilon_{v}^{p},r_{k}^{c}\right)=\tilde{q}_{k}^{c}\left(\boldsymbol{\sigma},\mathbf{X}_{(k)},\varepsilon_{v}^{p},r_{k}^{c}\right)-\left(r_{k}^{c}+r_{éla}^{dc}\right)\leq 0\,.$$

Note:

When one reaches the first cyclic loading after a first monotonous way in the history given, one a: $\mathbf{X}_{(k)}^{H}=\frac{\mathbf{S}_{(k)H}}{p_{kH}\,.\mathrm{F}\left(p_{kH},\varepsilon_{vH}^{p}\right)}$ $\mathbf{S}_{(k)H}^{c}=\mathbf{S}_{(k)}(\boldsymbol{\sigma})$ *, therefore:*

$\mathbf{S}_{(k)}^{c}\left(\boldsymbol{\sigma},\varepsilon_{v}^{p},r_{k}^{c}\right)=-p_{k}(\boldsymbol{\sigma})\,.\mathrm{F}\left(p_{k}(\boldsymbol{\sigma}),\varepsilon_{v}^{p}\right).\frac{\mathbf{S}_{(k)H}^{c}}{\left\|\mathbf{S}_{(k)H}^{c}\right\|_{VM}^{2D}}.\left(r_{k}^{c}+r_{éla}^{dc}\right)$ *with the initialization of the cyclic way.*

1.1.2.4 Models of flow and hardening in cyclic loading

the contribution of these cyclic mechanisms deviatoric at the plastic strainrate has the same form as for the monotonous mechanism [éq 1.1.2]:

$$\left(\dot{\boldsymbol{\varepsilon}}^{p}\right)_{(k)}^{c}=\dot{\lambda}_{k}^{c}.\boldsymbol{\Psi}_{(k)}^{c}=\dot{\lambda}_{k}^{c}.\left(\frac{\mathbf{S}_{(k)}^{c}}{2\,q_{k}^{c}}-\frac{\zeta_{0}.\zeta\left(r_{k}^{c}+r_{éla}^{dc}\right)}{2}.\left(\sin\psi+\frac{\mathbf{S}_{(k)}.\mathbf{S}_{(k)}^{c}}{2\,p_{k}\,q_{k}^{c}}\right).\mathbf{I}_{(k)}\right)$$ **éq 1.1.2-14**

oùest $\mathbf{S}_{(k)}^{c}$ defined by [éq1.1.2] and the function $\zeta(r)$ defined by [éq1.1.2]. The tensor $\boldsymbol{\Psi}_{(\mathbf{k})}^{\mathbf{c}}$ indicates the flow direction, which is *not standard*.

The law of evolution associated with the variables with hardening r_{k}^{c} is identical to that stated for the factor of monotonous mobilization r_{k}^{m} [éq1.1.2]:

$$\dot{r}_{k}^{c}=\dot{\lambda}_{k}^{c}.\rho_{k}^{c}=\dot{\lambda}_{k}^{c}\frac{\left(1-r_{k}^{c}-r_{éla}^{dc}\right)^{2}}{a_{c}+\zeta\left(r_{k}^{c}+r_{éla}^{dc}\right).\left(a_{m}-a_{c}\right)}\cdot\gamma_{k}^{c}$$ **éq 1.1.2-15**

with:
$$\gamma_{k}^{c}=\frac{2\mathrm{q}_{k}^{c}.\|\mathbf{S}_{(\mathbf{k})\mathbf{H}}^{\mathbf{c}}\|_{VM}^{2D}}{2\mathrm{q}_{k}^{c}.\|\mathbf{S}_{(\mathbf{k})\mathbf{H}}^{\mathbf{c}}\|_{VM}^{2D}-\mathbf{S}_{(\mathbf{k})\mathbf{H}}^{\mathbf{c}}.\mathbf{S}_{(\mathbf{k})}^{\mathbf{c}}}$$

Note:

The last factor appearing with the denominator of [éq 1.1.2 -15] is not noted in the references [bib3], [bib4], but is present in the Gefdyn software; this factor allows that the predictions are closer to the experimental results . The denominator in γ_{k}^{c} *should not be null in practice (* $\mathbf{S}_{(k)}^{c}$ *not being identical to* $\mathbf{S}_{(k)H}^{c}$ *); however before convergence of the iterations, if that occurs, one makes simply* $\gamma_{k}^{c}=1$ *, as in the monotonous case, cf [éq*

1.1.2 -5] . This choice of γ_k^c *makes it possible to obtain the equality of the plastic moduli* $f^c_{k,r^c_k}\cdot\rho^c_k=f^m_{k,r^m_k}\cdot\rho^m_k$ *, in accordance with the experimental observation .*
One must always have $\dot{r}^c_k\geq 0$ *, moreover, [éq 1.1.2 -15] imposes that* $r^c_k+r^{dc}_{éla}\leq 1$ *.*

The plastic multipliers $\dot{\lambda}^c_k$, which must be positive, are obtained by resolution of the equation of complementarity of Kühn-Tücker, jointly with the condition of coherence:

$$\dot{\lambda}^c_k . f^c_k\left(\boldsymbol{\sigma},\varepsilon^p_v,r^c_k\right)=0 \text{ and } \dot{f}^c_k\left(\boldsymbol{\sigma},\varepsilon^p_v,r^c_k\right)=0 \quad \textbf{éq 1.1.2-16}$$

from where, while combining with [éq 1.1.1], if only this mechanism is activated:

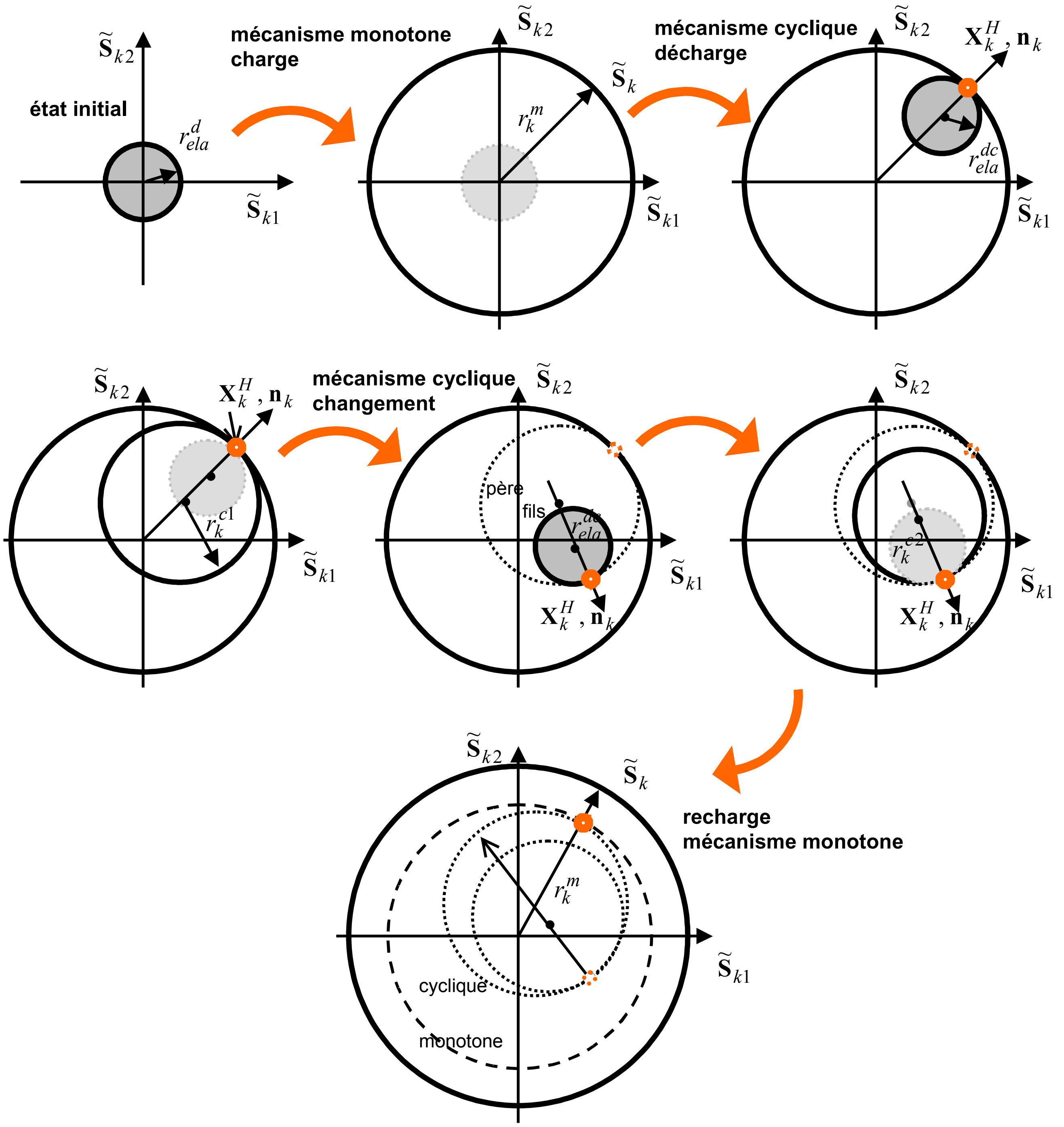
$$\dot{\lambda}^c_k=-\frac{f^c_{k,\boldsymbol{\sigma}}.\dot{\boldsymbol{\sigma}}}{f^c_{k,\varepsilon^p_v}.\boldsymbol{\Psi}^c_{(k)}.\mathbf{I}+f^c_{k,r^c_k}.\rho^c_k}=\frac{\left\langle f^c_{k,\boldsymbol{\sigma}}.\mathbf{C}.\dot{\boldsymbol{\varepsilon}}\right\rangle_+}{f^c_{k,\boldsymbol{\sigma}}.\mathbf{C}.\boldsymbol{\Psi}^c_{(k)}-f^c_{k,\varepsilon^p_v}.\boldsymbol{\Psi}^c_{(k)}.\mathbf{I}-f^c_{k,r^c_k}.\rho^c_k} \quad \textbf{éq 1.1.2-17}$$

and in the general case, it is necessary to take account of the contribution of all the active mechanisms on yielding $\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}$, cf [éq1.1.5]:

$$\dot{\lambda}^c_k=-\frac{f^c_{k,\boldsymbol{\sigma}}.\dot{\boldsymbol{\sigma}}+f^c_{k,\varepsilon^p_v}.\left(\sum_{(K,t)\neq(c,k)}\dot{\lambda}^K_t\boldsymbol{\Psi}^K_{(t)}.\mathbf{I}\right)}{f^c_{k,\varepsilon^p_v}.\boldsymbol{\Psi}^c_{(k)}.\mathbf{I}+f^c_{k,r^c_k}.\rho^c_k}=\frac{\left\langle f^c_{k,\sigma}.\mathbf{C}.\dot{\boldsymbol{\varepsilon}}+\left(\sum_{(K,t)\neq(c,k)}\dot{\lambda}^K_t\boldsymbol{\Psi}^K_{(t)}\right).\left(f^c_{k,\sigma}.\mathbf{C}-f^c_{k,\varepsilon^p_v}.\mathbf{I}\right)\right\rangle_+}{f^c_{k,\sigma}.\mathbf{C}.\boldsymbol{\Psi}^c_{(k)}-f^c_{k,\varepsilon^p_v}.\boldsymbol{\Psi}^c_{(k)}.\mathbf{I}-f^c_{k,r^c_k}.\rho^c_k}$$

éq 1.1.2-18

here appearing Les various terms are computed in [éq 7], [éq 7], [éq 7]. The statement [éq1.1.2] contributes to the computation of the increase in stresses $\dot{\boldsymbol{\sigma}}$, from where the tangent operator is fired, cf [§2].



Appear 1.1-b: Evolution of surfaces of load déviatoires for an unspecified way of loading (in the plane of the stresses standardized by $p_k(\boldsymbol{\sigma}) \cdot F\left(p_k(\boldsymbol{\sigma}), \varepsilon_v^p\right)$). The points of the way marked of a small round indicate the places of the changes of mechanisms.

1.1.3 Elastoplastic mechanisms of consolidation spherical

Sur of the paths of consolidation (forced spherical), the various mechanisms déviatoires do not take part. However, the géomatériaux ones have a strong non-linearity for this kind of loadings. One thus identifies, in the field of compressions $p(\boldsymbol{\sigma})<0$, a valid behaviour of consolidation during the ways of local monotonous load, and another behaviour of consolidation, controlled by a variable of history in cyclic way of loading (as of the discharge starting from a monotonous way). They thus act on the voluminal behavior of the material on the spherical terms of the tensors. These mechanisms are coupled with the mechanisms déviatoires to take account of the phenomenon of hardening of density related to the behavior déviatoire of the material.

1.1.3.1 Criterion of consolidation in monotonic loading

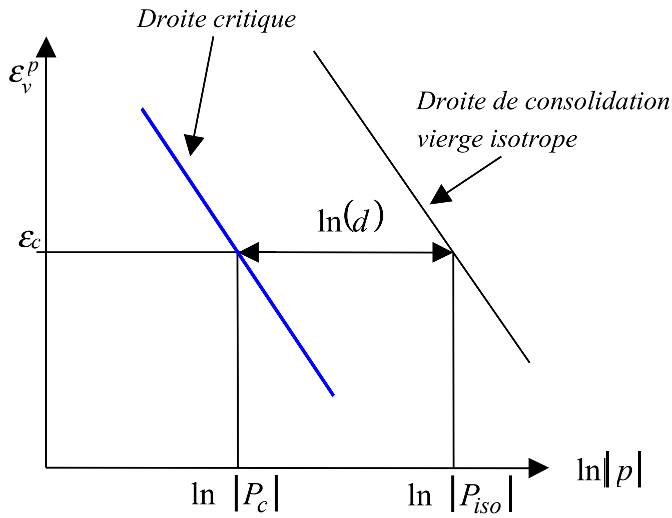
the mechanism of consolidation for a monotonous way is governed by the criterion:

$$f_4^m\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ r_4^m\right)=\left|p(\boldsymbol{\sigma})\right|+d\ .P_{c0}.\ e^{-\beta\varepsilon_v^p}.\left(r_4^m+r_{éla}^s\right)\ \leq\ 0 \quad \textbf{éq 1.1.3-1}$$

where two new parameters material intervene: d (positive), distance enters the line of critical condition and the isotropic line of consolidation, in the plane $\left(\ln|p|,\ \varepsilon_v^p\right)$, and $r_{éla}^s$, cf [fig. 1.1]. The parameter $r_{éla}^s\in]\ 0,1\ [$ characterizes the size of the threshold in an initial state.

Note:

One observes in [éq 1.1.3 -1] that if one passes from a contracting phase $\dot{\varepsilon}_v^p<0$ to a dilating phase $\dot{\varepsilon}_v^p>0$, hardening passes from a positive state to a negative state (attenuation of the threshold): this point of transition during the evolution thus constitutes a "delicate" point for the integration of the model of Hujeux in a structural analysis.



Appear 1.1-c: straight line of critical condition and isotropic line of consolidation.

1.1.3.2 Models of flow and hardening in Pour

monotonic loading the mechanism of consolidation, by respecting a flow model associated, one defines a contribution at the plastic strainrates:

$$\left(\dot{\boldsymbol{\varepsilon}}^p\right)_4^m=\dot{\lambda}_4^m.\boldsymbol{\Psi}_{(4)}^m=\dot{\lambda}_4^m.\frac{p}{3\ |p|}.\mathbf{I}=\frac{1}{3}\dot{\lambda}_4^m.\text{sgn}(p)\ .\mathbf{I} \quad \text{éq 1.1.3-2}$$

where $\dot{\lambda}_4^m$ indicates the monotonous spherical plastic multiplier, which must be positive. One will note: $sgn(p)=\frac{p}{|p|}$. In practice $\dot{\lambda}_4^m \cdot p \leq 0$.

The law of evolution followed by the variable of hardening r_4^m (factor of mobilization) is controlled by the same plastic multiplier $\dot{\lambda}_4^m$:

$$\dot{r}_4^m=\dot{\lambda}_4^m \rho_4^m=\dot{\lambda}_4^m \frac{\left(1-r_4^m-r_{éla}^s\right)^2}{c_m} \frac{P_{réf}}{P_c\left(\varepsilon_v^p\right)} \qquad \text{éq 1.1.3-3}$$

where c_m is a parameter (strictly positive) monotonous mechanism of consolidation. $P_{réf}$ is the confining pressure of reference and $P_c(\varepsilon_v^p)$ the critical pressure [éq 1.1.1]. One must always have $\dot{r}_4^m \geq 0$, moreover, [éq 1.1.3-3] imposes that $r_4^m + r_{éla}^s \leq 1$.

The plastic multiplier $\dot{\lambda}_4^m$ is obtained by resolution of the equation of complementarity of Kühn-Tücker, jointly with the condition of "coherence":

$$\dot{\lambda}_4^m . \mathrm{f}_4^m\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ r_4^m\right)=0 \quad \text{and} \quad \dot{\mathrm{f}}_4^m\left(\boldsymbol{\sigma}, \varepsilon_v^p, r_4^m\right)=0=\mathrm{f}_{4,\boldsymbol{\sigma}}^m . \dot{\boldsymbol{\sigma}}+\mathrm{f}_{4,\varepsilon_v^p}^m . \dot{\varepsilon}_v^p+\mathrm{f}_{4,r_4^m}^m . \dot{r}_4^m \qquad \text{éq 1.1.3-4}$$

from where, while combining with [éq 1.1.1], if only this mechanism is activated (note: $\mathbf{C} \cdot \mathbf{I}=3K \cdot \mathbf{I}$, cf [éq 6-25]):

$$\begin{aligned}\dot{\lambda}_4^m &= -\frac{\mathrm{f}_{4,\sigma}^m . \dot{\boldsymbol{\sigma}}}{\mathrm{f}_{4,\varepsilon_v^p}^m . \mathrm{sgn}(p)+\mathrm{f}_{4,r_4^m}^m . \rho_4^m}=\frac{\langle \mathrm{f}_{4,\sigma}^m . \mathbf{C} . \dot{\boldsymbol{\varepsilon}}\rangle_+}{K(p) . \mathrm{sgn}(p) . \mathrm{f}_{4,\sigma}^m . \mathbf{I}-f_{4,\varepsilon_v^p}^m . \mathrm{sgn}(p)-\mathrm{f}_{4,r_4^m}^m . \rho_4^m} \\ &= \frac{K(p) . \langle \mathrm{sgn}(p) . \mathrm{tr}\ \dot{\boldsymbol{\varepsilon}}\rangle_+}{K(p)+d . \beta . P_c\left(\varepsilon_v^p\right) . \mathrm{sgn}(p) . \left(r_4^m+r_{éla}^s\right)-\frac{d . P_{réf}}{c_m}\left(1-r_4^m-r_{éla}^s\right)^2}\end{aligned} \qquad \text{éq 1.1.3-5}$$

and in the general case, it is necessary to take account of the contribution of all the active mechanisms on yielding $\dot{\boldsymbol{\varepsilon}}^p$, cf [éq1.1.5]:

$$\begin{aligned}\dot{\lambda}_4^m &= -\frac{\mathrm{f}_{4,\boldsymbol{\sigma}}^m . \dot{\boldsymbol{\sigma}}+\mathrm{f}_{4,\varepsilon_v^p}^m . \left(\sum_{(K,k)\neq(m,4)} \dot{\lambda}_k^K \boldsymbol{\Psi}_{(k)}^K . \mathbf{I}\right)}{\mathrm{f}_{4,\varepsilon_v^p}^m . \boldsymbol{\Psi}_{(4)}^m . \mathbf{I}+\mathrm{f}_{4,r_4^m}^m . \rho_4^m}= \\ & \frac{\left\langle 3\mathrm{K}(p) . \mathrm{sgn}(p) . \mathrm{tr}\ \dot{\boldsymbol{\varepsilon}}+\left(\sum_{(K,k)\neq(m,4)} \dot{\lambda}_k^K \boldsymbol{\Psi}_{(k)}^K . \mathbf{I}\right) . \left(K(p) . \mathrm{sgn}(p)-\mathrm{f}_{4,\varepsilon_v^p}^m\right)\right\rangle_+}{K(p)-\mathrm{f}_{4,\varepsilon_v^p}^m . \mathrm{sgn}(p)-\mathrm{f}_{4,r_4^m}^m . \rho_4^m}\end{aligned} \qquad \text{éq 1.1.3-6}$$

here appearing Les various terms are computed in [éq 7], [éq 7], [éq 7]. The statement [éq 1.1.3-5] contributes to the computation of the increase in stresses $\dot{\boldsymbol{\sigma}}$, from where the tangent operator is fired, cf [§2].

1.1.3.3 Criterion of consolidation in cyclic loading

Quand the cyclic mechanism intervenes then the monotonous mechanism is "fixed". The cyclic mechanism of consolidation "wires" engages when:

$$f_4^m\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ r_4^m\right)=0 \quad ; \quad f_{4,\boldsymbol{\sigma}}^m\left(\boldsymbol{\sigma}(t),\varepsilon_v^p(t),r_4^m(t)\right).\mathbf{C}\left(\frac{p(\boldsymbol{\sigma})}{P_{réf}}\right).\dot{\boldsymbol{\varepsilon}}(t) < 0 \qquad \text{éq 1.1.3-7}$$

the cyclic criterion of consolidation is written same form as into monotonous [éq 1.1.3]:

$$f_4^c\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ p_H,\varepsilon_{vH}^p,\ r_4^c\right)=\left|p^c\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ p_H,\varepsilon_{vH}^p\right)\right|+d.P_{c0}.e^{-\beta\varepsilon_v^p}.\left(r_4^c+r_{éla}^{sc}\right) \leq 0 \qquad \text{éq 1.1.3-8}$$

oùdésigne p^c an alternative of $p(\boldsymbol{\sigma})$ for the cyclic mechanisms:

$$p^c\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ p_H,\varepsilon_{vH}^p\right)=\left|p(\sigma)\right|+p_H.e^{-\beta\left(\varepsilon_v^p-\varepsilon_{vH}^p\right)} \qquad \text{éq 1.1.3-9}$$

where p_H (variable mémoratrice) the value reached by at p the beginning of the way indicates where the cyclic mechanism engages, cf [fig. 1.1]. The parameter $r_{éla}^{sc}\in\left]0,1\right[$, having often the same value that for the monotonous criterion, the size of the threshold in an initial state characterizes.

Note:

The statement [éq 1.1.3-9] is that chosen by the ECP [bib3] today, while Hujeux [bib4] had proposed a formulation utilizing too r_4^c .

1.1.3.4 Models of flow and hardening in cyclic loading

the contribution of the cyclic mechanism of consolidation at the speed of the plastic strains has the same form as for the monotonous mechanism:

$$\left(\dot{\boldsymbol{\varepsilon}}^p\right)_4^c=\dot{\lambda}_4^c.\boldsymbol{\Psi}_{(4)}^{\mathbf{c}}=\dot{\lambda}_4^c.\frac{p^c\ .\ p}{3\left|p^c\right|.\left|p\right|}.\mathbf{I}=\frac{1}{3}\dot{\lambda}_4^c.\operatorname{sgn}(p).\operatorname{sgn}\left(p^c\right).\mathbf{I} \qquad \text{éq 1.1.3-10}$$

where $\dot{\lambda}_4^c$ indicates the cyclic spherical plastic multiplier, which must be positive, and where p^c is defined by [éq. 1.1.3].

The law of evolution followed by the variable of hardening r_4^c is controlled by the same plastic multiplier $\dot{\lambda}_4^c$:

$$\dot{r}_4^c=\dot{\lambda}_4^c\rho_4^c=\dot{\lambda}_4^c\frac{\left(1-r_4^c-r_{éla}^{sc}\right)^2}{2c_c}\frac{P_{réf}}{P_c\left(\varepsilon_v^p\right)} \qquad \text{éq 1.1.3-11}$$

c_c is a parameter (strictly positive) cyclic mechanism of consolidation. $P_{réf}$ is the confining pressure of reference and $P_c(\varepsilon_v^p)$ the critical pressure of reference [éq 1.1.1-13]. One must always have $\dot{r}_4^c\geq 0$, moreover, [éq 1.1.3-11] imposes that $r_4^c+r_{éla}^{sc}\leq 1$.

Note:

Factor 2 appearing with the denominator of [éq 1.1.3 -11] is not noted in the references [bib3], [bib4], but is present in the Gefdyn software; this factor allows that the predictions are closer to the experimental results .

The plastic multiplier $\dot{\lambda}_4^c$ is obtained by resolution of the equation of complementarity of Kühn-Tücker, jointly with the condition of coherence:

$$\dot{\lambda}_4^c.f_4^c\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ p_H,p_H^c,\ r_4^c\right)=0 \quad \text{and} \quad \dot{f}_4^c\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ p_H,p_H^c,\ r_4^c\right)=0 \qquad \text{éq 1.1.3-12}$$

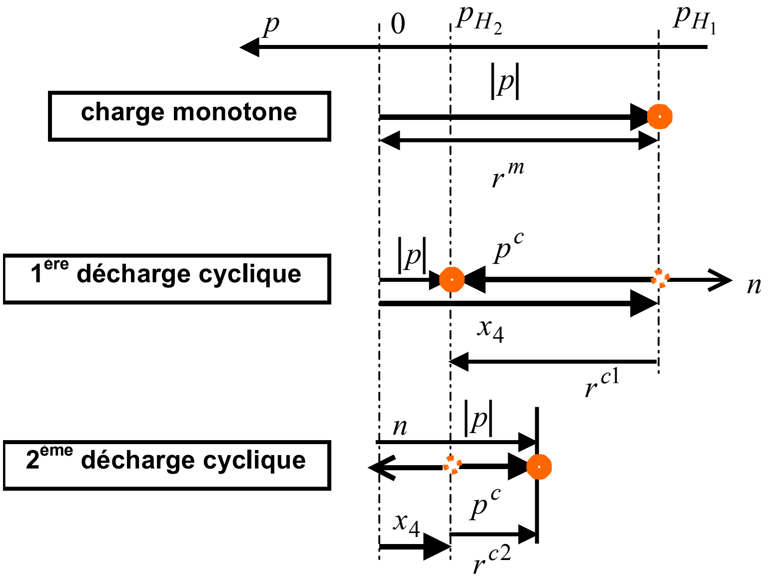
from where, while combining with [éq 1.1.1], if only this mechanism is activated:

$$\dot{\lambda}_4^c = -\frac{f^c_{4,\sigma}.\dot{\sigma}}{f^c_{4,\varepsilon_v^p}\frac{p^c}{|p^c|}\frac{p}{|p|}+f^c_{4,r_4^c}.\rho_4^c} = \frac{\langle f^c_{4,\sigma}.\mathbf{C}.\dot{\varepsilon}\rangle_+}{f^c_{4,\sigma}\frac{p^c}{3|p^c|}\frac{p}{|p|}\cdot\mathbf{C}.\mathbf{I}-f^c_{4,\varepsilon_v^p}\frac{p^c}{|p^c|}\frac{p}{|p|}-f^c_{4,r_4^c}.\rho_4^c}$$ **éq 1.1.3-13**

and in the general case, it is necessary to take account of the contribution of all the active mechanisms on yielding $\dot{\varepsilon}^p$, cf [éq1.1.5]:

$$\dot{\lambda}_4^c = -\frac{f^c_{4,\sigma}.\dot{\sigma}+f^c_{4,\varepsilon_v^p}\cdot\left(\sum_{(K,k)\neq(c,4)}\dot{\lambda}_k^K\Psi^K_{(k)}.\mathbf{I}\right)}{f^c_{4,\varepsilon_v^p}\cdot\Psi^c_{(4)}.\mathbf{I}+f^c_{4,r_4^c}.\rho_4^c}$$
$$=\frac{\left\langle 3K(p).\mathrm{sgn}(p).\mathrm{sgn}(p^c)\,\mathrm{tr}\,\dot{\varepsilon}+\left(\sum_{(K,k)\neq(c,4)}\dot{\lambda}_k^K\Psi^K_{(k)}.\mathbf{I}\right).\left(K(p).\mathrm{sgn}(p).\mathrm{sgn}(p^c)-f^c_{4,\varepsilon_v^p}\right)\right\rangle_+}{K(p)-f^c_{4,\varepsilon_v^p}\mathrm{sgn}(p).\mathrm{sgn}(p^c)-f^c_{4,r_4^c}.\rho_4^c}$$ **éq 1.1.3-14**

here appearing Les various terms are computed in [éq 7], [éq 7], [éq 7]. The statement [éq1.1.3] contributes to the computation of the increase in stresses $\dot{\sigma}$, from where the tangent operator is fired, cf [§2].



Appear 1.1-d: Evolution of the spherical surface of load under a cyclic monotonic loading then (visualized after normalization by $d.P_{c0}.e^{-\beta\varepsilon_v^p}$ **).**

The term n represents the direction of the variable memory $x_4=\frac{p_H}{d.P_{c0}.e^{-\beta\varepsilon_v^p}}$ and is equivalent to $\dot{x}_4/|\dot{x}_4|$

1.1.4 Traitement of the tension complementary to the model of Hujeux

Même if the usual mode of loading of the ground is compression, it is possible to be in tension, either locally (in space and/or time), or simply during iterations of resolution of the nonlinear system of the balance equations. It is thus necessary to define a behaviour in tension. He is thus proposed a perfect elastoplastic mechanism, with a null cohesion, in each plane k, as for the criteria deviatoric in compression.

The criterion is established on the confining pressure $p_k(\boldsymbol{\sigma})=\frac{1}{2}\mathrm{tr}\left(\boldsymbol{\sigma}_{(k)}\right)$ in the plane k :

$$f_k^{tr}(\boldsymbol{\sigma})=p_k(\boldsymbol{\sigma})-p_0^{tr} \leq 0 \quad \textbf{éq 1.1.4-1}$$

with $p_0^{tr}=10^{-6}.|P_{réf}|$.
The flow model is associated:

$$\left(\dot{\boldsymbol{\varepsilon}}^p\right)_k^{tr}=\dot{\lambda}_k^{tr}\left(\boldsymbol{\Psi}_{\boldsymbol{rs}}\right)_{(k)}^{tr(\sigma)}=\dot{\lambda}_k^{tr}.\frac{1}{2}\mathbf{I}_k \quad \textbf{éq 1.1.4-2}$$

the plastic multiplier $\dot{\lambda}_{tr}$ is obtained by resolution of the equation of complementarity of Kühn-Tücker, jointly with the condition of coherence.

1.1.5 Total elastoplastic evolution

the velocity of total elastoplastic evolution results from the contributions of all these mechanisms in ways monotonous and cyclic ([éq 1.1.2], [éq 1.1.2], [éq 1.1.3], [éq 1.1.3]), without forgetting the mechanisms associated with the tension, cf [§1.1.4]:

$$\dot{\boldsymbol{\varepsilon}}_{rs}^p=\sum_{K=m,c}\left(\sum_{k=1}^{3}\dot{\lambda}_k^K\left(\boldsymbol{\Psi}_{rs}\right)_{(k)}^{K(\sigma)}\right)+\frac{1}{3}\mathrm{sgn}(p)\,.\left(\dot{\lambda}_4^m+\dot{\lambda}_4^c.\mathrm{sgn}\left(p^c\right)\right)\mathbf{I}_{rs}+\sum_{k=1}^{3}\dot{\lambda}_k^{tr}\left(\boldsymbol{\Psi}_{rs}\right)_{(k)}^{tr(\sigma)} \quad \textbf{éq -1.1.5-1}$$

the voluminal plastic strainrate $\dot{\varepsilon}_v^p$ (model of dilatancy) is deducted directly using the relation $\varepsilon_v^p=\mathrm{tr}\,\boldsymbol{\varepsilon}^{\mathbf{p}}$ (it is not a variable of internal state independent of $\boldsymbol{\varepsilon}^{\mathbf{p}}$). It is specified nevertheless that the variable of internal state relating to the plastic voluminal strains does not take into account the share of the plastic strains from the plastic mechanisms of tension.

Note:

In situation 2D plane strains or axisymmetric, the plane $k=3$ corresponds to the plan $(\mathbf{e}_1,\mathbf{e}_2)$ of the model . By exploiting them [éq 1.1.2 -3] and [éq 1.1.2 -14], one notes that $\dot{\varepsilon}_{23}^p=\dot{\varepsilon}_{31}^p=0$. Thus one will always have: $\varepsilon_{23}^p=\varepsilon_{31}^p=0$ and $\sigma_{23}=\sigma_{31}=0$, even if all the plastic mechanisms can be activated.

1.2 Identification of the parameters characteristic of the material

Les parameters characteristic of the material are identified using several tests and ways of loading. One takes here stock of the parameters of the model:

- E_0 and ν_0 : elastic characteristics with the initial confining pressure $P_{réf}$, to fire from the data of the moduli of compressibility K_0 and shears G_0 ($E_0=\frac{9K_0G_0}{3K_0+G_0}$, $\nu_0=\frac{3K_0-2G_0}{6K_0+2G_0}$);
- $n\in[0,1[$ and $P_{réf}$: nonlinear elastic characteristic and initial confining pressure ($n=0$ corresponds to linear elasticity);
- P_{c0} : critical pressure of initial reference (negative);
- β : coefficient of plastic compressibility voluminal or critical state model, (positive);

- $M=\sin\phi_{pp}$: slope of the straight line of critical condition (or perfect plasticity; in the plane $\left(\ln|p|,\ \varepsilon_v^p\right)$), ϕ_{pp} being the internal friction angle;
- ψ : angle of dilatancy defining the border $q=|p|.\sin\psi$ in the plane $(p,\ q)$ between contracting field and dilating field (not to be confused with the angle appearing in the model of Mohr-Coulomb);
- b_h : plus coefficient, influencing the loading function in the plane (p',q). If $b_h=0$, one finds a threshold of the Mohr-Coulomb type, if $b_h=1$, one finds a threshold of the Camwood-Clay type;
- d : plus coefficient, distance enters the line of critical condition (perfect plasticity) and the isotropic line of consolidation, in the plane $\left(\ln|p|,\ \varepsilon_v^p\right)$, cf [fig. 1.1] ;
- $r_{éla}^d\in]\ 0{,}1\ [$: characterize the size of the threshold of the monotonous mechanisms déviatoires in an initial state;
- $r_{éla}^s\in]\ 0{,}1\ [$: characterize the size of the monotonous threshold of consolidation in an initial state;
- $r_{éla}^{dc}\in]\ 0{,}1\ [$: characterize the size of the threshold of the cyclic mechanisms déviatoires in an initial state;
- $r_{éla}^{sc}\in]\ 0{,}1\ [$: characterize the size of the cyclic threshold of consolidation in an initial state;
- a_m a_c : parameters (>0) of hardening of the plastic mechanisms déviatoires;
- c_m c_c : hardening parameters (>0) of the mechanisms of consolidation;
- ζ_0 : parameter defining the amplitude voluminal plastic strainrate;
- r_{hys} : parameter defining the size of the hysteretic field;
- r_{mob} : parameter defining the size of the mobilized field;
- x_m : parameter of the function $\zeta(r)$, cf [éq1.1.2].

The initial threshold values for an ordinary ground are often very low: $r_{éla}^s$ $r_{éla}^d$ and will be taken small, just like $r_{éla}^{dc}$ and $r_{éla}^{sc}$, which will be able to have by default the same value as the two precedents. By default, one will be able to take $\varphi_{pp}=\psi$. It will be noted that the natural equilibrium of a slope imposes that the friction angle ϕ_{pp} is higher than that of the slope.

For clay soils, the values of n are usually larger than for sandy grounds. Among these parameters, some are traditional: elasticity, initial state (for example the parameter OCR of a clay - *over-consolidation ratio* - provides the report $P_{c0}/\mathrm{tr}\,\sigma_0$ ...) ; one will be able to adjust the parameter β with a relevant experimental observation, compared to the line of critical condition, model Camwood-Clay cf…

One will be able to find in the literature [bib11]…, of the relations between "index of plasticity" I_p of a clay, the parameter OCR, the index of the vacuums and the modulus G_0. One finds relations enters β and the "index of compression" C_c and the index of the vacuums e_0. In the same way, ϕ_{pp} is connected to I_p.
For clay soils, the values of b_h are rather close to 1, while for sandy grounds, the values of b_h are rather enters $0{,}1$ and $0{,}2$.

As many many parameters above is "directly measurable" starting from tests, as much the parameters like b_h ζ_0 r_{mob}, r_{hys} must be deducted of the responses obtained by the model, cf [bib11].

The traditional tests being used to identify the parameters of the model are:

- a test in-situ (pressure gauge),
- a drained isotropic laboratory test,
- a test œdometric,
- a drained consolidated triaxial compression test with imposed, monotonous and cyclic strain,
- a consolidated triaxial compression test not drained with imposed, monotonous and cyclic strain.

For three sandy or argillaceous soil types, varied relative densities (the relative density is expressed according to the index of the vacuums e_0 by: $D_r=\frac{e_{\max}-e_0}{e_{\max}-e_{\min}}$) here of the characteristic values:

Paramètres	Sol sandy loose $D_r=38\%$ [bib11]	Sol sandy dense $D_r=93\%$ [bib9]	Limon sandy [bib10]
Élasticité			
K_0 modulates compressibility	296. MPa	296. MPa	275.0 MPa
G_0 shear modulus	222. MPa	222. MPa	128.0 MPa
E_0	532.8 MPa	532.8 MPa	332.4 MPa
ν_0	0.20	0.20	0.30
n	0.40	0.40	0.60
initial État			
$P_{réf}$	- 1. MPa	- 1. MPa	- 1. MPa
P_{c0} initial critical pressure	- 1800 kPa	- 4900 kPa	- 530. kPa
plastic États and critical			
β plastic compressibility	43.	17.	33.
ϕ_{pp} internal friction angle	30°	30°	28°
ψ angle of dilatancy	30°	30°	28°
b_h	0.2	0.22	0.9.3.5.3.5
d			2.
Initial thresholds			
$r^s_{éla}$ initial threshold of consolidation	0.0001	0.0001	0.0003
$r^d_{éla}$ initial threshold déviatoire	0,005	0,005	0.0001
$r^{sc}_{éla}$ initial threshold of consolidation	0.0001	0.0001	0.0003
$r^{dc}_{éla}$ initial threshold déviatoire	0,005	0,005	0.0001
Hardening			
a_m ; a_c	0.0003; 0.01	0.0001; 0.15	0,005; 0.01
c_m ; c_c	0.06; 0.03	0.06; 0.03	0.10; 0.05
ζ_0	1.	1.	1.
r_{hys}	0.03	0,003	0,002
r_{mob}	0.8.0.8		0.05
x_m	1.	1.	1.

Table 1.2-a: parameters of Hujeux for various grounds (sand of Toyoura; kernel of the dam El Infiernillo).

2 Numerical integration of the Rappel

2.1 behavior model of the problem

One employs the following notations: $A^{-}, A, \Delta A$ for a quantity evaluated at known time $t^{-}=t_{i-1}$, with time $t_i=t^{-}+\Delta t$ and its increment Δt respectively. The equations are discretized in an implicit way, i.e. expressed according to the unknown variables at time $t_i=t^{-}+\Delta t$.

For an increment of loading given and a set of variables given (initial field of displacement, stress and intern variable), one solves the discretized total system ([éq 2.2.2.2 - 1] of [bib1]) which seeks to satisfy the balance equations.

Code_Aster uses a method of Newton [bib1], initiated by a stage of prediction (shooting of Euler) which provides an estimate $\Delta \mathbf{U}_i^0$ of the displacement increment, from where the prediction $\mathbf{U}_i^0=\mathbf{U}_{i-1}+\Delta \mathbf{U}_i^0$, followed by iterations $n=1,...$ of correction, which give corrections $\delta \mathbf{U}_i^{n+1}$ of the displacement increment , from where upgraded $\mathbf{U}_i^{n+1}=\mathbf{U}_i^n+\delta \mathbf{U}_i^{n+1}$, which must converge towards the solution $\mathbf{U}_i=\mathbf{U}^{+}$.

Option `RIGI_MECA_TANG` computes the tangent operator $\mathbf{K}_{i-1}$, for the stage of prediction (linearization of the balance equations around the equilibrium at time t_{i-1}), starting from the relation velocities of stress reported at the strainrates, using the operator strain assembled ${}^{T}Q$:

$$\mathbf{K}_{i-1}=\frac{\partial \mathbf{R}}{\partial \mathbf{U}}\Big|_{(\mathbf{U}_{i-1})}=\frac{d\,{}^{T}\mathbf{Q}}{d\,\mathbf{U}}\Big|_{(\mathbf{U}_{i-1})}.\boldsymbol{\sigma}_{i-1} \qquad \textbf{éq 2.1}$$

Remarque:

> *One can decide, to save time computation, not to reactualize this matrix, and to take the elastic stiffness matrix, to see [U4.51.03], Newton* `KEY WORD`*, operand* `PREDICTION`*, value ` * `ELASTIQUE`*', rather than ` * `TANGENTE`*'. But that can increase the iteration count of correction, to even make convergence delicate, in particular because of the nonlinear elastic model constituting the model of Hujeux [éq*1.1.1*].*

Options `RAPH_MECA` and `FULL_MECA` compute the nodal $\boldsymbol{\sigma}_i^n$ stresses, intern variables and forces $\mathbf{R}\left(\mathbf{U}_i^n\right)={}^{T}\mathbf{Q}\,.\,\boldsymbol{\sigma}_i^n$, with the total iteration n .

Option `FULL_MECA` calculates moreover the tangent operator $\mathbf{K}_i^n$, with the iteration of total correction n (on request via the command `STAT_NON_LINE`, key word factor `NEWTON`, key word `MATRICE=` `"TANGENTE"` and `REAC_ITER`), starting from the stresses $\boldsymbol{\sigma}_i^n$ and of the tangent matrix of the local problem discretized in time:

$$\mathbf{K}_i^n=\frac{\partial \mathbf{R}}{\partial \mathbf{U}}\Big|_{(\mathbf{U}_i^n)} \qquad \textbf{éq 2.1}$$

the resolution of these total systems of balance equations gives us increments $\Delta \mathbf{U}_i^n$, therefore increments of strain $\Delta \boldsymbol{\varepsilon}_i^n$. One thus seeks locally (in each point of Gauss) the increment of the stresses and intern variables corresponding to $\Delta \boldsymbol{\varepsilon}$ and which satisfy the constitutive law. The evolution of the stresses and the intern variables is obtained by resolution of a system of equations differentials, by local integration, under the initial conditions at the beginning of the time step, described with [§ 2.2].

The equations of elastoplastic behavior multi-mechanism presented to [§1] do not make it possible to show by a simple analysis the unicity of the solution of the problem of structure equilibrium, discretized in time. The diagram of integration must have an unquestionable influence on the computed solutions.

2.2 General outline of local integration

the diagram of local integration selected for the model installation of of Hujeux in *Code_Aster* entirely implicit is formulated on the incremental problem, for reasons of accuracy of computation. It uses an elastic prediction then iterations of correction. The purpose of it is producing, an increment of strain $\Delta \boldsymbol{\varepsilon}$ being provided, the value of the stresses and intern variables at time $t_i = t^- + \Delta t$. [Fig. 2.2] the general organization of the algorithm of local integration presents.

Note:

The evolutions of the quantities computed of the model of Hujeux through an implicit integration are well-known being sensitive to disturbances of the data of the algorithm of integration (for example according to the choice of the platform of computation or the choice of spatial discretization…). This is why various provisions were taken in order to limit this sensitivity: see hereafter the description of the tolerances introduced on the increments. One must note that because of character multi-mechanism with variables mémoratrices of the model of Hujeux, the effective activation of the mechanisms not being a differentiable process, the formulation of velocity does not constitute the limit of the incremental formulation in time.

Code_Aster does not propose D ` integration explicit for this model, cf [R5.03.14], although this method is used in the literature, cf [bib3].

A process of local subdivision of the time step is available in order to significantly reduce the possible situations of not-convergence, and to save iterations of correction. It consists in solving the local behavior on linear subdivisions of the increment of strain $\Delta \boldsymbol{\varepsilon}$ proposed by the total resolution. The local subdivision is activable in the nonlinear operator of mechanics of *Code_Aster* (key word factor `COMP_INCR` and key word `ITER_INTE_PAS`, cf [U4.51.11].

2.2.1 Phase of prediction

One starts by establishing the elastic prediction $\boldsymbol{\sigma}^+_{ij\,é}$ (approximate tangent elastic shooting - Euler):

$$\sigma^+_{ij\,é} = \sigma^-_{ij} + C_{ijrs}\left(\hat{p}_é\right) \ . \ \Delta \varepsilon_{rs}$$ **éq 2.2.1**

the tensor $C_{ijrs}\left(\hat{p}_é\right)$, defined by [éq 1.1.1], is obtained starting from a statement discretized by an approximate diagram of the confining pressure:

$$\hat{p}_é = \frac{1}{2}\left(p^- + p^+_é\right) = p^- + \frac{1}{2} K_0 \cdot \left|\frac{p^-}{P_{réf}}\right|^n . \operatorname{tr} \Delta \boldsymbol{\varepsilon}$$ **éq 2.2.1**

This choices makes it possible to limit the risk to have a predictor of confining pressure producing of the stresses too far away from the hoped values, because of exponential nonlinear form of the elastic model [éq 1.1.1].

One starts with a first stage of “reprédiction”. A recutting of the increment of strain $\Delta \boldsymbol{\varepsilon}$ - additional of that possibly requested by the user via the key words of the nonlinear operator of mechanics of *Code_Aster* - is introduced so as to limit the following increments, associated with the prediction $\boldsymbol{\sigma}_é$, via a simple homothety:

$$\Delta p_k(\boldsymbol{\sigma}_{é})/p_k(\boldsymbol{\sigma}^-)\le 5\% \quad ; \quad \Delta q_k^K(\boldsymbol{\sigma}_{é})/q_k^K(\boldsymbol{\sigma}^-)\le 5\% \quad ; \quad \sum_{i,j}\left(\boldsymbol{\sigma}_{ij}^{é}-\boldsymbol{\sigma}_{ij}^{-}\right)/\sum_{i,j}\boldsymbol{\sigma}_{ij}^{-}\le 50\% \qquad \text{éq 2.2.1}$$

the finality is to avoid possible incursions during iterations of correction towards ways of loadings too moved away, being able to result in diverging (while leaving the field of convergence of the algorithm of Newton).

2.2.1.1 Potentially activated plastic mechanisms M^{pot}

Les elastoplastic mechanisms M_k^K , for $k=1,\ldots,4$ and $K=m,c$, are activated or inactivated according to the type of followed loading. A family of potentially active mechanisms is thus determined as a preliminary at the beginning of the pitch running starting from the state reaches previously, and is used for compute the really followed evolution. The initial choice of the mechanisms potentially activated with the boot of the time step considered results from the converged mechanical state reached at time t^- : the state of activation of the various mechanisms (monotonous or cyclic) to the instantet t^- values of the variables $\boldsymbol{\sigma}^-$ and $\boldsymbol{\alpha}^-$.

One notes by M^{pot} all the potentially activated plastic mechanisms cf [fig. 2.2], and by M^{act} all the plastic mechanisms really activated during integration, cf [fig. 2.2]. This definition makes it possible to reduce to the strict minimum the size of the system of equations nonlinear local to solve with each pitch. One uses the elastic prediction [éq 2.2.1] to establish a reappraisal of M^{pot} using the evolution $\dot{f}_k^K$ for $k=1,\ldots,4$ and $K=m,c$, of the thresholds of the mechanisms and to thus define the situations of loading in plastic mode or unloading.

One thus initializes $M^{pot}:=M^{act}(t^-)$, and one records the variables of history of the cyclic mechanisms "fathers" α_H^- , starting from the situation at previous time t^- . One computes for all the mechanisms M_k^K two quantities helping with the decision:

$$c_k^K=f_{k,\boldsymbol{\sigma}}^K\left(\boldsymbol{\sigma}^-+\Delta\boldsymbol{\sigma}_{élas}\right).C\left(\hat{p}_{é}\right).\Delta\boldsymbol{\varepsilon} \quad \text{and} \quad g_k^K=f_k^K\left(\boldsymbol{\sigma}^-+\Delta\boldsymbol{\sigma}_{élas}\right) \qquad \text{éq 2.2.1}$$

If it is about a cyclic mechanism M_k^c then, one also computes c_k^m for the associated monotonous M_k^m mechanism. In the event of difficulties, one carries out a recutting of the time step Δt . If the value of $c_k^m\ge 0$, and if two surfaces of load of M_k^c and M_k^m are "close", then $M^{pot}:=M^{pot}\cup\{M_k^m\}\backslash"\{M_k^c\}$ and the properties of hardening of M_k^c are reset with $r_{éla}^c$. This proximity is established by computing the difference between the factor of mobilization reached for the monotonous threshold r_k^m and the value of the factor of mobilization in a preceding converged state r_k^c ,

Puis one reviews all the possible mechanisms M_k^K while testing c_k^K and g_k^K . The situations thus are identified:

- $M_k^m\in M_{pot}$. Then, if $c_k^m\ge 0$, M_k^m remains in M^{pot} ; if not, the corresponding variables of history are memorized α_H^+ and so $g_k^c<0$ for the cyclic mechanism M_k^c associated with M_k^m , one deactivates M_k^c and M_k^m ; but if $c_k^m\ge 0$ one deactivates M_k^m with the profit of M_k^c put in M^{pot}

- $M_k^m \notin M_{pot}$. Then, if $c_k^m \geq 0$ and $g_k^m \geq 0$, then $M^{pot} := M^{pot} \cup \{ M_k^m \}$, but if $g_k^m < 0$ M_k^m remains out of M^{pot}
- $M_k^c \in M_{pot}$. Then, if $c_k^c \geq 0$, M_k^c remains in M^{pot} ; if not, the corresponding variables of history are memorized α_H^+ and if $g_k^c \geq 0$, one maintains M_k^c in M^{pot} , but if $g_k^c < 0$, M_k^c is withdrawn from M^{pot}
- $M_k^c \notin M_{pot}$. Then, if $c_k^c \geq 0$ and $g_k^c \geq 0$, then $M^{pot} := M^{pot} \cup \{ M_k^c \}$ and one manages the potentiality of microcomputer-discharges for the only mechanisms déviatoires: one tests the threshold of the mechanism “father” of M_k^c : so $g_k^{c'} \geq 0$ then the mechanism “wires” M_k^c is overridden by this mechanism “father” $M_k^{c'}$ with upgraded variables of history α_H^+ . If $c_k^c < 0$, the corresponding variables of history are memorized and if $r_k^c \geq r_{éla}^{dc}$ one creates a mechanism “wires” $M_k^{c'}$, whose surface of load is tangent with that of the mechanism “father”, who is put in M^{pot} if its threshold checks $g_k^{c'} \geq 0$, if not, $M_k^{c'}$ is deactivated.

Of course, so $M_k^K \in M^{pot}$ for $K = m$ or c , then the mechanism “brother” for $K = m$ or c is not in M^{pot} . So at the end of the analysis of the various situations above, no mechanism remains in M^{pot} then the mode will be elastic, to see cf [fig. 2.2 - B] .

The elastic mode occurs when no mechanism is activated.

One forces the prediction $\tilde{p}_{k_é}$ of each mechanism to be strictly negative to be able to respect the fields of definition of the various derivatives which intervene in the equations below: thus one obtains the prediction in stress $\tilde{\boldsymbol{\sigma}}_{ijé}^+ = \boldsymbol{\sigma}_{ij}^- + \mathbf{C}_{\boldsymbol{ijrs}}(\hat{p}_é) . \eta . \Delta \boldsymbol{\varepsilon}_{rs}$ oùest $\hat{p}_é$ defined in [éq2.2.1] and with η such as $\tilde{p}_{k_é} \leq 10^{-6} . P_{réf}$. One notes thereafter $\tilde{\mathbf{Y}}_é = (\tilde{\boldsymbol{\sigma}}_{ijé}^+ , \boldsymbol{\varepsilon}_v^{p^-} , r_k^{K^-} , \mathbf{0})$.

One seeks then an explicit solution of test by computing a candidate $(\Delta \lambda_k^K)_0$ using the potentially active mechanisms M_k^K , by solving the equations linearized around the state without plastic evolution $\tilde{\sigma}_é$:

$$\frac{\partial f_k^K}{\partial (\Delta \lambda_k^K)} |_{\hat{\mathbf{Y}}_é} . (\Delta \lambda_k^K)_0 = -f_k^K (\boldsymbol{\sigma}^- + \mathbf{C}(\hat{p}_é) . \Delta \boldsymbol{\varepsilon} , \varepsilon_v^{p-} , r_k^{K-}) \quad \text{éq 2.2.1}$$

with:

$$\frac{\partial f_k^K}{\partial (\Delta \lambda_k^K)} |_{\hat{\mathbf{Y}}_é} = -\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}_k^K} |_{\hat{\mathbf{Y}}_é} . \mathbf{C}(\hat{p}_é) . \boldsymbol{\Psi}_{(k)}^{K-} + \frac{\partial f_k^K}{\partial \varepsilon_v^p} |_{\hat{\mathbf{Y}}_é} . \boldsymbol{\Psi}_{(k)}^{K-} . \mathbf{I} + \frac{\partial f_k^K}{\partial r_k^K} |_{\hat{\mathbf{Y}}_é} . \rho_k^{K-} \quad \text{éq 2.2.1}$$

derivatives being given by [éq 7] with [éq 7]. From where then the solution of test $\Delta \mathbf{Y_0}$:

$$(\Delta \boldsymbol{\sigma})_0 = \boldsymbol{\sigma}^- + \mathbf{C}(\hat{p}_é) . \left(\Delta \boldsymbol{\varepsilon} - \sum_{k,K} (\Delta \lambda_k^K)_0 \boldsymbol{\Psi}_{(k)}^{K-} \right) ; (\Delta \varepsilon_v^p)_0 = \sum_{k,K} (\Delta \lambda_k^K)_0 \boldsymbol{\Psi}_{(k)}^{K-} . \mathbf{I} ; (\Delta r_k^K)_0 = (\Delta \lambda_k^K)_0 . \rho_k^{K-} \quad \text{éq 2.2.1}$$

Moreover, one imposes the condition $(\Delta \lambda_k^K)_0 \geq 0$; the factors of mobilization are also limited on the solution of test: $(r_k^m)_0 \leq 1$ and $(r_k^c)_0 \leq (r_k^m)_0$, in such a way that the solution of test respects the probability of the evolution of the intern variables. On the basis of these criteria, one also introduces a restriction on the evolution of the plastic voluminal strain also $\left|\left(\Delta \varepsilon_v^p\right)_0\right| \leqslant 10\%$.

Note:

The statement of the critical pressure function of the voluminal plastic strain utilizing exponential [éq 1.1.1], one prevents the overflowing risk of too high values of the field of convergence by engaging an increment division of strain $\Delta \boldsymbol{\varepsilon}$ given if necessary.

2.2.2 Phase of correction: nonlinear equations to solve

Cette stage consists in solving the system of equations local nonlinear established on the basis of potentially activated mechanism M^{pot}. After convergence, one revalues all the really activated mechanisms M^{act}, cf [§2.2.3.1], and if there is a difference with M^{pot}, one reiterates the nonlinear local resolution with M^{act}.

2.2.2.1 Iterations of correction of Newton

Les iterations of correction of Newton consist to solve the following equations, for $\Delta \boldsymbol{\varepsilon}$ given, to see [éq 6]:

- the incremental equation of state [éq 1.1.1], noted LE_{ij} (6 scalar equations),
- incremental evolution of the voluminal plastic strain $\Delta \varepsilon_v^p$ [éq 1.1.5]: $LEVP$,
- incremental evolution of the factors of mobilization Δr_k^K [éq1.1.2], [éq1.1.2], [éq1.1.3], and [éq1.1.3], notéset $LR.1$ $LR.2$ (1 for the mechanisms déviatoires, 2 for the mechanisms of consolidation; as many equations as of active mechanisms),
- criteria of the various potentially active mechanisms $f_k^K=0$ [éq1.1.2], [éq1.1.2], [éq1.1.3], and [éq1.1.3], noted $LF.1$ and $LF.2$ (1 for the mechanisms déviatoires, 2 for the mechanisms of consolidation; as many equations as of active mechanisms).

They constitute a square system $\mathbf{R}(\Delta \mathbf{Y})=0$, where the unknown factors are $\Delta \mathbf{Y}=\left(\Delta \boldsymbol{\sigma}_{\boldsymbol{ij}},\ \Delta \varepsilon_v^p,\ \Delta r_k^K,\ \Delta\left(\Delta \lambda_k^K\right)\right)$, which couple the equations (15 at the most and 9 at least). One solves in an implicit way the system $\mathbf{R}(\Delta \mathbf{Y})=0$ by a method of Newton, for $K=m, c$ and $k=1,\ldots,4$.

WITH the iteration j of the loop of on-the-spot correction of Newton, one solves the matric equation:

$$\frac{d\,\mathbf{R}}{d(\Delta \mathbf{Y})}\bigg|_{\Delta \mathbf{Y}_j} . \delta \boldsymbol{\Delta} Y_{j+1} = -R\left(\Delta \mathbf{Y}_j\right) \qquad \text{éq 2.2.2}$$

where the matrix tangent $\frac{d\,\mathbf{R}}{d(\Delta \mathbf{Y})}\big|_{\Delta \mathbf{Y}_j}$, asymmetric, is computed in the way presented to [§6], according to [éq6], with upgraded elasticity tensor with the value brought up to date of the confining pressure $\frac{1}{3}\mathrm{tr}\left(\boldsymbol{\sigma}^- + \Delta \boldsymbol{\sigma}_j\right)$ to the preceding iteration, according to [éq1.1.1]:

$$\frac{d\mathbf{R}}{d(\Delta\mathbf{Y})}\Big|_{\Delta\mathbf{Y}}=\begin{bmatrix}\frac{\partial LE_{ij}}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LE_{ij}}{\partial\varepsilon_v^p} & \frac{\partial LE_{ij}}{\partial r_k^K} & \frac{\partial LE_{ij}}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LEVP}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LEVP}{\partial\varepsilon_v^p} & \frac{\partial LEVP}{\partial r_k^K} & \frac{\partial LEVP}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LR.1}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LR.1}{\partial\varepsilon_v^p} & \frac{\partial LR.1}{\partial r_k^K} & \frac{\partial LR.1}{\partial(\Delta\lambda_k^K)}\\ 0 & \frac{\partial LR.2}{\partial\varepsilon_v^p} & \frac{\partial LR.2}{\partial r_k^K} & \frac{\partial LR.2}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LF.1}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LF.1}{\partial\varepsilon_v^p} & \frac{\partial LF.1}{\partial r_k^K} & 0\\ \frac{\partial LF.2}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LF.2}{\partial\varepsilon_v^p} & \frac{\partial LF.2}{\partial r_k^K} & 0\end{bmatrix}$$ **éq 2.2.2**

One put as a preliminary the various lines of this system "on the scale", by dividing the equations of étatet LE_{ij} the seuilspar LF the initial Young's modulus E_0 , the models of évolutionpar LR $P_{réf}/E_0$ and the stresses as well as the factors of mobilization r_k^K in the unknown vector $\Delta\mathbf{Y}_j$ are settings at the level of the strains via the initial Young's modulus E_0 and the pressure $P_{réf}$. This choice makes it possible to have equations of the same order of magnitude thus to ensure a "uniform" convergence more on the group of the system [éq 2.2.2].

Note:

The determinant of the tangent matrix $\frac{d\mathbf{R}}{d(\Delta\mathbf{Y})}\Big|_{\Delta\mathbf{Y}_j}$ is a priori positive in phase of positive hardening. However, it could become negative; no particular processing is envisaged.

Convergence famous is acquired since (tol is given by key word `RESI_INTE_RELA` of the key word factor `COMP_INCR` of the command `STAT_NON_LINE`, cf [U4.51.11]):

$$\|\mathbf{R}(\Delta\mathbf{Y}_{j+1})\|\le tol$$ **éq 2.2.2**

this norm $\|.\|$ on the residue also using the setting at the level of the various terms intervening.

Note:

One strongly recommends to use a value of tolerance lower or equal to 10^{-7} (key word `RESI_INTE_RELA`), at least in the "delicate" stages more of the way of loading.

In order to avoid possible incursions during iterations of correction towards ways of too distant loadings, being able to result in diverging (while leaving the field of local convergence of the algorithm of Newton), the algorithm of integration of Hujeux forces also a recutting of the increment of strain $\Delta\boldsymbol{\varepsilon}$ if the stage of correction leads to produce increments of factors of mobilization $\Delta r_k^K/r_k^K\ge 10$. Moreover, one imposes that the values predicted at this stage of the factors of mobilization déviatoires check: $r_k^{m^+}\le 1$ and $r_k^{c^+}\le r_k^{m^+}$ for the active mechanisms. Moreover, one records the cases where the plastic multipliers are negative. If $-\eta_{tolé}\le\Delta\lambda_k^K/\underset{K=1,..,4}{Max}(\Delta\lambda_k^K)\le 0$, where $\eta_{tolé}$ indicates the value of RESI_INTE_RELA provided by the user, then one imposes: $\Delta\lambda_k^K=0$. That makes it possible to limit the "harmful" effects of the mechanisms in "neutral" loading, i.e $\Delta\lambda_k^K\approx 0$.

The last task of management of the mechanisms described hereafter intervenes after a failure of the algorithm of local Newton (§ [2.2.2]) or so of the criteria, specified below, are violated. Any amendment made to the field of potentially active mechanisms M^{pot} will then result in reiterating with new M^{pot} the resolution of the nonlinear system of equations. This process of upgraded is however restricted with 5 attempts at rebuilding, or else one declares a not-convergence of local integration.

- The first cases treated are interested after each iteration of the algorithm of local Newton with the risk of overlapping of two surfaces of load déviatoires cyclic and monotonous or cyclic and cyclic within the space of stresses. These problems are avoided via tests of proximity between the position of the stress state in the déviatoire plane considered and surfaces it of load considered.
  - In the typical case of two surfaces of load déviatoires cyclic and monotonous, if the criterion of proximity is checked, the monotonous mechanism M_k^m is added directly to the field of the mechanisms M^{pot} while withdrawing the mechanism déviatoire cyclic M_k^c considered active until now in M^{pot} .
  - In the case of proximity of two surfaces of load of cyclic mechanisms déviatoires "father" and "wires", the criterion relates to the proximity of the stress state in the déviatoire plane regarded compared to the value recorded $\mathbf{X}_{(k)}^H$ as tangential point between surfaces of load "father" and "wires". Since the criterion of proximity is reached, the mechanism déviatoire "wires" M_k^c is withdrawn from the potential field of active mechanisms M^{pot} .
- After these tests of proximity of surfaces of load déviatoires, it is necessary to treat the stress states of tension causing very often the not-convergence of the local nonlinear system to solve.
  - If a state of tension is detected after failure of the method of resolution, for any plastic multiplier associated with a mechanism déviatoire considered in M^{pot} negative after the explicit shooting of Euler $\left(\Delta\lambda_k^K\right)_0$, this mechanism déviatoire will be withdrawn from M^{pot} . This solution to start again the resolution of the local nonlinear system does not impose any restriction on the value of the predictor of stresses. This predictor is established by considering a purely elastic increment of strain.
  - If all the plastic multipliers are positive, IE $\left(\Delta\lambda_k^K\right)_0\geq 0$, then all the mechanisms déviatoires are withdrawn from M^{pot} : the mechanisms of consolidation integrated only make it possible to prepare a new predictor for the stage of following local integration after updating of M^{act} .
- For the cases different from the two big classes described above (proximity and tension), if the maximum value of the vector residue $\mathbf{R}\left(\Delta\mathbf{Y}_j\right)$ is carried by a law of evolution of a factor of cyclic mobilization of mechanism déviatoire M_k^c , (either the equation $LR.1$ of [éq 6]), then the mechanism in question is withdrawn from M^{pot} .
- The following case is interested only in the cyclic mechanisms déviatoires ever hammer-hardened previously. These mechanisms are then withdrawn from M^{pot} if there were failure at the time of the resolution of the system of equations nonlinear.
- The last cases relate to the value of the plastic multipliers after the shooting of Euler $\left(\Delta\lambda_k^K\right)_0$.
  - The mechanisms associated with negative values are withdrawn from M^{pot} .
  - If all the plastic multipliers are positive, one withdraws the mechanism presenting the lowest value for $\left(\Delta\lambda_k^K\right)_0$ after the shooting of Euler.

If none the specific processing described above were requested, the not-convergence of the model to the local scales is assumed and this information is returned to the algorithm of total integration of the nonlinear equilibrium.

2.2.3 Phase of upgraded

Après resolution, one upgrades the vector solution:

$$\Delta\mathbf{Y}^{+}=\Delta\mathbf{Y}_{j+1}=\Delta\mathbf{Y}_{j}+\delta\Delta\mathbf{Y}_{j+1} \quad \textbf{éq 2.2.3}$$

what completes the stage of local integration, and one fixes $M^{act}=M^{pot}$ to start.

2.2.3.1 Really activated plastic mechanisms M^{act}

One describes hereafter the management of the real activation of the plastic mechanisms. After convergence of the nonlinear system of equations local, one is brought to check all the mechanisms really activated M^{act} according to the following procedure, on the basis of the prediction M^{pot}, cf [fig. 2.2]. This decision tree is a component of the model of Hujeux, since it describes the sequence of the cyclic mechanisms and the records of the variables discrete mémoratrices and the possible restorations of the variables of hardening, cf also [fig. 1.1] and [fig. 1.1].

Note:

With an explicit integration, cf [bib 5 , one would proceed rather with of recuttings (under-incrémention) managing the transitions from mechanisms.

One thus reviews all the mechanisms, any amendment of M^{act} leading to rependre a new nonlinear local resolution, cf [§ 2.2.2.1]:

•case of an active monotonous mechanism deviatoric M_k^m with a negative plastic $\Delta\lambda_k^K$ multiplier:
•if $r_k=r_{éla}^d$, then this mechanism M_k^m is deactivated: $M_{act}:=M_{act}\setminus\{M_k^m\}$;
•if not (reconsideration of the estimate made in M_{pot}):
•if there was a cyclic mechanism M_k^c prior to the stage of M_{pot}, the variables of memory of this mechanism are kept, one declares it created and one removes the monotonous mechanism M_k^m : $M_{act}:=M_{act}\setminus\{M_k^m\}$,
•if there did not exist cyclic mechanism, the variables of memory are memorized and a cyclic mechanism is created M_k^c and one removes M_k^m : $M_{act}:=M_{act}\setminus\{M_k^m\}$.
•case of an active monotonous isotropic mechanism M_4^m with a negative plastic multiplier:
• if $r_4=r_{éla}^s$, then this mechanism M_4^m is deactivated: $M_{act}:=M_{act}\setminus\{M_4^m\}$,
•if not:
•if there was a cyclic mechanism M_4^c prior to the stage of M_{pot}, the variables of memory of this mechanism are kept, one declares it created and one removes M_4^m : $M_{act}:=M_{act}\setminus\{M_4^m\}$,
•if there does not exist cyclic mechanism, the variables of memory are not memorized and one creates a cyclic mechanism M_4^c only if $\overline{\left(\frac{|\dot{p}|}{d|p_c|}\right)}<0$ (it is reminded the meeting that p_c depends on ε_v^p, which itself varies according to all the mechanisms), and in this case, one removes M_4^m .
•case of a monotonous mechanism deviatoric M_k^m created but inactive: if the threshold $f_k^m(t^+)\geq 0$ then this mechanism becomes active: $M_{act}:=M_{act}\cup\{M_k^m\}$, and the variables of hardening of the associated cyclic M_k^c mechanism take the virgin initial values.

•case of a monotonous isotropic mechanism M_4^m created but inactive: if the threshold $f_4^m(t^+)\geq 0$ then this mechanism becomes active: $M_{act}:=M_{act}\cup\{M_4^m\}$; if the threshold $f_4^m(t^+)<0$ then one does not memorize the variables of memory and one does not create a cyclic mechanism and one removes M_4^m only if $\left(\frac{\dot{\overline{|p|}}}{d|p_c|}\right)<0$.

•case of a mechanism cycliquecréé M_k^c before in M^{pot} starting from a monotonous mechanism:

•if the plastic multiplier is negative then the mechanism is deactivated: $M_{act}:=M_{act}\backslash\{M_k^c\}$; if not, one passes to:

•if the threshold of the then associated M_k^m monotonous $f_k^m(t^+)\geq 0$ mechanism:

•for a mechanism deviatoric, one passes into monotonous: $M_{act}:=M_{act}\backslash\{M_k^c\}\cup\{M_k^m\}$ and the variables of hardening of the cyclic mechanism M_k^c take again the virgin initial values,

•for an isotropic mechanism: if the variable of history of direction changes sign enters to t^- and t^+ following M^{pot} , then one remains into cyclic M_4^c ; if not one passes into monotonous: $M_{act}:=M_{act}\backslash\{M_4^c\}\cup\{M_4^m\}$ and the variables of hardening of the cyclic mechanism M_k^c take again the virgin initial values.

•if it is about a mechanism deviatoric M_k^c declared active with t^+, and if $\left(\left\| X_{(k)}^H+\frac{S_{(k)H}^c}{\|S_{(k)H}^c\|_{VM}^{2D}}.\left(r_k^c+r_{éla}^{dc}\right)\right\|+r_k^c-r_k^m\right)/r_k^m\geq 0$ (computed with t^+), then the variables mémoratrices of this mechanism M_k^c are upgraded, and the variables of hardening of the monotonous mechanism deviatoric “father” are reset in a virgin state. That prevents the crossing of the threshold of the monotonous mechanism by the threshold of the cyclic mechanism.

•if the cyclic mechanism M_k^c is declared created but inactive in M^{pot} then:

•if it is about a mechanism deviatoric M_k^c , then:

•if the threshold $f_k^c(t^+)\geq 0$ then M_k^c is declared active and:

•if $X_{(k)père}^H\neq 0$ (if there exists a mechanism “father”), then so with t^- $\frac{S_{(k)pèreH}^c}{\|S_{(k)pèreH}^c\|_{VM}^{2D}}.\left(X_{(k)père}^H-\frac{S_{(k)}^c}{p_k.F\left(p_k\varepsilon_v^p\right)}\right)<0$ and the threshold checks $f_k^c(t^-)\geq 0$ this mechanism M_k^c takes again to the variables mémoratrices of the mechanism “father” - who becomes again active - like its value of r_k^c, then the mechanism “father” is reset in a virgin state;

•if there does not exist mechanism “father” and so $\frac{S_{(k)filsH}^c}{\|S_{(k)filsH}^c\|_{VM}^{2D}}.\left(X_{(k)fils}^H-\frac{S_{(k)}^c}{p_k.F\left(p_k\varepsilon_v^p\right)}\right)\geq 0$ with t^+ , then the variables mémoratrices àde t^+ this mechanism M_k^c are stored in those of a mechanism “father” which one must keep the values, and the variables mémoratrices of this mechanism “wires” M_k^c - which becomes active - become: $X_{(k)fils}^H=X_{(k)père}^H+2r_k^c\frac{S_{(k)pèreH}^c}{\|S_{(k)pèreH}^c\|_{VM}^{2D}}$ and $\frac{\mathbf{S}_{(k)filsH}^c}{\left\|\mathbf{S}_{(k)filsH}^c\right\|_{VM}^{2D}}=\frac{-\mathbf{S}_{(k)pèreH}^c}{\left\|\mathbf{S}_{(k)pèreH}^c\right\|_{VM}^{2D}}$;

- •if the variables mémoratrices estimated before the stage of M^{pot} check $X_{(k)}^{H} \neq X_{(k)}^{H}(t^{-})$ and $X_{(k)}^{H}(t^{-}) \neq 0$, then so with t^{+} $\frac{S_{(k)H}^{c}}{\| S_{(k)H}^{c} \|_{VM}^{2D}} \cdot \left(X_{(k)}^{H} - \frac{S_{(k)}^{c}}{p_k . F(p_k , \varepsilon_v^p)} \right) \geq 0$ one equalizes the variables mémoratrices with t^{+} these estimated values, and moreover if the threshold $f_k^c(t^{+}) \geq 0$, then M_k^c is activated and one equalizes also the variables mémoratrices with t^{-} and with t^{+} , in the same way the values of r_k^c , and if not, it is pointless reiterate a local nonlinear resolution;
- •if the variables mémoratrices estimated before the stage of M^{pot} check $X_{(k)}^{H} \neq X_{(k)}^{H}(t^{-})$ and $X_{(k)}^{H}(t^{-}) = 0$ (the "father" is a monotonous mechanism), then so $\frac{S_{(k)H}^{c}}{\| S_{(k)H}^{c} \|_{VM}^{2D}} \cdot \left(X_{(k)}^{H} - \frac{S_{(k)}^{c}}{p_k . F(p_k , \varepsilon_v^p)} \right) \geq 0$ with t^{+} the variables mémoratrices of this mechanism M_k^c are reset in a virgin state and M_k^m is activated;
- •so with t^{-} $r_k^c \neq r_{éla}^{dc}$, then one records the variables mémoratrices of this mechanism M_k^c at time t^{+} (a new "father" is created) and one resets in a virgin state this mechanism $r_k^{c+} = r_{éla}^{dc}$, which is created inactive. In this case, it is pointless reiterate a local nonlinear resolution.
- •if it is about an isotropic mechanism M_4^c and:
- •if $\left(\frac{|p|}{d|p_c|} \right)^{+} \leq \left(\frac{|p|}{d|p_c|} \right)^{-}$ and $p_H^c / |p_H^c| = 1$ with t^{-} (reduction of compression): if the threshold $f_4^c(t^{+}) \geq 0$ then M_4^c is declared active if not the variables mémoratrices of this isotropic mechanism àreprennent t^{+} the values before the stage of M^{pot} . Moreover, so $p_H^c = 0$ at the stage M^{pot} , then the associated mechanisms M_4^m and M_4^c are inactivated;
- •if $\left(\frac{|p|}{d|p_c|} \right)^{+} > \left(\frac{|p|}{d|p_c|} \right)^{-}$ and $p_H^c / |p_H^c| = -1$ with t^{-} (increase in compression): if the threshold $f_4^c(t^{+}) \geq 0$ then M_4^c is declared active if not the variables mémoratrices of this isotropic mechanism àprennent t^{+} the values before the stage of M^{pot} ;
- •if $\left(\frac{|p|}{d|p_c|} \right)^{+} > \left(\frac{|p|}{d|p_c|} \right)^{-}$ and $p_H^c / |p_H^c| = 1$ with t^{-} (reduction of compression):
- •if $p_H^c / |p_H^c|$ changes sign to this stage, then the variables mémoratrices of this cyclic isotropic mechanism with t^{+} take the values obtained before the stage of M^{pot} ; more if there does not exist deà p_H^c t^{+} , then the mechanisms M_4^m and M_4^c are declared inactive, and if not, if the threshold $f_4^c(t^{+}) \geq 0$ then M_4^c - declared active with t^{-} - remains created with t^{+} and the variables mémoratrices with t^{-} take the values obtained before the stage of M^{pot} with t^{-} ;

•if $p_H^c / |p_H^c|$ does not change sign with this stage, then one reactualizes the variables mémoratrices of this isotropic mechanism (creation of "wires") and if the threshold f_4^c computed with the stresses with t^{+} exceeds the initial criterion ($r_{éla}^c$), then M_4^c is declared active with t^{-} and the variables mémoratrices with t^{-} take the values obtained with t^{+} ;

•if $\left(\frac{|p|}{d|p_c|} \right)^{+} \leq \left(\frac{|p|}{d|p_c|} \right)^{-}$ and $p_H^c / |p_H^c| = -1$ with t^{-} (increase in compression):

- •if $p_H^c / | p_H^c |$ changes sign to this stage, then the variables mémoratrices of this isotropic mechanism with t^+ this mechanism take the values obtained during the estimate of M^{pot}, and if the threshold f_c^4 computed with the stresses with t^+, then the variables mémoratrices with t^- take the values obtained before the stage of M^{pot} and M_4^c is declared active with t^- and remains created with t^+ ;
- •if $p_H^c / | p_H^c |$ does not change sign since the stage of M^{pot}, then one reactualizes the variables mémoratrices of this isotropic mechanism (creation of “wires”) and if the threshold f_4^c computed with the stresses with t^+ exceeds the initial criterion ($r_{éla}^c$), then M_4^c is declared active with t^- and remains created with t^+, and the variables mémoratrices with t^- take the values obtained with t^+.

Note:

In the case where $M^{act} \neq M^{pot}$, one restarts the algorithm of local Newton by not using the elastic predictor of the stresses $\boldsymbol{\sigma}^{-} + \mathbf{C}(\hat{p}_{\dot{e}}) . \Delta \boldsymbol{\varepsilon}$, cf [éq 2.2.1 -1], but the result obtained at the end of the preceding failed stage $\sigma_{échoué}^+$, cf [éq 2.2.3 -1], associated with the values obtained of the intern variables α^-. One thus hopes to avoid inopportune loopings and while booting in a way more “close” to the sought solution, to accelerate convergence.

2.2.4 Computation of the incremental tangent stiffness matrix

One finally establishes (on request) the local tangent matrix $\mathbf{C}^T$ of the incremental behavior, with the current total iteration, which connects the variation of total stress to the variation of total deflection :

$$\Delta \sigma_{ij} = C_{ijrs}^T . \Delta \varepsilon_{rs} = C_{ijrs}(p^+) . (\Delta \varepsilon_{rs} - \Delta \varepsilon_{rs}^p) \quad \textbf{éq 2.2.4}$$

Pour that, one exploits the conditions of “coherence”: $\dot{f}_k^K(\sigma^+, \varepsilon_v^{p+}, r_k^{K+}) = 0$ for the mechanisms active, monotonous or cyclic $K = m,c$, the quantities intervening in these statements being computed at current $t^+ = t_i$ time. While combining with [éq1.1.1], one fires:

$$\Delta \lambda_k^K = \frac{\left\langle \frac{\partial f_k^K}{\partial \boldsymbol{\sigma}} |_{Y¿+} . \mathbf{C}(p^+) . \Delta \boldsymbol{\varepsilon} \right\rangle_+}{\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}} |_{\mathbf{Y}^+} . \mathbf{C}(p^+) . \boldsymbol{\psi}_{(k)}^{K+} - \frac{\partial f_k^K}{\partial \varepsilon_v^p} |_{\mathbf{Y}^+} . \boldsymbol{\psi}_{(k)}^{K+} . \mathbf{I} + \frac{\partial f_k^K}{\partial r_k^K} |_{\mathbf{Y}^+} . \rho_k^{K+}} \geq 0 \quad \textbf{éq 2.2.4}$$

cf [éq1.1.2], [éq1.1.2], [éq1.1.3], [éq1.1.3], various derivatives being given by [éq7] with [éq 7]. The sign of $\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}} |_{Y^+} . \mathbf{C}(\hat{p}_{\dot{e}}) . \Delta \dot{\boldsymbol{\varepsilon}}$ is compared to a tolerance of reference (`R8PREM`).

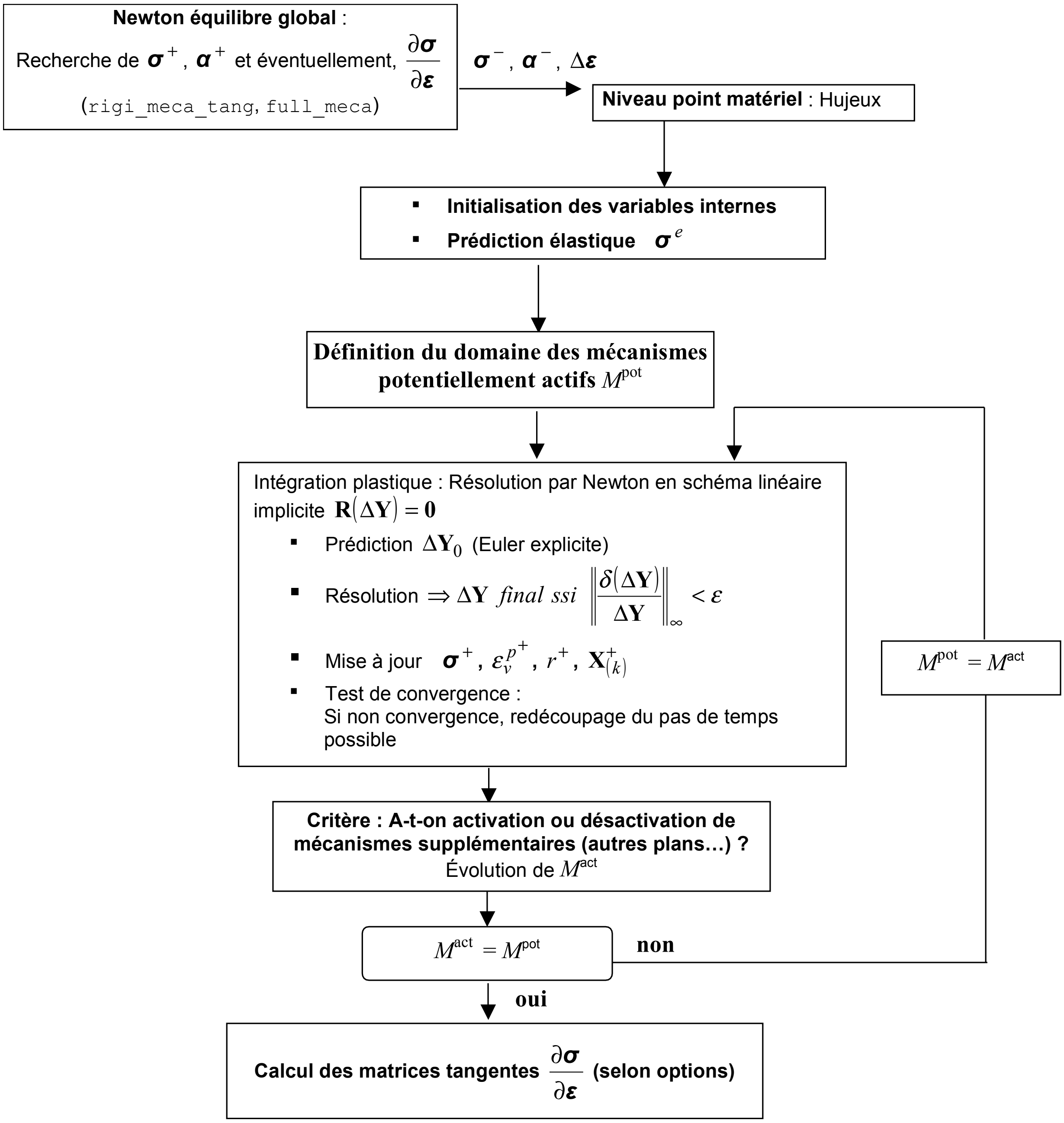
One of deducted the form of the incremental tangent matrix (option `FULL_MECA`), on the mechanisms of M^{act} :

$$\mathbf{C}^T_{ijrs}=\mathbf{C}_{ijlt}\left(p^+\right).\left(\mathbf{I}_{rslt}-\sum_{M^{act}}\frac{\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}}|_{\mathbf{Y}^{+}_{pq}}.\mathbf{C}_{pqrs}\left(p^+\right).\left(\boldsymbol{\Psi}_{lt}\right)^{K+}_{(k)}}{\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}}|_{\mathbf{Y}^+}.\mathbf{C}\left(p^+\right).\boldsymbol{\Psi}^{K+}_{(k)}-\frac{\partial f_k^K}{\partial \varepsilon_v^p}|_{\mathbf{Y}^+}.\boldsymbol{\Psi}^{K+}_{(k)}.I+\frac{\partial f_k^K}{\partial r_k^K}|_{\mathbf{Y}^+}.\rho_k^{K+}}\right) \quad \text{éq 2.2.4}$$

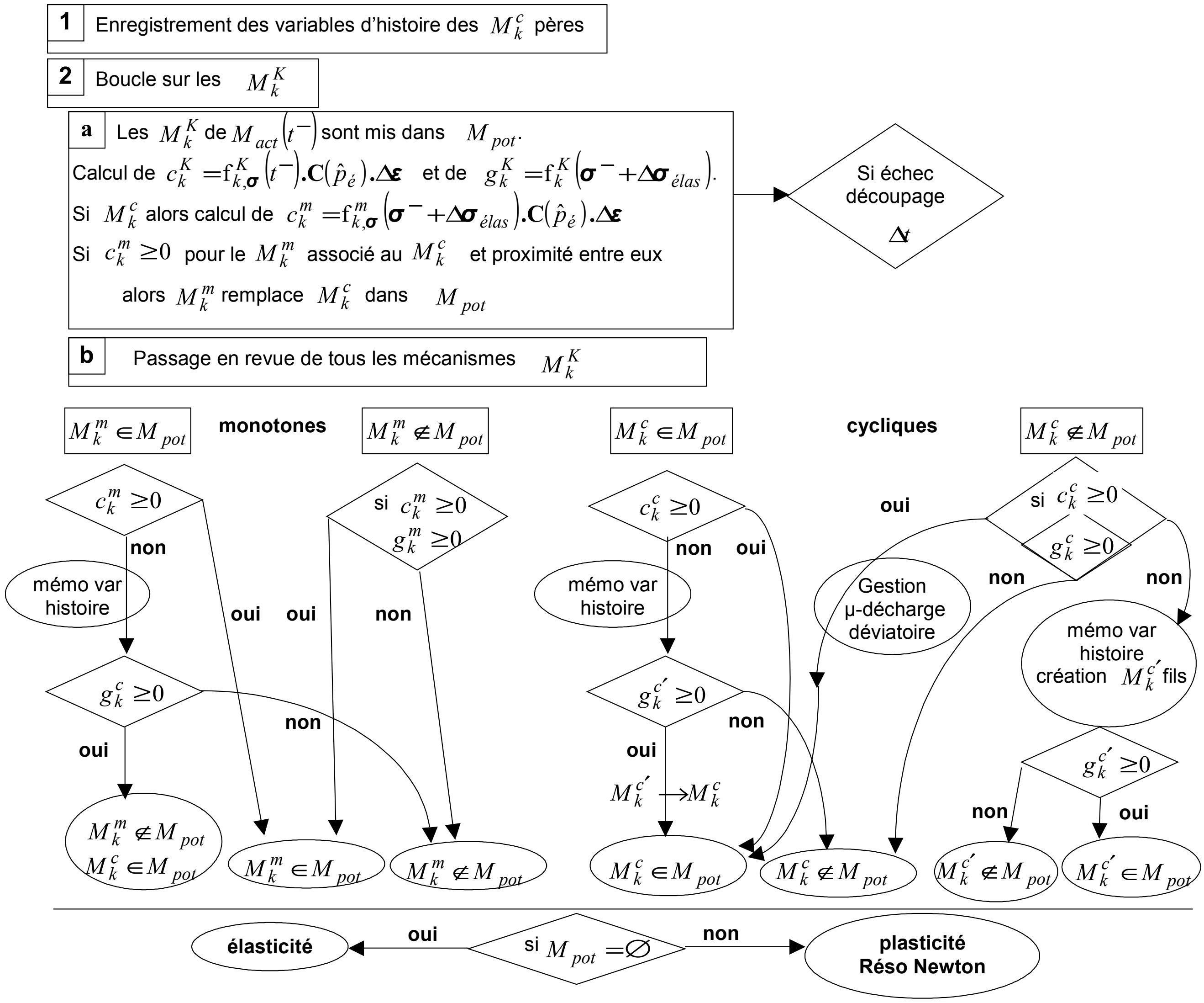
the incremental tangent matrix is thus established analytically, cf [§2.4], the various derivatives present in [éq 2.2.3] being computed with [§7]. The determinant of the incremental tangent matrix $\det\left(C^T_{ijrs}\right)$, in option FULL_MECA, is stored among the intern variables: VARI_33, cf [§3.1].

Note:

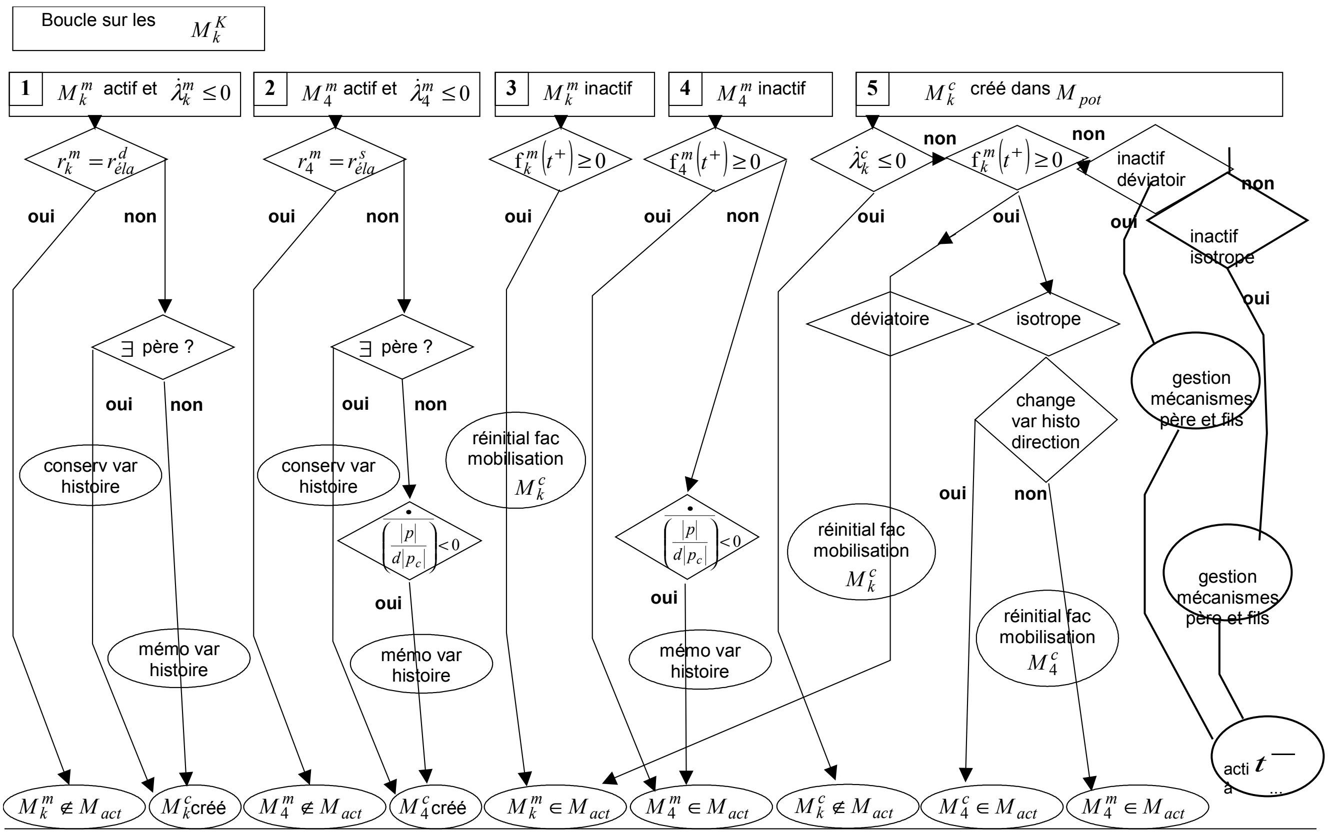
The rating of the incremental tangent matrix using a technique of disturbances (by finite differences, as checking) is not possible because of character multi-mechanism of the model of Hujeux: the effective activation of the mechanisms not being a differentiable process.



Appear 2.2-a: Diagram of local integration of the constitutive law of Hujeux in *Code_Aster*.



Appear 2.2-b: Algorithm of evolution of the field M^{pot} of the potentially active mechanisms.



Appear 2.2-c: Algorithm of definition of the field M^{act} of the activated mechanisms.

2.3 Tangent operator of velocity: option `RIGI_MECA_TANG`

Pour the option `RIGI_MECA_TANG` , which is used at the time of the total prediction, called with the first iteration of a new increment of load $\Delta\hat{\mathbf{L}}(t_i)$, the total tangent operator, noted $\mathbf{K_{i-1}}$, is computed starting from the results known at time $t^- = t_{i-1}$ [bib3].

The total tangent operator is assembled starting from the contributions of the tangent matrix in each point of Gauss, called "out of velocities":

$$\dot{\sigma}_{ij} = C_{ijrs}^{elp} . \dot{\varepsilon}_{rs} \qquad \textbf{éq 2.3-1}$$

Comme the constitutive law is elastic nonlinear [éq 1.1.1], one builds a nonlinear elastic tangent operator.

♦ Si in a preceding state the tensor of the stresses is not at the border of any plastic threshold, the elastic prediction $\sigma_{ij\,é}^{+}$ is written according to [éq 2.2.1], using the confining pressure estimated $\hat{p}_é$ [éq2.2.1].

In this case, one builds the matrix $\mathbf{K}_{i-1}$ using the elasticity tensor computed for the confining pressure $\hat{p}_é$. This choice makes it possible to limit the risk to have a matrix producing of the predictions of stresses too far away from the hoped values, because of exponential nonlinear form of the elastic model [éq1.1.1].

♦ Si in a preceding state the tensor of the stresses is on the border of a plastic threshold , one exploits the conditions of "coherence": $\dot{f}_k^K(\sigma^-, \varepsilon_v^{p-}, r_k^{K-}) = 0$, for the mechanisms active, monotonous or cyclic $K=m,c$, the quantities intervening in these statements being computed at previous $t^{-}=t_{i-1}$ time. While combining with [éq 1.1.1 -9], one obtains the statement of the plastic multipliers $\dot{\lambda}_k^K$:

$$\dot{\lambda}_k^K=\frac{\left\langle \left.\frac{\partial \mathbf{f}_k^K}{\partial \boldsymbol{\sigma}}\right|_{\hat{\mathbf{Y}}^-} .\mathbf{C}\left(\hat{p}_{\acute{e}}\right).\dot{\boldsymbol{\varepsilon}}\right\rangle_+}{\left.\frac{\partial \mathbf{f}_k^K}{\partial \boldsymbol{\sigma}}\right|_{\hat{\mathbf{Y}}^-}.\mathbf{C}\left(\hat{p}_{\acute{e}}\right).\boldsymbol{\Psi}_{(k)}^{K-}-\left.\frac{\partial f_k^K}{\partial \varepsilon_v^p}\right|_{\hat{\mathbf{Y}}^-}.\boldsymbol{\Psi}_{(k)}^{K-}.\mathbf{I}+\frac{\partial f_k^K}{\partial r_k^K}|_{\hat{\mathbf{Y}}^-}.\rho_k^{K-}} \geq 0 \qquad \textbf{éq 2.3-2}$$

cf [éq1.1.2], [éq1.1.2], [éq1.1.3], [éq1.1.3], various derivatives being given in appendix by [éq7] with [éq 7].

One of deducted the form of the tangent matrix “out of velocities”, on the mechanisms of M^{act} :

$$C_{ijrs}^{elp}=C_{ijlt}\left(\hat{p}_{\acute{e}}\right).\left(I_{rslt}-\sum_{M^{act}}\frac{\left.\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}}\right|_{\hat{\mathbf{Y}}^{-}}.C_{pqrs}\left(\hat{p}_{\acute{e}}\right).\left(\boldsymbol{\Psi}_{lt}\right)_{(k)}^{K-}}{\frac{\partial f_k^K}{\partial \boldsymbol{\sigma}}|_{\hat{\mathbf{Y}}^-}.\mathbf{C}\left(\hat{p}_{\acute{e}}\right).\boldsymbol{\Psi}_{(k)}^{K-}-\frac{\partial f_k^K}{\partial \varepsilon_v^p}|_{\hat{\mathbf{Y}}^-}.\boldsymbol{\Psi}_{(k)}^{K-}.\mathbf{I}+\frac{\partial f_k^K}{\partial r_k^K}|_{\hat{\mathbf{Y}}^-}.\rho_k^{K-}}\right) \qquad \textbf{éq 2.3-3}$$

Remarque:

One must note that character nonassociated with the flow models [éq 1.1.2] and [éq 1.1.2] *of the mechanisms deviatoric makes lose major symmetries in the tangent matrix. In the same way, these flow models introduce couplings between the components C_{iijj}^{elp} and the components C_{klkl}^{elp} for $k \neq l$, which is closely related to dilatancy.*

Note:

One must also note that, as the elasticity tensor $\mathbf{C}\left(\hat{p}_{\acute{e}}\right)$ is isotropic, the situations of loading where only the diagonal directions are requested, then the tensors $\left(\boldsymbol{\Psi}_{\mathrm{lt}}\right)_{(k)}^K$ comprise only diagonal terms, and thus the elasticity tensor tangent $\mathbf{C}^{\mathbf{elp}}$ does not comprise coupling between diagonal terms and extra-diagonal terms.

2.4 Incremental tangent operator: options RAPH_MECA and FULL_MECA

Les options RAPH_MECA and FULL_MECA are used in the iterations of correction of the algorithm of Newton applied to the resolution of the total equilibrium at the level of structure. Option RAPH_MECA is limited to the processing of the iterations of correction of Newton without upgrading the total tangent operator.

Option FULL_MECA provides the reactualization of the total tangent operator $\mathbf{K}_{\mathbf{i}}^{\mathbf{n}}$ to each total iteration (on request). It is built by assembling at each point of Gauss the incremental tangent matrix C_{ijrs}^T established in [éq 2.2.4], with the fields obtained at the end of the total iteration i :

$$\Delta \sigma_{ij}=C_{ijrs}^T.\Delta \varepsilon_{rs}=C_{ijrs}.\left(\Delta \varepsilon_{rs}-\Delta \varepsilon_{rs}^p\right) \qquad \textbf{éq 2.4}$$

WITH convergence of the total iterations, option FULL_MECA upgrades the stresses and the intern variables, cf [R5.03.01].

3 Establishment in *Code_Aster*

3.1 Un certain nombre de

Intern variables “intern variables” of the model within the meaning of *Code_Aster* are created and stored. The tensor of the plastic strains $\boldsymbol{\varepsilon}^{\mathbf{p}}$ is not stored because it is obtained by computation in postprocessing starting from the stresses and of the total deflections. The values of the intern variables at Gauss points (VARI_ELGA) are for all the modelizations:

VARI_1	r_1^m (factor of mobilization of the monotonous mechanism déviatoire, $k=1$)
VARI_2	r_2^m (factor of mobilization of the monotonous mechanism déviatoire, $k=2$)
VARI_3	r_3^m (factor of mobilization of the monotonous mechanism déviatoire, $k=3$)
VARI_4	r_4^m (factor of mobilization of the monotonous mechanism of consolidation)
VARI_5	r_1^c (factor of mobilization of the cyclic mechanism déviatoire, $k=1$)
VARI_6	r_2^c (factor of mobilization of the cyclic mechanism déviatoire, $k=2$)
VARI_7	r_3^c (factor of mobilization of the cyclic mechanism déviatoire, $k=3$)
VARI_8	r_4^c (factor of mobilization of the cyclic mechanism of consolidation)
VARI_9 with variable	VARI_12 mémoratrices $\left(X_{(1)}^H\right)_{1111}$ $\sqrt{2}\left(X_{(1)}^H\right)_{1212}$ $-\left(S_{(1)H}^c\right)_{1111}/\|S_{(1)H}^c\|_{VM}^{2D}$, $-\sqrt{2}\left(S_{(1)H}^c\right)_{1212}/\|S_{(1)H}^c\|_{VM}^{2D}$ (entering norm), for the cyclic mechanism déviatoire of plane $k=1$
VARI_13 with VARI_16	idem for the cyclic mechanism déviatoire of plane $k=2$
VARI_17 with VARI_20	idem for the cyclic mechanism déviatoire of variable $k=3$
plane	VARI_21 discontinuous mémoratrice $p_H/\left(d.P_{c0}.e^{-\beta\varepsilon_{vH}^p}\right)$ of the mechanism of variable
consolidation	VARI_22 discontinuous mémoratrice $p_H^c/\|p_H^c\|$ of norm on the surface of load of the mechanism of consolidation
VARI_23	voluminal plastic strain ε_v^p
VARI_24 with indicating	VARI_27 of activation (1) or not (0) of the monotonous mechanisms or of transition to cyclic (- 1)
the VARI_28 with indicating	VARI_31 of activation (1) or not (0) of cyclic mechanisms
VARI_32	density standardized for work of the second order $\dot{\boldsymbol{\sigma}}.\dot{\boldsymbol{\varepsilon}}$ (obtained by discretization: $\Delta\boldsymbol{\sigma}.\Delta\boldsymbol{\varepsilon}/\|\Delta\boldsymbol{\sigma}\|.\|\Delta\boldsymbol{\varepsilon}\|$)
VARI_33	Déterminant of the tangent matrix $\frac{d\boldsymbol{\sigma}}{d\boldsymbol{\varepsilon}}$ }
VARI_34	Indicateur of state of the active mechanisms after convergence (see below)
VARI_35	Compteur of local iterations

VARI_36 with vectorial	VARI_41 coordinated of the tangential point $\left(X_{(1)}^{H}\right)_{1111}$, $\sqrt{2}\left(X_{(1)}^{H}\right)_{1212}$ the mechanism "father" on the surface of load of the mechanism déviatoire of the plane $k=1$; vectorial coordinates of the norm entering on the surface of load of the mechanism "father" of the mechanism déviatoire; radius of the threshold déviatoire reaches by the surface of load before the discharge of this mechanism déviatoire
VARI_42 with VARI_47	idem for the cyclic mechanism déviatoire of plane $k=2$
VARI_48 with VARI_50	idem for the cyclic mechanism déviatoire of the plane $k=3$

the indicator of state VARI_34 of the active mechanisms after convergence is a byte (8 digits) obtained by binary number system. Each digit is associated with mechanisms plastic, by affecting value 1 for an active mechanism and value 0 for a mechanism not requested.
This makes it possible to easily visualize with a simple scalar the active mechanisms. For example, if VARI_34 = 10000010, then the active criteria are: déviatoire XZ monotonous (10^1) and isotropic cyclic (10^7).

4 Functionalities and checking

the model of HUJEUX (behavior HUJEUX for key word COMP_INCR) is usable in *Code_Aster* with various modelizations:

- •conventional version: 3D, D_PLAN,
- •under-integrated finite elements: 3D SI, D_PLAN_SI,
- •coupled with the models of THM (cf [R7.01.11]): 3D_HMD, D_PLAN_HMD…

Here the list of the cases of validation available:

SSNV197 abcd	[V6.04.197]	triaxial compression test drained (mechanical pure) with the model of Hujeux; 2 pressures of pre-consolidation: 50 and 200 kPa and introduction of parameters to allow a comparison with *Gefdyn*. Benchmark SSNV197D is treated with a rotation of the local coordinate system of 45° compared to the vertical (direction of loading). One compares the solution obtained with a computation where the mesh is turned.
SSNV204 ABC	[V6.04.204]	compression test drained (mechanical pure) with the model of Hujeux; interest: test the consolidation then the cyclic one in representative conditions “pure mechanics”. SSNV204b: isotropic compression of an elastic orthotropic material. One degenerates the model of Hujeux out of orthotropic elastic model, and one validates compared to a true elastic design. SSNV204c: one carries out a purely isotropic loading leading to the setting in tension of the ground. One thus validates the activation of the plastic mechanisms of tension.
SSNV205 has	[V6.04.205]	drained cyclic shear test controlled in strain. Interest: test the cyclic one in representative conditions “pure mechanics”. Comparison with *Gefdyn*.
SSNV207 has	[V6.04.207]	cyclic shears controlled in stresses with microcomputer-discharge.
SSNV208 has	[V6.04.208]	Cas biaxial test in conditions drained on dense sand of Hostun (D_PLAN). Computation during the postprocessing of the criterion of Rice. (option INDL_ELGA)
WTNV132 ABC	[V7.31.132]	Test of pure construction by layers in conditions drained “mechanical” with the model of Hujeux. WTNV132b: Simulation identical to the overloaded preceding one of a modelization D_PLAN_DIL. WTNV132c: Simulation identical to modelization A with a total resolution out of secant matrix.
WTNV133 ab	[V7.31.133]	triaxial compression test in conditions not drained with the model of Hujeux; interest: test the consolidation then the cyclic one in representative hydraulic conditions.
WTNV134 ab	[V7.31.134]	triaxial compression test in not drained conditions cyclic hydromechanics. Comparison with *Gefdyn*. WTNV134b: loading applied identical using operator SIMU_POINT_MAT
WDNP101ab	[V7.34.101]	seismic stress of a construction by layers with the model of Hujeux, modelization D_PLAN_HM. WDNP101b: Simulation identical on physical base and nonmodal like the preceding case.

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History of the versions of the document

Version Aster	Auteur (S) or contributor (S), Description	organization of the amendments
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6 Annexe 1: analytical computation of the tangent matrix of local integration

One must solve by a method of Newton the system of equations nonlinear local following: $\mathbf{R}(\Delta\mathbf{Y})=0$, written at the end of the studied increment, where $\Delta\mathbf{Y}=\left(\Delta\boldsymbol{\sigma}_{ij},\ \Delta\varepsilon_v^p,\ \Delta r_k^K,\ \Delta\left(\Delta\lambda_k^K\right)\right)$. This is written:

$$\left\{\begin{array}{l}
\boldsymbol{\sigma}_{ij}^{+}-\boldsymbol{\sigma}_{ij}^{-}-\mathbf{C}_{ijrs}\left(\boldsymbol{\sigma}^{+}\right).\left(\Delta\boldsymbol{\varepsilon}_{rs}-\sum_{K=m,c}\left(\sum_{k=1}^{3}\Delta\lambda_k^K\left(\boldsymbol{\Psi}_{rs}\right)_{(k)}^{K(\sigma^+)}\right)-\dfrac{p}{3|p|}.\left(\Delta\lambda_4^m+\dfrac{\Delta\lambda_4^c p^c}{|p^c|}\right)\mathbf{I}_{rs}\right)=0\quad\left(LE_{ij}\right)\\
\boldsymbol{\varepsilon}_v^{p+}-\boldsymbol{\varepsilon}_v^{p-}+\sum_{K=m,c}\left(\sum_{k=1}^{3}\Delta\lambda_k^K\zeta_0.\zeta\left(r_k^K+r_{éla}^{dK}\right).\left(\sin\psi+\dfrac{q_k^{K+}}{p_k^+}\right)\right)-\dfrac{p}{|p|}.\left(\Delta\lambda_4^m+\dfrac{\Delta\lambda_4^c p^c}{|p^c|}\right)=0\ \left(LEVP\right)\\
r_k^{K+}-r_k^{K-}-\Delta\lambda_k^K\dfrac{\left(1-r_k^{K+}-r_{éla}^{dK}\right)^2}{a_c+\zeta\left(r_k^{K+}+r_{éla}^{dK}\right).\left(a_m-a_c\right)}\gamma_k^{K+}=0\quad K=m,c\quad k=1,2,3\quad\left(LR.1\right)\\
r_4^{K+}-r_4^{K-}-\Delta\lambda_4^K\dfrac{\left(1-r_4^{K+}-r_{éla}^{sK}\right)^2}{c_K}\left(\dfrac{P_{réf}}{P_c\left(\varepsilon_v^{p+}\right)}\right)=0\quad K=m,c\quad\left(LR.2\right)\\
q_k^{K+}+Mp_k^+\left(1-b_h.\ln\left(\dfrac{p_k^+}{P_{c0}.e^{-\beta\varepsilon_v^{p+}}}\right)\right).\left(r_k^{K+}+r_{éla}^{dK}\right)=0\quad K=m,c\quad\left(LF.1\right)\\
|p^{K+}|+d.P_{c0}.e^{-\beta\varepsilon_v^{p+}}.\left(r_4^{K+}+r_{éla}^{sK}\right)=0\quad K=m,c\quad\left(LF.2\right)
\end{array}\right.$$

éq 6

with: $\gamma_k^m=1$ and $\gamma_k^c=\dfrac{2q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}{2q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}-\mathbf{S}_{(k)H}^c.\mathbf{S}_{(k)}^c}$, cf *[éq 1.1.2-5] and [éq 1.1.2-15].*

It is thus necessary to establish the computation of the tangent matrix $\left.\dfrac{d\mathbf{R}}{d\left(\Delta\mathbf{Y}\right)}\right|_{\Delta\mathbf{Y}}$, which requires the analytical computation of various following derivatives compared to $\mathbf{Y}=\left(\boldsymbol{\sigma}_{ij},\ \varepsilon_v^p,\ r_k^K,\ \Delta\lambda_k^K\right)$:

$$\frac{d\,\mathbf{R}}{d(\Delta\mathbf{Y})}\Big|_{\Delta\mathbf{Y}}=\begin{bmatrix}\frac{\partial LE_{ij}}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LE_{ij}}{\partial\varepsilon_v^p} & \frac{\partial LE_{ij}}{\partial r_k^K} & \frac{\partial LE_{ij}}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LEVP}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LEVP}{\partial\varepsilon_v^p} & \frac{\partial LEVP}{\partial r_k^K} & \frac{\partial LEVP}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LR.1}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LR.1}{\partial\varepsilon_v^p} & \frac{\partial LR.1}{\partial r_k^K} & \frac{\partial LR.1}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LR.2}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LR.2}{\partial\varepsilon_v^p} & \frac{\partial LR.2}{\partial r_k^K} & \frac{\partial LR.2}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LF.1}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LF.1}{\partial\varepsilon_v^p} & \frac{\partial LF.1}{\partial r_k^K} & \frac{\partial LF.1}{\partial(\Delta\lambda_k^K)}\\ \frac{\partial LF.2}{\partial\boldsymbol{\sigma}_{rs}} & \frac{\partial LF.2}{\partial\varepsilon_v^p} & \frac{\partial LF.2}{\partial r_k^K} & \frac{\partial LF.2}{\partial(\Delta\lambda_k^K)}\end{bmatrix}\quad , K=m,c\ ;\ k=1,\ldots,4 \qquad \text{éq 6}$$

Ces derived are given in [§6.1] with [§6.6]. They exploit in particular the results establish by [éq7] with [éq 7].

Note:

All the mechanisms appear in the summation in [éq 6 -1], but in practice only appear the active mechanisms.
The directions of yielding $(\boldsymbol{\Psi})_k^K$ are expressed after injection in the Cartesian total reference in the shape of a tensor of order 2.

6.1 Derived from the Elles

equation of state come from the equation of state [éq 1.1.1] and the plastic strain [éq 1.1.5]:

$$\sigma_{ij}^{+}-\sigma_{ij}^{-}-C_{ijrs}(\boldsymbol{\sigma}^{+}).\left(\Delta\varepsilon_{rs}-\sum_{K=m,c}\left(\sum_{k=1}^{3}\Delta\lambda_k^K\left(\boldsymbol{\Psi}_{rs}\right)_{(k)}^{K_{(\sigma^+)}}\right)-\frac{p}{3\,|p|}.\left(\Delta\lambda_4^m+\frac{\Delta\lambda_4^c\,p^c}{|p^c|}\right)\mathbf{I}_{rs}\right)=0\quad(LE_{ij})$$

with:

$$C_{ijrs}^{+}=C_{ijrs}^{0}\,.\left(\frac{p}{P_{réf}}\right)^n \quad\text{and}\quad \begin{aligned}\left(\boldsymbol{\Psi}_{(k)}^m\right)_{rs}&=\frac{\left(\mathbf{S}_{(k)}\right)_{rs}}{2q_k}-\frac{\zeta_0.\zeta\left(r_k^m+r_{éla}^d\right)}{2}.\left(\sin\psi+\frac{q_k}{p_k}\right).\left(\mathbf{I}_{(k)}\right)_{rs}\\ \left(\boldsymbol{\Psi}_{(k)}^c\right)_{rs}&=\frac{\left(\mathbf{S}_{(k)}^K\right)_{rs}}{2q_k^c}-\frac{\zeta_0.\zeta\left(r_k^c+r_{éla}^{dc}\right)}{2}.\left(\sin\psi+\frac{\mathbf{S}_{(k)}.\mathbf{S}_{(k)}^c}{2\,p_k\,q_k^c}\right).\left(\mathbf{I}_{(k)}\right)_{rs}\end{aligned}\qquad\text{éq 6.1}$$

Computation of the components $\frac{\partial LE_{ij}}{\partial\sigma_{rs}}$ (tensor of order 4):

$$\frac{\partial LE_{ij}}{\partial \sigma^+_{rs}} = \frac{\partial \sigma^+_{ij}}{\partial \sigma^+_{rs}} + \frac{\partial C^+_{ijkl}}{\partial \sigma^+_{rs}} \cdot \left(\Delta \varepsilon_{kl} - \sum_{K=m,c} \left(\sum_{t=1}^{3} \Delta \lambda^K_t \left(\boldsymbol{\Psi}^K_{(t)} \right)_{kl} \left(\sigma^+ \right) \right) - \frac{p}{3|p|} \cdot \left(\Delta \lambda^m_4 + \frac{\Delta \lambda^c_4 p^c}{|p^c|} \right) I_{kl} \right) - C^+_{ijkl} \cdot \sum_{K=m,c} \left(\sum_{t=1}^{3} \Delta \lambda^K_t \frac{\partial \left(\boldsymbol{\Psi}^K_{(t)} \right)_{kl}}{\partial \sigma^+_{rs}} \right) \quad \textbf{éq 6.1}$$

the contribution of the mechanisms of consolidation being null in the last term. The statement of the various terms concerned in the computation of this derivative is, by exploiting the results [éq 7] and [éq7]:

$$\frac{\partial C^+_{ijkl}}{\partial \sigma^+_{rs}} = \frac{\partial C^+_{ijkl}}{\partial p} \cdot \frac{\partial p}{\partial \sigma^+_{rs}} = \frac{n}{P_{réf}} \cdot \left(\frac{p}{P_{réf}} \right)^{n-1} \cdot C^0_{ijkl} \cdot \frac{1}{3} \cdot I_{rs} = \frac{n}{3p} \cdot C^+_{ijkl} \cdot I_{rs}$$, cf [éq 6.1];

$$\frac{\partial \left(\boldsymbol{\Psi}^m_{(t)} \right)_{kl}}{\partial \sigma^+_{rs}} = \frac{1}{2 q_t} \cdot \frac{\partial \left(\mathbf{S}_{(t)} \right)_{kl}}{\partial \sigma^+_{rs}} - \frac{\left(\mathbf{S}_{(t)} \right)_{kl}}{2 \left(q_t \right)^2} \cdot \frac{\partial q_t}{\partial \sigma^+_{rs}} - \frac{\zeta_0 . \zeta \left(r^m_t + r^d_{éla} \right)}{2} \cdot \left(\mathbf{I}_{(t)} \right)_{kl} \cdot \left(\frac{1}{p_t} \cdot \frac{\partial q_t}{\partial \sigma^+_{rs}} - \frac{q_t}{p^2_t} \cdot \frac{\partial p_t}{\partial \sigma^+_{rs}} \right)$$

for each monotonous mechanism déviatoire $t=1,\dots,3$, cf [éq 1.1.2-3] and:

$$\frac{\partial \left(\boldsymbol{\Psi}^c_{(t)} \right)_{kl}}{\partial \sigma^+_{rs}} = \frac{1}{2 q^c_t} \cdot \frac{\partial \left(\mathbf{S}^c_{(t)} \right)_{kl}}{\partial \sigma^+_{rs}} - \frac{\left(\mathbf{S}^c_{(t)} \right)_{kl}}{4 \left(q^c_t \right)^3} \cdot \left(\mathbf{S}^c_{(t)} \right)_{ij} \cdot \frac{\partial \left(\mathbf{S}^c_{(t)} \right)_{ij}}{\partial \sigma_{rs}} - \frac{\zeta_0 . \zeta \left(r^c_t + r^{dc}_{éla} \right) . \left(\mathbf{I}_{(t)} \right)_{kl}}{4 p_t . q^c_t} \left(\frac{\partial \left(\mathbf{S}^c_{(t)} \right)_{ij}}{\partial \sigma^+_{rs}} . \left(\mathbf{S}_{(t)} \right)_{ij} + \frac{\partial \left(\mathbf{S}_{(t)} \right)_{ij}}{\partial \sigma^+_{rs}} . \left(S^c_{(t)} \right)_{ij} - \frac{\mathbf{S}_{(t)} . \mathbf{S}^c_{(t)}}{p_t . q^c_t} \left(q^c_t \frac{\partial p_t}{\partial \sigma^+_{rs}} + p_t \frac{\partial q^c_t}{\partial \sigma^+_{rs}} \right) \right)$$

for each cyclic mechanism déviatoire $t=1,\dots,3$, cf [éq 1.1.2-14].

From where for the monotonous mechanisms déviatoires:

$$\frac{\partial \left(\boldsymbol{\Psi}^m_{(t)} \right)_{kl}}{\partial \sigma^+_{rs}} = \frac{1}{2 q_t} \cdot \left(\mathbf{Dev}_{(t)} \right)_{klrs} - \frac{\left(\mathbf{S}_{(t)} \right)_{kl}}{4 \left(q_t \right)^3} \cdot \left(\mathbf{S}_{(t)} \right)_{rs} - \frac{\zeta_0 . \zeta \left(r^m_t + r^d_{éla} \right)}{4 p_t} \cdot \left(\mathbf{I}_{(t)} \right)_{kl} \cdot \left(\frac{\left(\mathbf{S}_{(t)} \right)_{rs}}{q_t} - \frac{q_t}{p_t} \cdot \left(\mathbf{I}_{(t)} \right)_{rs} \right) \quad \textbf{éq 6.1}$$

and for the cyclic mechanisms déviatoires:

$$\frac{\partial \left(\boldsymbol{\Psi}^c_{(t)}\right)_{kl}}{\partial \sigma^+_{rs}} = \frac{1}{2 q^c_t} \cdot \left(\left(\mathbf{Dev}_{(t)}\right)_{klrs} - \frac{F\left(p_t, \varepsilon^v_p\right) - b_h M}{2} \left(\mathbf{X}^H_{(t)} + \frac{\mathbf{S}^c_{(t)H}}{\|\mathbf{S}^c_{(t)H}\|^{2D}_{VM}} \cdot \left(r^c_t + r^{dc}_{éla}\right) \right)_{kl} \cdot \left(\mathbf{I}_{(t)}\right)_{rs} \right)$$

$$- \frac{\left(\mathbf{S}^c_{(t)}\right)_{kl}}{4 \left(q^c_t\right)^3} \cdot \left(\left(\mathbf{S}^c_{(t)}\right)_{rs} - \frac{F\left(p_t, \varepsilon^p_v\right) - b_h M}{2} \left(\mathbf{X}^H_{(t)} + \frac{\mathbf{S}^c_{(t)H}}{\|\mathbf{S}^c_{(t)H}\|^{2D}_{VM}} \cdot \left(r^c_t + r^{dc}_{éla}\right) \right)_{ij} \cdot \left(\mathbf{S}^c_{(t)}\right)_{ij} \cdot \left(\mathbf{I}_{(t)}\right)_{rs} \right)$$

$$+ \frac{\zeta_0 . \zeta\left(r^c_t + r^{dc}_{éla}\right) . \left(\mathbf{I}_{(t)}\right)_{kl}}{4 \, p_t . q^c_t} \left(\frac{F\left(p_t, \varepsilon^p_v\right) - b_h M}{2} \left(\mathbf{X}^H_{(t)} + \frac{\mathbf{S}^c_{(t)H}}{\|\mathbf{S}^c_{(t)H}\|^{2D}_{VM}} \cdot \left(r^c_t + r^{dc}_{éla}\right) \right)_{ij} \cdot \left(\mathbf{S}^c_{(t)}\right)_{ij} \cdot \left(\mathbf{I}_{(t)}\right)_{rs} \right. \quad \text{éq 6.1}$$

$$+ \frac{\mathbf{S}_{(t)} \cdot \mathbf{S}^c_{(t)}}{2 \mathrm{q}^{c2}_t} \left(\left(\mathbf{S}^c_{(t)}\right)_{rs} - \frac{F\left(p_t, \varepsilon^p_v\right) - b_h M}{2} \left(\mathbf{X}^H_{(t)} + \frac{\mathbf{S}^c_{(t)H}}{\|\mathbf{S}^c_{(t)H}\|^{2D}_{VM}} \cdot \left(r^c_t + r^{dc}_{éla}\right) \right) . \mathbf{S}^c_{(t)H} \left(\mathbf{I}_{(t)}\right)_{rs} \right)$$

$$\left. \frac{\mathbf{S}_{(t)} \cdot \mathbf{S}^c_{(t)}}{2 \, p_t} . \left(\mathbf{I}_{(t)}\right)_{rs} - \left(\mathbf{S}_{(t)}\right)_{rs} - \left(\mathbf{S}^c_{(t)}\right)_{rs} \right)$$

various terms being given by [éq 7], [éq 7], [éq 7], [éq 7] and [éq 7].

Computation of the components $\frac{\partial LE_{ij}}{\partial \varepsilon^p_v}$ **(tensor of order 2):**

$$\frac{\partial LE_{ij}}{\partial \varepsilon^p_v} = 3 K_0 \cdot \left| \frac{p(\boldsymbol{\sigma})}{P_{réf}} \right|^n . \mathbf{I}_{ij} \quad \text{éq 6.1}$$

Computation of the components $\frac{\partial LE_{ij}}{\partial r^K_k}$ **(tensors of order 2):**

It is noted that, for; $K = m, c \;\; ; \;\; k = 1, \ldots, 3$, cf [éq 6.1]:

$$\frac{\partial LE_{ij}}{\partial r^K_k} = C_{ijrs} \cdot \sum_{K=m,c} \left(\sum_{t=1}^{3} \Delta \lambda^K_t \frac{\partial \left(\Psi^K_{(t)}\right)_{rs}}{\partial r^K_t} \right) = \Delta \lambda^K_k . C_{ijrs} \frac{\partial \left(\Psi^K_{(k)}\right)_{rs}}{\partial r^K_k}$$

From where, fonctionétant it $\zeta(r)$ defined in [éq 1.1.2] eten $\mathbf{S}^{\mathbf{K}}_{(\mathbf{k})}$ [éq 1.1.1] and [éq 1.1.2]:

Monotonous mechanisms déviatoires

$$\frac{\partial LE_{ij}}{\partial r_k^m}=\begin{cases}0 & si\ r_k^m+r_{éla}^d\le r_{hys}\\ \frac{-\zeta_0}{2}\Delta\lambda_k^m.\left(\sin\psi+\frac{q_k}{p_k}\right).\left.\frac{d\zeta}{dr}\right|_{\left(r_k^m+r_{éla}^d\right)}C_{ijrs}.\left(\mathbf{I}_{(k)}\right)_{rs} & si\ r_{hys}<r_k^m+r_{éla}^d<r_{mob}\\ 0 & si\ r_k^m+r_{éla}^d>r_{mob}\end{cases}\quad \textbf{éq 6.1 -6}$$

Mécanismes déviatoires cyclic

$$\frac{\partial LE_{ij}}{\partial r_k^c}=\frac{\Delta\lambda_k^c}{2}C_{ijrs}.\Bigg(\frac{p_k.F_k}{q_k^c\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}\left(\frac{\left(\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)H}^c\right)\left(\mathbf{S}_{(k)}^c\right)_{rs}}{2\left(q_k^c\right)^2}-\left(\mathbf{S}_{(k)H}^c\right)_{rs}\right)-\zeta_0\left(\sin\psi+\frac{\mathbf{S}_{(k)}.\mathbf{S}_{(k)}^c}{2\,p_k\,q_k^c}\right).\frac{d\zeta}{dr}\Big|_{\left(r_k^c+r_{éla}^{dc}\right)}\left(\mathbf{I}_{(k)}\right)_{rs}$$
$$-\frac{\zeta_0.\zeta\left(r_k^c+r_{éla}^{dc}\right).F_k}{2\,q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}\left(\frac{\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)H}^c}{2\left(q_k^c\right)^2}\mathbf{S}_{(k)}.\mathbf{S}_{(k)}^c-\mathbf{S}_{(k)}.\mathbf{S}_{(k)H}^c\right).\left(\mathbf{I}_{(k)}\right)_{rs}\Bigg)$$

éq 6.1 the -7

function $F_k=F\left(p_k,\varepsilon_v^p\right)$ being defined in [éq 1.1.2].

Moreover, for the mechanisms of consolidation: $\frac{\partial LE_{ij}}{\partial r_4^K}=0$, for $K=m,c$.

Computation of the components $\frac{\partial LE_{ij}}{\partial\Delta\lambda_k^K}$ (tensors of order 2):

For $K=m,c\ ;\ k=1,\dots,3$:

$$\frac{\partial LE_{ij}}{\partial\Delta\lambda_k^K}=C_{ijrs}^+.\left(\Psi_{(k)}^K\right)_{rs}\quad;\quad\frac{\partial LE_{ij}}{\partial\Delta\lambda_4^m}=C_{ijrs}^+.\frac{p}{3|p|}I_{rs}\quad;\quad\frac{\partial LE_{ij}}{\partial\Delta\lambda_4^c}=C_{ijrs}^+.\frac{p.p^c}{3|p||p^c|}I_{rs}\quad \textbf{éq 6.1}$$

6.2 Derivatives of the equation of evolution of the Elles

plastic strain come from the evolution of the voluminal plastic strain, cf [éq 2.2.4 -6.2-1]:

$$\varepsilon_v^{p+}-\varepsilon_v^{p-}+\sum_{k=1}^3\Delta\lambda_k^m.\zeta_0.\zeta\left(r_k^m+r_{éla}^d\right).\left(\sin\psi+\frac{q_k^+}{p_k^+}\right)$$
$$+\sum_{k=1}^3\Delta\lambda_k^c.\zeta_0.\zeta\left(r_k^c+r_{éla}^{dc}\right).\left(\sin\psi+\frac{\mathbf{S}_{(k)}^+.\mathbf{S}_{(k)}^{c+}}{2\,p_k^+\,q_k^{c+}}\right)-\frac{p}{|p|}.\left(\Delta\lambda_4^m+\frac{\Delta\lambda_4^c\,p^c}{|p^c|}\right)=0\qquad(LEVP)$$

Computation of the components $\frac{\partial LEVP}{\partial \sigma_{ij}}$ **(tensor of order 2):**

$$\frac{\partial LEVP}{\partial \sigma_{ij}} = \sum_{k=1}^{3} \frac{\Delta\lambda_k^m}{2\,p_k}.\zeta_0.\zeta\left(r_k^m + r_{éla}^d\right).\left(\frac{\left(\mathbf{S}_{(k)}\right)_{ij}}{q_k} - \frac{q_k.\left(\mathbf{I}_{(k)}\right)_{ij}}{p_k}\right) + \sum_{k=1}^{3} \frac{\Delta\lambda_k^c}{2\,p_k\,q_k^c}.\zeta_0.\zeta\left(r_k^c + r_{éla}^{dc}\right).\left(\left(\mathbf{S}_{(k)}\right)_{ij} + \left(\mathbf{S}_{(k)}^c\right)_{ij}\left(1 - \frac{\mathbf{S}_{(k)}.\mathbf{S}_{(k)}^c}{\left(q_k^c\right)^2}\right)\right) \quad \text{éq 6.2 -1}$$

by using the results [éq 7], the interventions of the mechanisms of consolidation being null.

Computation of the component $\frac{\partial LEVP}{\partial \varepsilon_v^p}$ **(scalar):**

$$\frac{\partial LEVP}{\partial \varepsilon_v^p} = 1 - \sum_{k=1}^{3} \frac{\Delta\lambda_k^c}{2\,q_k^c}.b_h \beta M\,\zeta_0.\zeta\left(r_k^c + r_{éla}^{dc}\right). \left[\mathbf{S}_{(k)}.\left(\mathbf{X}_{(k)}^H + \frac{\mathbf{S}_{(k)H}^c}{\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}.\left(r_k^c + r_{éla}^{dc}\right)\right) - \frac{\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)}}{2\,q_k^c}\mathbf{S}_{(k)}^c.\left(\mathbf{X}_{(k)}^H + \frac{\mathbf{S}_{(k)H}^c}{\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}.\left(r_k^c + r_{éla}^{dc}\right)\right)\right] \quad \text{éq 6.2}$$

(cf [éq 1.1.2-3] and [éq 1.1.2-14]).

Computation of the components $\frac{\partial LEVP}{\partial r_k^K}$ **(scalar):**

For $K = m, c\,; k = 1, \ldots, 3$, by using the function $\zeta(r)$, cf [éq 1.1.2]:

Monotonous mechanisms déviatoires

$$\frac{\partial LEVP}{\partial r_k^m} = \begin{cases} 0 & si\ r_k^m + r_{éla}^d \le r_{hys} \\ \frac{\Delta\lambda_k^m.x_m.\zeta_0}{r_{mob} - r_{hys}}.\left(\frac{r_k^m + r_{éla}^d - r_{hys}}{r_{mob} - r_{hys}}\right)^{x_m - 1}.\left(\sin\psi + \frac{q_k}{p_k}\right) & si\ r_{hys} < r_k^m + r_{éla}^d \le r_{mob} \\ 0 & si\ r_k^m + r_{éla}^d > r_{mob} \end{cases} \quad \text{éq 6.2 -3}$$

Mécanismes déviatoires cyclic

$$\frac{\partial LEVP}{\partial r_k^c} = \Delta\lambda_k^c \zeta_0 \left(\left.\frac{\partial \zeta}{\partial r_k^c}\right|_{r_k^c + r_{éla}^{dc}} \left(\sin\psi + \frac{\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)}}{2\,p_k\,q_k^c}\right) - \frac{F_k.\zeta\left(r_k^c + r_{éla}^{dc}\right)}{2\,q_k^c}\left(\frac{\mathbf{S}_{(k)H}^c.\mathbf{S}_{(k)}}{\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}} - \frac{\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)}}{2\left(q_k^c\right)^2}.\mathbf{S}_{(k)H}^c.\mathbf{S}_{(k)}^c\right)\right) \quad \text{éq 6.2 -4}$$

where $F_k = F\left(p_k, \varepsilon_v^p\right)$ is defined in [éq 1.1.2]. Moreover $\frac{\partial LEVP}{\partial r_4^m} = \frac{\partial LEVP}{\partial r_4^c} = 0$.

Computation of the components $\frac{\partial LEVP}{\partial \Delta\lambda_k^K}$ **(scalar):**

For; $K = m, c\,; k = 1, \ldots, 3$:

Monotonous mechanisms déviatoires: $$\frac{\partial LEVP}{\partial \Delta \lambda_k^m}=\zeta_0 . \zeta\left(r_k^m+r_{éla}^d\right) . \left(\sin\psi+\frac{q_k}{p_k}\right)$$ **éq 6.2**

Mécanismes déviatoires cyclic: $$\frac{\partial LEVP}{\partial \Delta \lambda_k^c}=\zeta_0 . \zeta\left(r_k^c+r_{éla}^{dc}\right) . \left(\sin\psi+\frac{\mathbf{S}_{(k)} . \mathbf{S}_{(k)}^c}{2 p_k q_k^c}\right)$$ **éq 6.2**

Mécanismes of consolidation: $$\frac{\partial LEVP}{\partial \Delta \lambda_4^m}=-\frac{p}{|p|} \quad ; \quad \frac{\partial LEVP}{\partial \Delta \lambda_4^c}=-\frac{p}{|p|} . \frac{p^c}{|p^c|}$$ **éq 6.2**

6.3 Derivatives of the equation of evolution of hardening déviatoire

Elles come from the evolution of the hardening of the mechanisms déviatoires cf [éq1.1.2] and [éq1.1.2]:

$$r_k^{K+}-r_k^{K-}-\Delta \lambda_k^K \frac{\left(1-r_k^{K+}-r_{éla}^{dK}\right)^2 . \gamma_k^K}{a_c+\zeta\left(r_k^{K+}-r_{éla}^{dK}\right) . \left(a_m-a_c\right)}=0 \qquad K=m,c \quad k=1,2,3 \quad (LR.1)$$

Computation of the components $\frac{\partial LR.1}{\partial \sigma_{ij}}$ **(tensor of order 2):**

For the monotonous mechanisms déviatoires $k=1,\ldots,3$: $\gamma_k^m=1$, therefore:

$$\frac{\partial LR.1}{\partial \sigma_{ij}}=0$$ **éq 6.3**

Pour the cyclic mechanisms déviatoires $k=1,\ldots,3$:

$$\frac{\partial LR.1}{\partial \sigma_{ij}}=-\Delta \lambda_k^c \frac{\left(1-r_k^{c+}-r_{éla}^{dc}\right)^2}{a_c+\zeta\left(r_k^{c+}+r_{éla}^{dc}\right) . \left(a_m-a_c\right)} \cdot \frac{\|\mathbf{S}_{(t)H}^c\|_{VM}^{2D}}{\left(2 q_k^c . \|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}-\mathbf{S}_{(k)}^c . \mathbf{S}_{(k)H}^c\right)^2} \frac{\partial\left(\mathbf{S}_{(k)}^c\right)_{rs}}{\partial \sigma_{ij}} \cdot \left(2 q_k^c . \left(\mathbf{S}_{(k)H}^c\right)_{rs}-\frac{\mathbf{S}_{(k)}^c . \mathbf{S}_{(k)H}^c}{q_k^c}\left(\mathbf{S}_{(k)}^c\right)_{rs}\right)$$ **éq 6.3**

with the operator $\frac{\partial (\mathbf{S}_{(k)}^c)_{rs}}{\partial \sigma_{ij}}$ defined by [éq 6-8].

Computation of the components $\frac{\partial LR.1}{\partial \varepsilon_v^p}$ **(scalar):**

For the monotonous mechanisms déviatoires $k=1,\ldots,3$: $\gamma_k^m=1$, therefore:

$$\frac{\partial LR.1}{\partial \varepsilon_v^p}=0$$ **éq 6.3**

Pour the cyclic mechanisms déviatoires $k=1,\ldots,3$:

$$\frac{\partial LR.1}{\partial \varepsilon_v^p}=-\frac{\Delta\lambda_k^c.\left(1-r_k^c-r_{éla}^{dc}\right)}{a_c+\zeta\left(r_k^c+r_{éla}^{dc}\right).\left(a_m-a_c\right)}\cdot\frac{\|S_{(t)H}^c\|_{VM}^{2D}}{\left(2q_k^c.\|S_{(k)H}^c\|_{VM}^{2D}-\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)H}^c\right)^2}$$
$$\frac{\partial\left(S_{(k)}^c\right)_{ij}}{\partial\varepsilon_v^p}\cdot\left(2q_k^c.\left(S_{(k)H}^c\right)_{ij}-\frac{\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)H}^c}{q_k^c}\left(S_{(k)}^c\right)_{ij}\right)$$

éq 6.3

with $\frac{\partial\left(\mathbf{S}_{(k)}^c\right)_{ij}}{\partial\varepsilon_v^p}=\beta M\, b_h.p_k.\left(\mathbf{X}_{(k)}^H+\frac{\mathbf{S}_{(k)H}^c}{\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}.\left(r_k^c+r_{éla}^{dc}\right)\right)_{ij}$, cf [éq1.1.2] and [éq1.1.2].

Computation of the components $\frac{\partial LR.1}{\partial r_k^K}$ **(scalar):**

For the monotonous mechanisms déviatoires $k=1,\dots,3:\gamma_k^m=1$, therefore:

$$\frac{\partial LR.1}{\partial r_k^m}=1+\frac{\Delta\lambda_k^m.\left(1-r_k^m-r_{éla}^d\right)}{a_c+\zeta\left(r_k^m+r_{éla}^d\right).\left(a_m-a_c\right)}\cdot\left(2+\frac{\left(1-r_k^m-r_{éla}^d\right).\left(a_m-a_c\right)}{a_c+\zeta\left(r_k^m+r_{éla}^d\right).\left(a_m-a_c\right)}.\frac{\partial\zeta}{\partial r_k^m}\Big|_{\left(r_k^m+r_{éla}^d\right)}\right)$$

that is to say:

$$\frac{\partial LR.1}{\partial r_k^m}=\begin{cases}1+\Delta\lambda_k^m.\dfrac{2\left(1-r_k^m-r_{éla}^d\right)}{a_c} & si\ \ r_k^m+r_{éla}^d<r_{hys}\\ 1+\dfrac{\Delta\lambda_k^m\left(1-r_k^m-r_{éla}^d\right)}{a_c+\zeta\left(r_k^K+r_{éla}^{dK}\right).\left(a_m-a_c\right)}\cdot\left(2+\dfrac{\left(1-r_k^m-r_{éla}^d\right).\left(a_m-a_c\right)}{a_c+\zeta\left(r_k^m+r_{éla}^d\right).\left(a_m-a_c\right)}.\dfrac{\partial\zeta}{\partial r_k^m}\Big|_{\left(r_k^m+r_{éla}^d\right)}\right) & si\ \ r_{hys}<r_k^m+r_{éla}^d<r_{mob}\\ 1+\Delta\lambda_k^m.\dfrac{2\left(1-r_k^m-r_{éla}^d\right)}{a_m} & si\ \ r_k^m+r_{éla}^d>r_{mob}\end{cases}$$

éq 6.3 -5

Pour the cyclic mechanisms déviatoires $k=1,\dots,3$:

$$\frac{\partial LR.1}{\partial r_k^c}=1+$$
$$\frac{\Delta\lambda_k^c.\left(1-r_k^c-r_{éla}^{dc}\right)}{a_c+\zeta\left(r_k^c+r_{éla}^{dc}\right).\left(a_m-a_c\right)}\cdot\frac{2q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}{2q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}-\mathbf{S}_{(k)H}^c.\mathbf{S}_{(k)}^c}\cdot\left(2+\frac{\left(1-r_k^c-r_{éla}^{dc}\right).\left(a_m-a_c\right)}{a_c+\zeta\left(r_k^c+r_{éla}^{dc}\right).\left(a_m-a_c\right)}.\frac{\partial\zeta}{\partial r_k^c}\Big|_{\left(r_k^c+r_{éla}^{dc}\right)}\right)$$
$$-\Delta\lambda_k^c\frac{\left(1-r_k^{c+}-r_{éla}^{dc}\right)^2}{a_c+\zeta\left(r_k^{c+}+r_{éla}^{dc}\right).\left(a_m-a_c\right)}\cdot\frac{\|\mathbf{S}_{(t)H}^c\|_{VM}^{2D}}{\left(2q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}-\mathbf{S}_{(k)}^c.S_{(k)H}^c\right)^2}\cdot\frac{\partial\left(\mathbf{S}_{(k)}^c\right)_{ij}}{\partial r_k^c}\cdot\left(2q_k^c.\left(\mathbf{S}_{(k)H}^c\right)_{ij}-\frac{\mathbf{S}_{(k)}^c.\mathbf{S}_{(k)H}^c}{q_k^c}\left(\mathbf{S}_{(k)}^c\right)_{ij}\right)$$

éq 6.3 -6

with $\frac{\partial\left(\mathbf{S}_{(k)}^c\right)_{ij}}{\partial r_k^c}=-p_k(\boldsymbol{\sigma}).F\left(p_k(\boldsymbol{\sigma}),\varepsilon_v^p\right).\frac{\left(\mathbf{S}_{(k)H}^c\right)_{ij}}{\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}$, cf [éq1.1.2].

Moreover: $\frac{\partial LR.1}{\partial r_4^K}=0$.

Computation of the components $\frac{\partial LR.1}{\partial \Delta \lambda_k^K}$ **(scalar):**

For $K=m,c\,;k=1,\ldots,3$:

$$\frac{\partial LR.1}{\partial \Delta \lambda_k^K}=-\frac{\left(1-r_k^K-r_{éla}^{dK}\right)^2}{a_c+\zeta\left(r_k^K+r_{éla}^{dK}\right).\left(a_m-a_c\right)}\cdot\gamma_k^K \qquad \text{éq 6.3}$$

with: $\gamma_k^m=1$ and $\gamma_k^c=\frac{2\,q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}{2q_k^c.\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}-\mathbf{S}_{(k)H}^c.\mathbf{S}_{(k)}^c}$.

Moreover: $\frac{\partial LR.1}{\partial \Delta \lambda_4^K}=0$.

6.4 Derived from the equation of evolution of spherical hardening

Elles come from the evolution of the hardening of the mechanisms from consolidation spherical, cf [éq1.1.3] and [éq 1.1.3]:

$$r_4^{K+}-r_4^{K-}-\Delta\lambda_4^K\frac{\left(1-r_4^{K+}-r_{éla}^{sK}\right)^2}{c_K}\left(\frac{P_{réf}}{P_c\left(\varepsilon_v^{p+}\right)}\right)=0 \qquad K=m,c \quad (LR.2)$$

Computation of the components $\frac{\partial LR.2}{\partial \sigma_{ij}}$ **(tensor of order 2):**

$$\frac{\partial LR.2}{\partial \sigma_{ij}}=0 \qquad \text{éq 6.4}$$

Computation of the components $\frac{\partial LR.2}{\partial \varepsilon_v^p}$ **(scalar):**

For $K=m,c$:

$$\frac{\partial LR.2}{\partial \varepsilon_v^p}=\frac{\partial LR.2}{\partial P_c}\cdot\frac{\partial P_c}{\partial \varepsilon_v^p}=-\frac{\Delta\lambda_4^K\,.\,P_{réf}\,.\,\beta\,.\left(1-r_4^K-r_{éla}^{sK}\right)^2}{c_K\,.\,P_c\left(\varepsilon_v^p\right)} \qquad \text{éq 6.4}$$

Computation of the components $\frac{\partial LR.2}{\partial r_4^K}$ **(scalar):**

For $K=m,c$:

$$\frac{\partial LR.2}{\partial r_4^K}=1+2\,\Delta\lambda_4^K\,.\,\frac{\left(1-r_4^K-r_{éla}^{sK}\right)}{c_K}\,.\left(\frac{P_{réf}}{P_c\left(\varepsilon_v^p\right)}\right) \qquad \text{éq 6.4}$$

Computation of the components $\frac{\partial LR.2}{\partial \Delta\lambda_4^K}$ **(scalar):**

For $K=m,c$:

$$\frac{\partial LR.2}{\partial \Delta \lambda_4^K} = -\frac{\left(1-r_4^K-r_{éla}^{sK}\right)^2}{c_K} \cdot \left(\frac{P_{réf}}{P_c\left(\varepsilon_v^p\right)}\right)$$ **éq 6.4**

Moreover: $\frac{\partial LR.2}{\partial r_k^K}=0$, for the mechanisms déviatoires $k=1,\ldots,3$.

6.5 Derived from the criteria of the mechanisms deviatoric

Elles come from the criteria [éq 1.1.2] and [éq 1.1.2]:

$$q_k^{K+} + Mp_k^+\left(1-b_h.\ln\left(\frac{p_k^+}{P_{c0}.e^{-\beta\varepsilon_v^{p+}}}\right)\right).\left(r_k^{K+}+r_{éla}^{dK}\right)=0 \qquad K=m,c \quad (LF.1)$$

Computation of the components $\frac{\partial LF.1}{\partial \sigma_{ij}}$ (tensor of order 2):

For the monotonous mechanisms, cf [éq 7]:

$$\frac{\partial LF.1}{\partial \sigma_{ij}} = \frac{1}{2q_k}\left(\mathbf{S}_{(k)}\right)_{ij} + \frac{M}{2}.\left(r_k^m+r_{éla}^d\right).\left(1-b_h.\left(1+\ln\left(\frac{p_k}{P_{c0}.e^{-\beta\varepsilon_v^p}}\right)\right)\right).\left(\mathbf{I}_{(k)}\right)_{ij}$$ **éq 6.5**

Pour the mechanisms déviatoires, cf [éq 7]:

$$\frac{\partial LF.1}{\partial \sigma_{ij}} = \frac{1}{2q_k^c}\left(\mathbf{S}_{(k)}\right)_{ij} + \frac{M}{2}.\left(1-b_h\left(1+\ln\left|\frac{p_k(\boldsymbol{\sigma})}{P_c\left(\varepsilon_v^p\right)}\right|\right)\right).\left(\left(r_k^c+r_{éla}^{dc}\right)-\left(\mathbf{X}_{(t)}^H+\frac{\mathbf{S}_{(t)H}^c}{\|\mathbf{S}_{(t)H}^c\|_{VM}^{2D}}.\left(r_t^c+r_{éla}^{dc}\right)\right).\frac{S_{(k)}^c}{2q_k^c}\right).\left(\mathbf{I}_{(k)}\right)_{ij}$$ **éq 6.5**

Computation of the components $\frac{\partial LF.1}{\partial \varepsilon_v^p}$ (scalar):

For $k=1,\ldots,3$, cf [éq 1.1.2] and [éq 1.1.2] and [éq 7]:

Monotonous mechanisms déviatoires: $$\frac{\partial LF.1}{\partial \varepsilon_v^p} = -\beta Mb_h.p_k.\left(r_k^m+r_{éla}^d\right)$$ **éq 6.5**

Mécanismes déviatoires cyclic:

$$\frac{\partial LF.1}{\partial \varepsilon_v^p} = -\beta Mb_h.p_k.\left(\left(r_k^m+r_{éla}^{dc}\right)-\frac{\mathbf{S}_{(k)}^c}{2q_k^c}.\left(\mathbf{X}_{(k)}^H+\frac{\mathbf{S}_{(k)H}^c}{\|\mathbf{S}_{(k)H}^c\|_{VM}^{2D}}.\left(r_k^c+r_{éla}^{dc}\right)\right)\right)$$ **éq 6.5 -8**

with $\mathbf{X}_{(k)}^H$ defined in [éq 1.1.2].

Computation of the components $\frac{\partial LF.1}{\partial r_k^K}$ (scalar):

For $k=1,\ldots,3$:

Monotonous mechanisms déviatoires: $$\frac{\partial LF.1}{\partial r_k^m}=Mp_k(\sigma).\left(1-b_h.\ln\left(\frac{p_k(\sigma)}{P_{c0}.e^{-\beta\varepsilon_v^{p+}}}\right)\right)$$ **éq 6.5**

Mécanismes déviatoires cyclic:

$$\frac{\partial LF.1}{\partial r_k^c}=Mp_k(\sigma).\left(1-b_h.\ln\left(\frac{p_k(\sigma)}{P_{c0}.e^{-\beta\varepsilon_v^{p+}}}\right)\right).\left(1-\frac{S_{(k)}^c.S_{(k)H}^c}{2q_k^c.\|S_{(k)H}^c\|_{VM}^{2D}}\right)$$ **éq 6.5**

Computation of the components $\frac{\partial LF.1}{\partial\Delta\lambda_k^K}$ **(scalar):**

$$\frac{\partial LF.1}{\partial\Delta\lambda_k^K}=0$$ **éq 6.5**

6.6 Derivatives of the criteria of the spherical mechanisms of consolidation

Elles come from the criteria [éq 1.1.3] and [éq 1.1.3]:

$$|p^{K+}|+d.P_{c0}.e^{-\beta\varepsilon_v^{p+}}.\left(r_4^{K+}+r_{éla}^{sK}\right)=0 \qquad K=m,c \quad (LF.2)$$

with in the cyclic case: $p^c\left(\boldsymbol{\sigma},\ \varepsilon_v^p,\ p_H,\varepsilon_{vH}^p\right)=|p(\boldsymbol{\sigma})|+p_H.e^{-\beta\left(\varepsilon_v^p-\varepsilon_{vH}^p\right)}$, cf [éq1.1.3].

Computation of the components $\frac{\partial LF.2}{\partial\sigma_{ij}}$ **(tensor of order 2):**

Monotonous spherical mechanism: $$\left(\frac{\partial LF.2}{\partial\boldsymbol{\sigma}}\right)_{ij}=\frac{1}{3}.\frac{p}{|p|}.\mathbf{I}_{ij}$$ **éq 6.6**

cyclic spherical Mécanisme: $$\left(\frac{\partial LF.2}{\partial\boldsymbol{\sigma}}\right)_{ij}=\frac{p}{3|p|}.\frac{p^c}{3|p^c|}.\mathbf{I}_{ij}$$ **éq 6.6**

Computation of the components $\frac{\partial LF.2}{\partial\varepsilon_v^p}$ **(scalar):**

Monotonous spherical mechanism: $$\frac{\partial LF.2}{\partial\varepsilon_v^p}=-d.\beta.P_{c0}.e^{-\beta\varepsilon_v^p}.\left(r_4^m+r_{éla}^s\right)$$ **éq 6.6**

cyclic spherical Mécanisme:

$$\frac{\partial LF.2}{\partial\varepsilon_v^p}=-\beta.e^{-\beta\varepsilon_v^p}\left(d.P_{c0}.\left(r_4^c+r_{éla}^{sc}\right)+p_H.e^{-\beta\varepsilon_{vH}^p}.\frac{|p|+p_H.e^{-\beta\left(\varepsilon_v^p-\varepsilon_{vH}^p\right)}}{\left||p|+p_H.e^{-\beta\left(\varepsilon_v^p-\varepsilon_{vH}^p\right)}\right|}\right)$$ **éq 6.6 -4**

(cf [éq 7] and [éq 7])

Computation of the components $\frac{\partial LF.2}{\partial r_4^K}$ **(scalar):**

Monotonous spherical mechanism: $$\frac{\partial LF.2}{\partial r_4^m}=d.P_{c0}e^{-\beta\varepsilon_v^p}$$ **éq 6.6 -5**

cyclic spherical Mécanisme: $$\frac{\partial LF.2}{\partial r_4^c}=d.P_{c0}.e^{-\beta\varepsilon_v^p}$$ **éq 6.6**

cf [éq 7] and [éq 7], with p^c defined in [éq 1.1.3].

Computation of the components $\frac{\partial LF.2}{\partial\Delta\lambda_4^K}$ **(scalar):**

$$\frac{\partial LF.2}{\partial\Delta\lambda_4^K}=0$$ **éq 6.6**

7 Annexe 2: notation of the tensors, their invariants and statements various derivatives

One is placed for more facility in orthonormal base $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ of space at 3 dimensions. One defines the tensor identity of order 2, here by using the notation of Walpole-Cowin:

$$\mathbf{I}=\delta_{ij}.\mathbf{e}_i\otimes\mathbf{e}_j=\begin{pmatrix}1\\1\\1\\0\\0\\0\end{pmatrix}$$ **éq 7**

One notes, for each mechanism in the plane k :

$$p_k(\boldsymbol{\sigma})=\frac{1}{2}\,\mathrm{tr}(\boldsymbol{\sigma}_{(k)}) \quad ; \quad q_k(\boldsymbol{\sigma})=\|\mathbf{S}_{(k)}(\boldsymbol{\sigma})\|_{VM}^{2D}$$ **éq 7**

with $$\mathbf{S}_{(k)}(\boldsymbol{\sigma})=\boldsymbol{\sigma}_{(k)}-p_k(\boldsymbol{\sigma})\,.\mathbf{I}_{(k)}$$ **éq 7**

and $$\|\mathbf{S}_{(k)}(\boldsymbol{\sigma})\|_{VM}^{2D}=\frac{\sqrt{2}}{2}.\sqrt{\mathbf{S}_{(k)_{11}}^2+\mathbf{S}_{(k)_{22}}^2+2\mathbf{S}_{(k)_{12}}^2}=\frac{1}{2}\sqrt{\left(\boldsymbol{\sigma}_{(k)_{11}}-\boldsymbol{\sigma}_{(k)_{22}}\right)^2+4\boldsymbol{\sigma}_{(k)_{12}}^2}$$ **éq 7**

$\boldsymbol{\sigma}_{(k)}$ being defined by [éq 1.1.1-3], for $i_k=1+mod(k,3)$ and $j_k=1+mod(k+1,3)$.
One has the following relation between the terms of the tensor $\mathbf{S}_{(k)}$:

$$\mathbf{S}_{i_k i_k}=-\mathbf{S}_{j_k j_k}$$

One defines moreover tensors identity of order 2 (for each plane) and 4:

$$\mathbf{I}_{(k)}=\delta_{i_k j_k}\mathbf{e}_{i_k}\otimes\mathbf{e}_{j_k} \quad ; \quad \mathbf{I}=\delta_{ij}\mathbf{e}_i\otimes\mathbf{e}_j$$ **éq 7**

One gives below the statements of several derivatives which intervene several times in the analytical statements of the model.

$$\frac{\partial \boldsymbol{\sigma}_{ij}}{\partial \boldsymbol{\sigma}_{rs}}=\mathbf{I}_{ijrs} \quad ; \quad \frac{\partial p}{\partial \boldsymbol{\sigma}_{rs}}=\frac{1}{3}\,.\,\mathbf{I}_{rs} \quad ; \quad \frac{\partial p_k}{\partial \boldsymbol{\sigma}}=\frac{1}{2}\,.\,\mathbf{I}_{(k)} \qquad \text{éq 7}$$

$$\frac{\partial q_k}{\partial \mathbf{S}_{(k)}}=\frac{1}{2\,q_k}\mathbf{S}_{(k)} \quad ; \quad \frac{\partial \mathbf{S}_{(k)}}{\partial \boldsymbol{\sigma}}=\mathbf{Dev}_{(k)} \quad ; \qquad \text{éq 7}$$

One notices that: $\mathbf{Dev}_{(k)} \underline{\underline{\otimes}}\, \mathbf{S}_{(k)}=\mathbf{S}_{(k)}$. Moreover:

$$\frac{\partial q_k^c}{\partial (\mathbf{S}_{(k)}^c)_{ij}}=\frac{\left(\mathbf{S}_{(k)}^c\right)_{ij}}{2\,q_k^c} \quad ; \quad \frac{\partial q_k^c}{\partial \boldsymbol{\sigma}_{rs}}=\frac{\left(\mathbf{S}_{(k)}^c\right)_{ij}}{2\,q_k^c}\cdot\frac{\partial\left(\mathbf{S}_{(k)}^c\right)_{ij}}{\partial \boldsymbol{\sigma}_{rs}} \quad ;$$

$$\frac{\partial\left(\mathbf{S}_{(k)}^c\right)_{ij}}{\partial \boldsymbol{\sigma}_{rs}}=\left(\mathbf{Dev}_{(k)}\right)_{ijrs}-\frac{F\left(p_t,\varepsilon_v^p\right)-b_h M}{2}\left(\mathbf{X}_{(t)}^H+\frac{\mathbf{S}_{(t)H}^c}{\|\mathbf{S}_{(t)H}^c\|_{VM}^{2D}}\,.\left(r_t^c+r_{éla}^{dc}\right)\right)_{ij}.\left(\mathbf{I}_{(t)}\right)_{rs} \qquad \text{éq 7}$$

One also has: $\mathbf{Dev}_{(k)} \underline{\underline{\otimes}}\, \mathbf{S}_{(k)}^c=\mathbf{S}_{(k)}^c$. Moreover, in $q_k=0$ (or $q_k^c=0$), the derivative $\frac{\partial q_k}{\partial \mathbf{S}_{(k)}}$ (or $\frac{\partial q_k^c}{\partial \mathbf{S}_{(k)}^c}$) is not defined. In practice, the value will be taken $\mathbf{0}$.

One uses, so as to reduce the writings, the notation of Walpole-Cowin of the tensors of order 2 and 4.

$$\mathbf{S}_{(k)}=\begin{pmatrix}\mathbf{S}_{i_k i_k}\\ \mathbf{S}_{j_k j_k}\\ \sqrt{2}.\mathbf{S}_{i_k j_k}\end{pmatrix}=\begin{pmatrix}\mathbf{S}_{i_k i_k}\\ -\mathbf{S}_{i_k i_k}\\ \sqrt{2}.\mathbf{S}_{i_k j_k}\end{pmatrix} \quad ; \quad \mathbf{I}_{(k)}=\begin{pmatrix}1\\1\\0\end{pmatrix}^{(k)} \quad ; \quad \mathbf{Dev}_{(k)}=\frac{1}{2}\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 2\end{pmatrix}^{(k)} \qquad \text{éq 7}$$

because $\mathbf{S}_{j_k j_k}=-\mathbf{S}_{i_k i_k}$, this tensor being deviative in the plane k . One obtains in particular:

$$\left(\mathbf{S}_{(k)}\right)_{ij}.\left(\mathbf{S}_{(k)}\right)_{rs}=\begin{pmatrix} \mathbf{S}^2_{i_k i_k} & -\mathbf{S}^2_{i_k i_k} & \sqrt{2}.\mathbf{S}_{i_k i_k}\mathbf{S}_{i_k j_k} \\ -\mathbf{S}^2_{i_k i_k} & \mathbf{S}^2_{i_k i_k} & -\sqrt{2}.\mathbf{S}_{i_k i_k} S_{i_k j_k} \\ \sqrt{2}.\mathbf{S}_{i_k i_k}\mathbf{S}_{i_k j_k} & -\sqrt{2}.\mathbf{S}_{i_k i_k}\mathbf{S}_{i_k j_k} & 2\mathbf{S}^2_{i_k j_k} \end{pmatrix} \quad \text{éq 7}$$

$$\left(\mathbf{S}_{(k)}\right)_{ij}.\left(\mathbf{I}_{(k)}\right)_{rs}=\begin{pmatrix} \mathbf{S}_{i_k i_k} & \mathbf{S}_{i_k i_k} & 0 \\ -\mathbf{S}_{i_k i_k} & -\mathbf{S}_{i_k i_k} & 0 \\ \sqrt{2}.\mathbf{S}_{i_k j_k} & \sqrt{2}.\mathbf{S}_{i_k j_k} & 0 \end{pmatrix} \quad \text{éq 7}$$

$$\left(\mathbf{I}_{(k)}\right)_{ij}.\left(\mathbf{I}_{(k)}\right)_{rs}=\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{éq 7}$$

Par ailleurs, for the computation of the plastic multipliers and the tangent operators, one needs:

$$\mathrm{f}^{\mathrm{m}}_{\mathrm{k},\boldsymbol{\sigma}}=\frac{1}{2}q_{k,S_{(k)}}\underline{\underline{\otimes}}\,\mathbf{Dev}_{(\boldsymbol{k})}+\frac{1}{2}f^{m}_{k,p_k}.I_{(k)}=\frac{\mathbf{S}_{(k)}}{2\mathrm{q}_k}+\frac{M}{2}.\left(r^m_k+r^d_{éla}\right).\left(1-b_h\left(1+\ln\left|\frac{p_k(\sigma)}{P_c\left(\varepsilon^p_v\right)}\right|\right)\right).\mathbf{I}_{(k)} \quad \text{éq 7}$$

In $q_k=0$, one withdraws the first term of this statement, cf [éq 7].

$$\mathrm{f}^{\mathrm{m}}_{\mathrm{k},\varepsilon^{\mathrm{p}}_{\mathrm{v}}}=-b_h\beta M\,.\,p_k(\sigma)\,.\,\left(r^m_k+r^d_{éla}\right) \quad \text{éq 7}$$

$$\mathrm{f}^{\mathrm{m}}_{k,r^m_k}=p_k(\boldsymbol{\sigma})\,.\,F\left(p_k(\boldsymbol{\sigma}),\varepsilon^p_v\right)=p_k(\boldsymbol{\sigma})\,.\,M.\left(1-b_h\left(1+\ln\left|\frac{p_k(\boldsymbol{\sigma})}{P_c\left(\varepsilon^p_v\right)}\right|\right)\right) \quad \text{éq 7}$$

$$\mathrm{f}^{\mathrm{c}}_{\mathrm{k},\sigma}=\frac{\mathbf{S}^c_{(k)}}{2\mathrm{q}^c_k}+\frac{M}{2}\left(1-b_h\left(1+\ln\left|\frac{p_k(\boldsymbol{\sigma})}{P_c\left(\varepsilon^p_v\right)}\right|\right)\right).\left(\left(r^c_k+r^{dc}_{éla}\right)-\left(\mathbf{X}^H_{(t)}+\frac{\mathbf{S}^c_{(t)H}}{\|\mathbf{S}^c_{(t)H}\|^{2\mathrm{D}}_{VM}}\,.\,\left(r^c_t+r^{dc}_{éla}\right)\right).\frac{\mathbf{S}^c_{(k)}}{2\mathrm{q}^c_k}\right).\mathbf{I}_{(k)} \quad \text{éq 7}$$

$$\begin{aligned}\mathrm{f}^{\mathrm{c}}_{\mathrm{k},\varepsilon^{\mathrm{p}}_{\mathrm{v}}}&=\mathrm{f}^{\mathrm{c}}_{k,q^c_k}.q^c_{k,\varepsilon^p_v}-b_h\beta M\,.\,p_k(\sigma)\,.\,\left(r^c_k+r^{dc}_{éla}\right)\\&=-\beta M\,b_h.\,p_k.\left(\left(r^c_k+r^{dc}_{éla}\right)-\frac{\mathbf{S}^c_{(k)}}{2\mathrm{q}^c_k}.\left(X^H_{(k)}+\frac{\mathbf{S}^c_{(k)H}}{\|\mathbf{S}^c_{(k)H}\|^{2\mathrm{D}}_{VM}}\,.\,\left(r^c_k+r^{dc}_{éla}\right)\right)\right)\end{aligned} \quad \text{éq 7}$$

$$\mathrm{f}^{\mathrm{c}}_{k,r^c_k}=\mathrm{f}^{\mathrm{c}}_{k,q^c_k}\cdot q^c_{k,r^c_k}+p_k(\sigma)\ .\ F\left(p_k(\sigma),\varepsilon^p_v\right)$$
$$=M.p_k(\sigma).\left(1-b_h.\ln\left(\frac{p_k(\sigma)}{P_{c0}.e^{-\beta\varepsilon^p_v}}\right)\right).\left(1-\frac{\mathbf{S}^c_{(k)}.\mathbf{S}^c_{(k)H}}{2\mathrm{q}^c_k.\|\mathbf{S}^c_{(k)H}\|^{2D}_{VM}}\right) \quad \textbf{éq 7}$$

$$f^m_{4,\sigma}=\frac{p}{3\,|p|}.I=\frac{1}{3}\,\mathrm{sgn}(p)\ .I \quad \textbf{éq 7}$$

$$f^m_{4,\varepsilon^p_v}=-d\,\beta\,.P_{c0}.e^{-\beta\varepsilon^p_v}.\left(r^m_4+r^s_{éla}\right) \quad \textbf{éq 7}$$

$$f^m_{4,r^m_4}=d\,.P_c\left(\varepsilon^p_v\right)=d\,.P_{c0}.e^{-\beta\varepsilon^p_v} \quad \textbf{éq 7}$$

$$f^c_{4,\sigma}=\frac{p}{3\,|p|}.\frac{p^c}{3\,|p^c|}.I \quad \textbf{éq 7}$$

$$f^c_{4,\varepsilon^p_v}=-\beta.e^{-\beta\varepsilon^p_v}\left(d\,.P_{c0}.\left(r^c_4+r^{sc}_{éla}\right)+p_H.e^{-\beta\varepsilon^p_{vH}}.\frac{|p|+p_H.e^{-\beta\left(\varepsilon^p_v-\varepsilon^p_{vH}\right)}}{\left\||p|+p_H.e^{-\beta\left(\varepsilon^p_v-\varepsilon^p_{vH}\right)}\right|}\right) \quad \textbf{éq 7}$$

$$f^c_{4,r^c_4}=d\,.P_{c0}.e^{-\beta\varepsilon^p_v} \quad \textbf{éq 7}$$

Enfin, one notes that:

$$\mathbf{C}\cdot\mathbf{I}=3K\cdot\mathbf{I}\ ;\quad \left(\mathbf{C}.\mathbf{I}_{(k)}\right)_{ij}=\frac{3\mu K}{E}.\left(\left(\mathbf{I}_{(k)}\right)_{ij}+2\nu\,\delta_{ki}\,\delta_{kj}\right)\quad ;\qquad \mathbf{C}.\mathbf{S}_{(k)}=2\mu\,\mathbf{S}_{(k)} \quad \textbf{éq 7}$$