Machine learning for assessing liquefaction potential of soils

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2011 Pan-Am CGS Geotechnical Conference

ABSTRACT
This paper presents a new simplified method for assessing the liquefaction potential of soils based on geotechnical, geometrical and seismic load parameters. A relatively large database consisting of CPT, SPT and V_s measurements and field liquefaction performance observations of historical earthquakes is analyzed. This database is used to construct a nonlinear environment where the occurrence and nonoccurrence of liquefaction can be predicted using *Machine Learning* tools. The successfully trained and tested scheme is composed of i) an artificial neural network to map some index properties to resistance values, ii) a neurofuzzy system to estimate the liquefaction occurrence and a multidimensional fuzzy-liquefaction index and iii) a regression tree to generate or complete the seismic load information. The data points, measured and estimated, collectively define the liquefaction boundary surface, the fuzzy limit state nonlinear-function. Based on this newly developed cognitive method, intelligent analyses of the cases in the database are conducted using a simple mapping approach. The *Machine Learning* models, no necessarily expressed as functions, provide a simple means for knowledge-based evaluation of the liquefaction potential. The newly developed simplified method compares favorably to a widely used existing methods.

RÉSUMÉ
Ce document présent une nouvelle simplifiée méthode pour évaluer la résistance de liquéfaction des sols basée dans des paramètres géotechniques, géométriques et de charge sismique. Une base de donnés relativement large constituée par des mesures CPT, STP et V_s et des observations de performance de liquéfaction de terrain des séismes a été analysée. Cette information est utilisée pour construire un environnement pas-linéal où l'occurrence et la non-occurrence de la liquéfaction peut été prévue en utilisant des outils de *Machine Learning*. Le schème traîné avec de succès et prouvé est composé de i) un réseau de neurones artificiel pour tracer quelques propriétés index à valeurs de résistance, ii) un système neuro-brouillé pour estimer l'occurrence de liquéfaction et un index brouillé-multidimensionnel de liquéfaction et iii) un arbre de régression pour générer ou compléter l'information de la charge sismique. Les coordonnés, mesurées et estimées, collectivement défient la surface frontière de liquéfaction non-linéale multidimensionnelle, la fonctionne brouillé d'état limite. En se basant dans cette récemment développé méthode, des analyses intelligentes des cas dans la base de donnés ont été réalisés en utilisant une approximation de mapping simple. Les *machine learning* modéles, pas nécessairement expressés comme fonctionnes, donnent des moyens simples pour l'évaluation, en se basant dans la connaissance du potentiel de liquéfaction. La récemment développée simplifiée méthode est comparable à des autres méthodes déjà existantes et largement utilisées.

1 INTRODUCTION

Over the past forty years, scientists have conducted extensive research and have proposed many methods to predict the occurrence of liquefaction. In the beginning, undrained cyclic loading laboratory tests had been used to evaluate the liquefaction potential of a soil (Castro et al., 1982) but due to difficulties in obtaining undisturbed samples of loose sandy soils, many researchers have preferred to use *in situ* tests (Seed et al., 1983).

In a semi-empirical approach the theoretical considerations and experimental findings provides the ability to make sense out of the field observations, tying them together, and thereby having more confidence in the validity of the approach as it is used to interpolate or extrapolate to areas with insufficient field data to constrain a purely empirical solution.

Empirical field-based procedures for determining liquefaction potential have two critical constituents: i) the analytical framework to organize past experiences, and ii) an appropriate in-situ index to represent soil liquefaction characteristics. The original simplified procedure (Seed and Idriss 1971) for estimating earthquake-induced cyclic shear stresses continues to be an essential component of the analysis framework. The refinements to the various elements of this context include improvements in the in-situ index tests (e.g., SPT, CPT, BPT, V_s), and the compilation of liquefaction/no-liquefaction cases.

The objective of the present study is to produce a simplified machine-learning ML method for evaluating liquefaction potential. ML, a branch of cognitive computation, is a scientific discipline concerned with the design and development of algorithms that allow computers to evolve behaviours based on empirical data, such as from sensor data or databases. Data can be seen as examples that illustrate relations between observed variables. A major focus of ML research is to automatically learn to recognize complex patterns and make intelligent decisions based on data; the difficulty lies in the fact that the set of all possible behaviours given all possible inputs is too large to be covered by the set of observed examples (training data). Hence the learner must generalize from the given examples, so as to be able to produce a useful output in new cases.

In this investigation, and following the findings of a previous work (García et al., 2010), some ML tools (Neural Networks NNs, Fuzzy Logic FL, and Regression Trees RTs) are used to evaluate liquefaction potential and to find out what parameters control liquefaction occurrence including: earthquake parameters, soil properties, and stress conditions. For each of these parameters, the emphasis has been on developing relations that capture the essential physics while being as simplified as possible. The proposed cognitive environment permits an improved-simple definition of i) the *loading* to a soil induced by an earthquake (the cyclic stress ratio CSR), and ii) the *resistance* of the soil to triggering of liquefaction (the cyclic resistance ratio CRR).

2 LIQUEFACTION POTENTIAL- AN OVERVIEW

The factor of safety FS against the initiation of liquefaction of a soil under a given seismic loading is commonly described as the ratio of cyclic resistance ratio (CRR), which is a measure of liquefaction resistance, over cyclic stress ratio (CSR), which is a representation of seismic loading that causes liquefaction, symbolically, FS=CRR/CSR. The reader is referred to Seed and Idriss (1971), Youd et al. (2001), and Idriss and Boulanger (2004) for a historical perspective of this approach. The term CSR

$$CSR = f(0.65, \sigma_{vo}, a_{max}, \sigma'_{vo}, r_d, MSF) \qquad [1]$$

is function of the vertical total stress of the soil σ_{vo} at the depth considered, the vertical effective stress σ'_{vo}, the peak horizontal ground surface acceleration a_{max}, a depth-dependent shear stress reduction factor r_d (dimensionless), a magnitude scaling factor MSF (dimensionless). For CRR, different in situ-resistance measurements and overburden correction factors are included in its determination; both terms operate depending of the geotechnical conditions. Details about the theory behind this topic in Idriss and Boulanger,(2004) and Youd *et al.* (2001).

2.1 Correction Framework for Semi-empirical Procedures

Talking about CSR, the stress reduction coefficient accounts for the flexibility of the soil column (e.g., $r_d = 1$ corresponds to rigid body behavior). The factor of 0.65 is used to convert the peak cyclic shear stress ratio to a cyclic stress ratio that is representative of the most significant cycles over the full duration of loading. The values of CSR calculated using equation (1) pertain to the equivalent uniform shear stress induced by an earthquake of magnitude M (moment magnitude). It has been customary to adjust these values so that they would pertain to ground motions generated by an earthquake having a $M = 7.5$.

On the other hand, for CRR, the purpose of the overburden normalization is to obtain quantities that are independent of σ'_{vo}, and thus more uniquely relate to the sand's relative density. The correlation of the cyclic stress ratio required to cause liquefaction (CRR) to in-situ resistance is thus directly affected by the choice of the correction expression, as has been illustrated for many researchers (Idriss and Boulanger, 2004).

The correction factors have been included in the conventional analytical frameworks to organize and to interpret the historical data. The correction factors improve the consistency between the geotechnical/seismological parameters and the observed liquefaction behavior, but they are a consequence of a constrained analysis space: a 2D plot [CSR *vs* CRR] where regression formulas (simple equations) intend to relate complicated nonlinear/multidimensional information. In this investigation the ML methods are applied to discover unknown, valid patterns and relationships between geotechnical, seismological and engineering descriptions using the relevant available information of liquefaction phenomena (expressed as empirical prior knowledge and/or input-output data). These ML techniques "work" and "produce" accurate predictions based on few logical conditions and they are not restricted for the mathematical/analytical environment. The ML techniques establish a *natural* connection between experimental and theoretical findings.

3 EVALUATION OF THE LIQUEFACTION POTENTIAL: A ML REFORMULATION

The study of the liquefaction is made based on information that is incomplete, often ambiguous, plagued with imperfect or inexact knowledge, and it involves the handling of large sets of competing constraints that can tolerate close enough solutions. The outcome (evaluation of the liquefaction potential) depends on many inputs and their statistical variations, and there is no clear logical method for arriving at the answer. The seismic-related phenomena are inherently ill-posed and ill-conditioned so they cannot be satisfactorily addressed using the traditional computational paradigms.

Following the format of the simplified method pioneered by Seed and Idriss (1971), in this investigation a nonlinear and adaptative *limit state* (a fuzzy-boundary that separates liquefied cases from nonliquefied cases) is proposed (Figure 1).

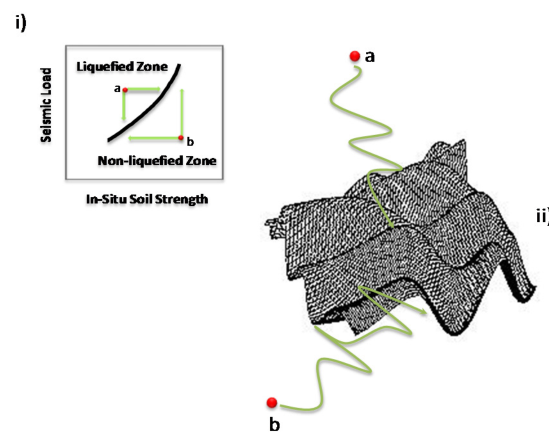


Figure 1. The schematic view of the nonlinear-multidimensional limit state: i) the traditional graph, ii) the proposed ML surface

3.1 Data base used in the re-formulation

The database used in the present study was constructed using the information included in Table 1. The information was compiled by Aghda et al., (1988), Juang et al., (1999), Juang (2003), Baziar, (2003) and Chern and Lee (2009). A summary of the parameters included in these datasets is presented in Table 1. From the 1035 patterns, the contradictory cases and duplicated events have been removed. The final number of patterns is around 400 different cases, 70% of them are liquefied cases and the other 30% cases are nonliquefied ones.

The cases are derived from CPT, SPT and V_s measurements and different seismic conditions (U.S., China, Taiwan, Romania, Canada and Japan). The soils types ranges from clean sand and silty sand to silt mixtures (sandy and clayey silt). Diverse geological and geomorphological characteristics are included. The reader is referred to the citations in Table 1 for details.

Table 1. Databases used in this study

Set	Input Parameters	Number .of Patterns	Ref.
A	Z, Z_{NAF}; H, Soil Class, Geomorphological units, Geological units, Site amplification, a_{max}	56	Fatemi-Agdha et al., 1988
B	Z, q_c, F_s, σ_0, σ_0', a_{max}, M	21	Juang et al., 1999
C	Z, q_c, F_s, σ_0, σ_0', a_{max}, M	242	Juang, 2003
D	D_{50}, a_{max}, σ_0', σ_0, M, F_s, q_c, SPT, Z	170	Baziar, 2003
E	M, σ_0, σ_0', q_c, a_{max}	466	Chern and Lee, 2009
F	Z_{NAF}, Z, H, σ_0, σ_0', Soil Class, V_s	80	Andrus and Stokoe, 1997; 2000
	Total:	1035	

Symbology: Z: top of layer depth, Z_{NAF}: Water table depth, H:layer thickness, a_{max}: max acceleration PGA, q_c: cone penetration resistance, F_s: fine content, σ_0: total vertical stress, σ_0': effective vertical stress, M: magnitude, D_{50}: 50% part. Diameter, SPT: standard penetration test values, V_s: shear wave velocity.

3.2 Reformulation of CSR/CRR

There are many examples that show that biological machinery is capable of providing satisfactory solutions to ill-structured problems with remarkable ease and flexibility. Between these attempts, some NNs have been developed to determine the liquefaction potential but no significant results have been achieved because of the conflicting and redundant data and the difficult for detecting what examples are really contradictory or superfluous.

For example, the back-propagation algorithm (for training neural networks) is not flexible enough for prediction purposes, particularly when the data set contains inconsistent information as in the liquefaction database (the minimum-error searching diverges). In view of these shortcomings, it was deemed necessary to develop an alternative procedure. In this paper is proposed a neurofuzzy system.

The basic idea in this reformulation is merging fuzzy systems and neural networks to design a computing scheme that uses the FL to represent the data in an interpretable manner and the profits from the learning ability of a NN to optimize the empirical knowledge.

This blending constitutes a decoded model that is capable of learning and using problem-specific prior knowledge. The new formulation uses subjective categories to evaluate the CSR/CRR items and to derive the conclusion according to the experience.

The seismic load that causes liquefaction is expressed thorough eleven items that were selected from a comprehensive review of the literature, past experience, and engineering judgment. Each item has 2-3 fuzzy-labels. The seismic and geotechnical characteristics constitute the mesh where the different positions of CSR can be determined.

Once the variables (input/output) were chosen and the fuzzy sets to represent these variables were picked, the fuzzy rules and the membership functions are modified by the Takagi-Sugeno neurofuzzy process (see Sugeno and Kang, 1998; and Takagi and Sugeno 1985), resulting a fuzzy IF-THEN rules that model the qualitative analyses. The typical values of the labels as the if-then rules are changed in the neuro-training phase until the minimum difference between the evaluated and the observed behavior is achieved. The schematic representation of the liquefaction neurofuzzy model is shown in Figure 2. In the initial fuzzy system the following input linguistic variables were booked:

- Geotechnical – “qc_Cone”, “SPT-N”, “Velo_V_s”, “Volumetric_W”, “Soil_Class”, “Esf_total”, “Esf_effec”,
- Geometrical – “Layer_H”, “Z_{NAF}”, “Z_{TOP}”
- Sesimic Load – “Magnitude_M”, “Amax”, “Signal(t)”, “Signal(f)”

The resistance measurements “qc_Cone”/ “SPT_N”/”Velo_V_s” and the volumetric weight “Volumetric_W”, are opened for learning, the training process will modify the membership functions and the fuzzy rules defined by the experts until the system response is optimized, which is achieved when the system obtained minimizes the error mapping (observed *vs* estimated). The soil classification “Soil_Class”, total and effective stresses “Esf_total”/“Esf_effec” and the Geometrical variables – “Layer_H”, “Z_{NAF}”, “Z_{TOP}” are not opened for learning, they are control labels that characterize and help to relate the different databases.

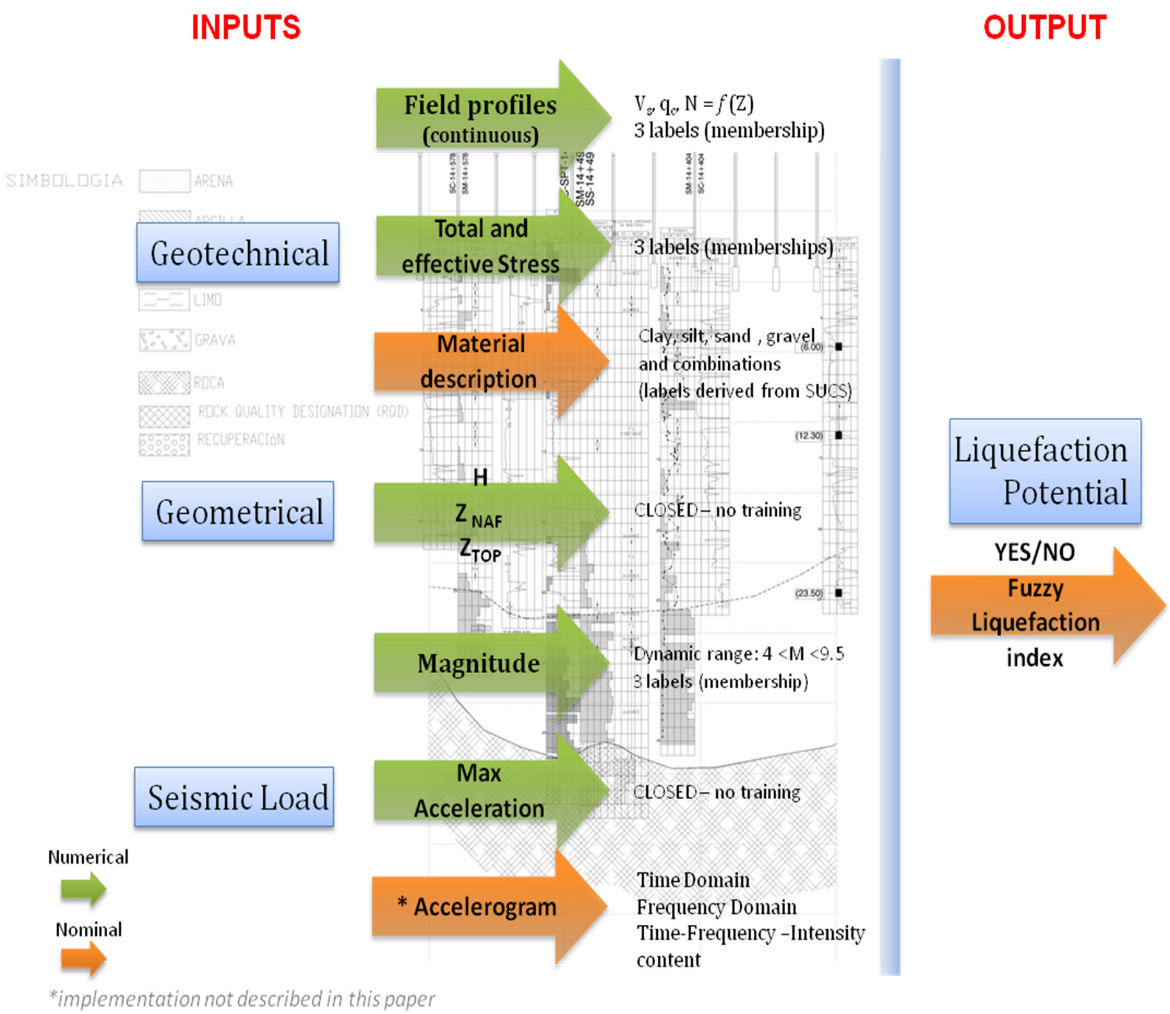


Figure 2. Liquefaction Potential Neurofuzzy Model

The seismic definition can be done through one of two categories: i) raw nominal values "Magnitude_M" and "A_max" or ii) a regression tree that estimates the peak ground acceleration PGA for different magnitudes, epicentral distances and focal depths, as being required in the analysis. In a posterior stage of this ML model (not presented here), the analyst will be able to load an acceleration vector (time or frequency domain, "Signal(t)"/"Signal(f)") to take into account the time/frequency/intensity event distinctiveness. Finally, the output variable is "Liquefies?" and it can take the linguistic values "YES"/"NO".

By definition, if the factor of safety against triggering liquefaction (FS=CRR/CSR) is less than 1, the occurrence of liquefaction is "predicted", no liquefaction is forecasting if FS.≥1. But using ML there are no simple equations for determining the nominal values of CRR and CSR and the FS. The resulting ML functions are multidimensional and nonlinear and a simple ratio between these two components is not adequate. In this investigation a fuzzy-liquefaction index LI_F is proposed for a lexical description of the liquefaction potential of the soils. Using the resulting rules that optimize the "IF CSR=X and CRR=Y, THEN the soil Liquefies?→YES"/"NO" mapping, a fuzzy linguistic conclusion, the index LI_F, is generated (Figure 3).

3.3 ML-data analysis

The numerical and categorical vectors are loaded into the inputs grid. The patterns in each mesh (geotechnical, geometrical and seismic) are fuzzyfied and the premises are constructed automatically. Warning messages are displayed about missed values or redundant/conflicting sentences and they can be estimated mechanically, the former, or removed, the latter. The rule interpretation is done by the Takagi-Sugeno-*inference mechanism* using the crisp inputs and outputs. It implements a nonlinear mapping from the input space to the output space. This mapping is achieved by a number of IF-THEN rules, each of which describes the local behavior of the mapping. In particular, the antecedent of a rule (statements before the conclusion THEN) defines a fuzzy region in the input space, while the consequent (conclusion, statement after THEN) is the output in the fuzzy region. For the ML scheme, a fuzzy IF-THEN rule with three groups of antecedents is written as

IF

Geotechnical:

qc_Cone is *LOW* and Volumetric_W is *MEDIUM* and Soil_Class is *SILTY_SAND* and Esf_total is.. and Esf_efec is …

Geometrical:

Layer_H= 10 m and Z_{NAF} =… and Z_{TOP}=…

Seismic Load:

Magnitude_M= 6.2 and A_max= …

THEN

Liquefies? is *NO*

When a prediction task is posed to the ML scheme, the rules R_i activated (significant for the specific inputs) are partially applied and the result of the inference is a combination of their propositions *i*. The labels for describing "Liquefies?" are sufficient to express the variations of the liquefaction potential between the whole range of soil materials, geometries and seismic loads.

3.4 The evaluation of recognition

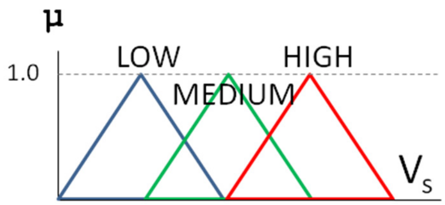
An example of the results obtained during testing with "unseen" data is presented in Figure 4. It is observed that the neurofuzzy system is capable of predicting the in situ measurements with a high degree of accuracy. Furthermore, and if the neurofuzzy results are compared with those obtained by commonly used semi-empirical methods (not shown here because of space limitations), we can conclude that the neurofuzzy system yields safety predictions using a significantly less expensive (faster and easier to get) seismic, geotechnical and geometrical descriptions.

An evaluation of the predicted "Liquefies?" versus the known output is shown in Figure 5. The goodness of this fit is reflected by the coefficient of determination is above 0.9. The overall uncertainty of this ML model, including additional error from the trained neural network, is estimated to be less than 10%. It should be noted that the idea of introducing the correction parameters in the traditional models, originated from concepts of equivalent resistances, seismic intensities, or soil classes, it is merely an intermediate adjustment derived from the conventional regression processes; no physical meaning is implied herein.

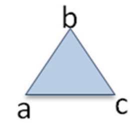
The CRR and CSR are not explicitly calculated when using neurofuzzy techniques; the inferred values are *compared* in the nonlinear environment and the behavior rules discovered are used to locate them in the complex-fuzzy space. Although in ML models it is not possible to locate a 2D position and to define a distance from the limit function (a line in a XY plot), the intelligent-empirical expression (via a fuzzy index) translates the multidimensional conditions that originates the output "Liquefies?YES/Liquefies?NO" into a linguistic label that better relates the phenomena with the human concept of risk.

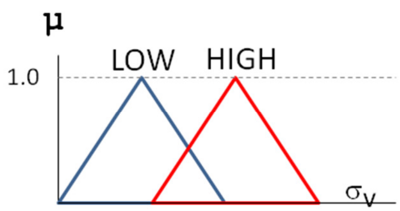
A prediction is considered a success if it agrees with the field observation. The success rate in predicting liquefied cases using the empirical model developed is 93%, whereas the success rate in predicting nonliquefied cases is 88%. The overall success rate is 91% for all cases in the database. The empirical method developed is shown to be quite accurate. In fact, it is approximately more accurate as the more sophisticated published NNs models (Juang et al., 2003; Aghda et al., 1998; Baziar and Nilipour, 2003). The ML model is much easier to use and it helps better to understand the conditions that makes a soil to trigger liquefaction, also it is very flexible and adaptative .The proposed ML method can be *calibrated* so that the meaning of the fuzzy-index calculated can be understood within the framework of probability.

INPUTS

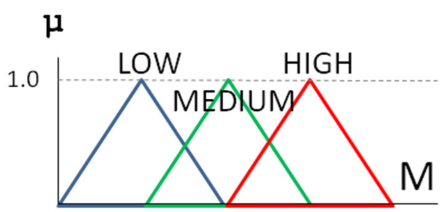


V_s (m/s)	a	b	c
LOW	20	54	85
MEDIUM	54	178	223
HIGH	178	290	370

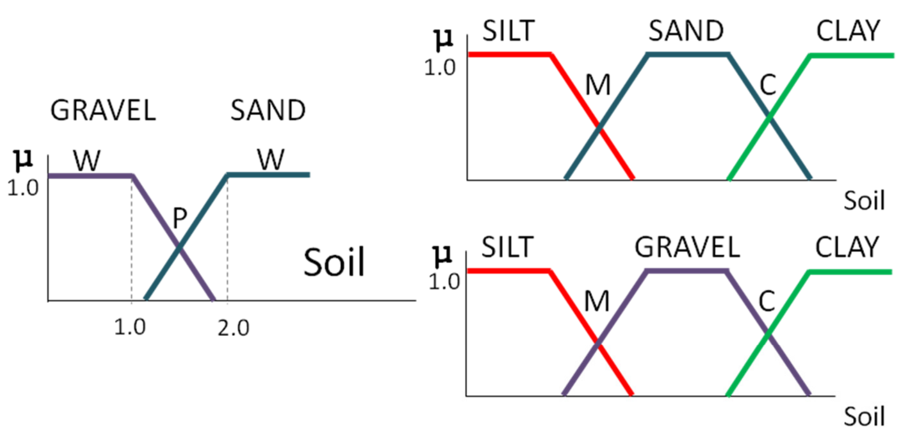




σ_v (kPa)	a	b	c
LOW	20	89	196
HIGH	92	212	323



M	a	b	c
LOW	3.0	4.7	5.5
MEDIUM	4.1	6.8	7.0
HIGH	6.4	8.0	9.8



OUTPUT

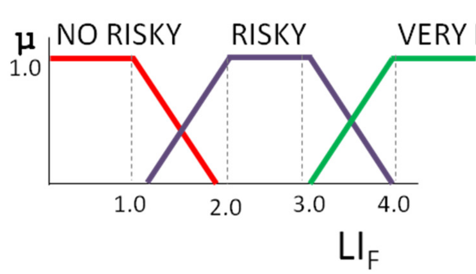


Figure 3. Concepts, labels, membership functions and their dynamic range

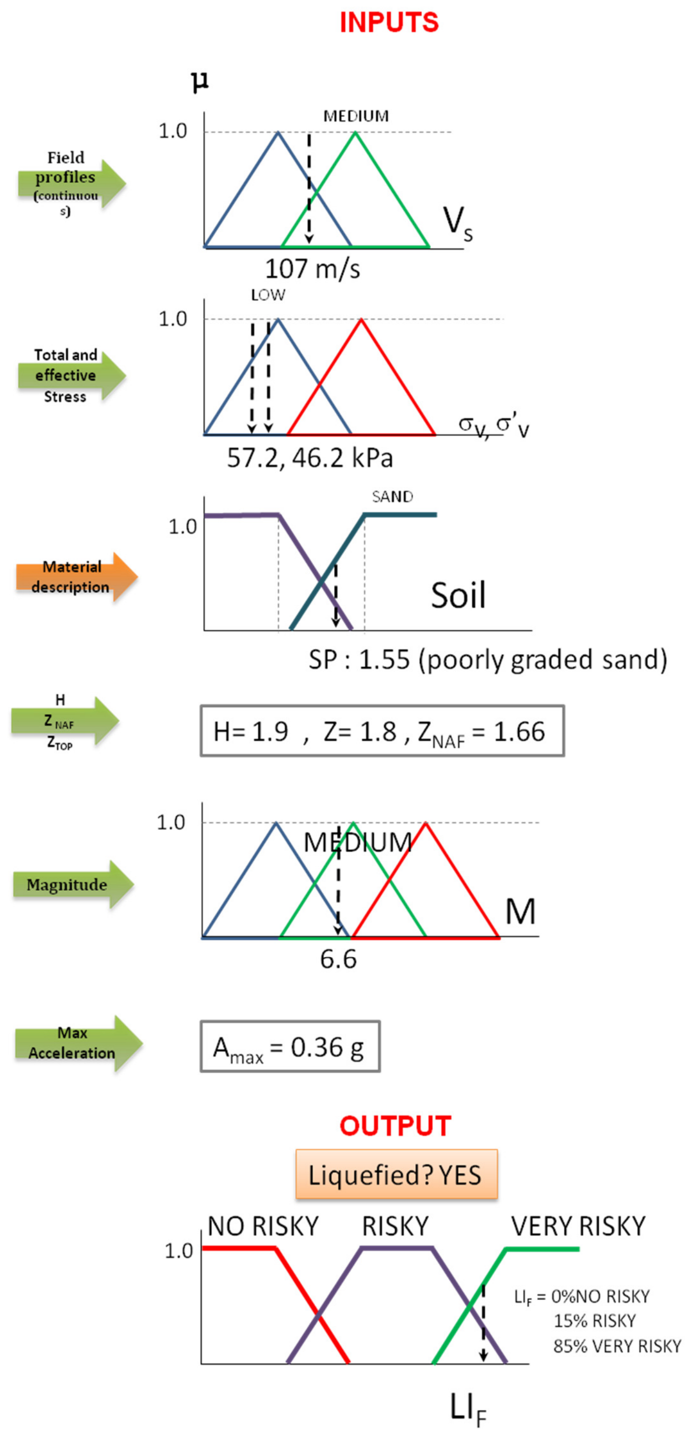


Figure 4. Neurofuzzy model: a calaculation example

Z_{NAF} (m)	Z (m)	H (m)	σ_v (kPa)	σ'_v (kPa)	Soil Type	V_s (m/s)	A_{max} (g)	**Liquefied?** 0- YES 1- NO Neuro Fuzzy	Observed
1.7	1.8	1.9	57.2	46.2	Sandy	107	0.36	0	0
1.5	1.5	2.8	52.7	40.5	Gravel	94	0.36	0	0
1.4	1.4	1.8	44.5	36		102	0.36	0	0
1.8	1.8	2.8	62.1	49.4	Gravel	109	0.36	0	0
1.5	1.5	1.9	60.5	45.6	Sandy	122	0.36	0	0
2	2	1.7	57.5	46.3		134	0.36	0	0
1.5	1.5	1.7	38.8	32.9		128	0.36	0	0
1.5	1.5	1.9	38.4	32.4		107	0.36	0	0
1.5	1.5	1.7	39.4	33.8		131	0.36	0	0
1.7	1.7	1.5	43.3	38.3		122	0.36	0	0
1.5	1.5	2.3	48.5	38.1		154	0.36	0	0
1.2	1.2	2	47.3	36	Sandy Gravel	122	0.3	1	0
1.2	1.2	2	41.1	32.7	Sandy Gravel	105	0.3	0	0
0.8	0.8	2.4	40.6	28.7	Sandy Gravel	106	0.29	0	0
0.8	0.8	2.4	39	27.8		105	0.29	0	0
0.8	2.2	1.3	59.9	39	Sandy-silty Gravel	176	0.5	0	0
0.8	2.2	1.3	55.4	38.4		153	0.5	0	0
0.8	2.2	1.3	59.9	40.5		183	0.5	0	0
0.8	1.8	2.2	59.1	38.2	Sandy-clayey Gravel	181	0.5	0	0
0.8	1.8	2.2	45.6	31.7	Silty Gravel	210	0.5	0	0
1	1.8	1.2	51	36	Sandy Gravel	206	0.46	1	1
3	3	1.3	75.2	53.5	Sandy Gravel	274	0.46	1	1
2.3	2.3	2.7	66.6	57.4	Sandy-silty Gravel	271	0.23	0	1

Figure 5. Work stage of ML model : predictions on “unseen” cases

4 CONCLUSIONS

Machine learning is used in the study to evaluate liquefaction potential subjected to earthquake loadings. To achieve this object, a total 400 limit state patterns were analyzed. These data patterns are then used to construct the cyclic stress ratio CSR/cyclic resistance ratio CRR fuzzy-space (concepts, labels, and categories). Subsequently, *Takagi-Sugeno* inference is used to determine a set of behavior rules that relates the inputs to the output. The computed liquefaction resistance is compared with earthquake induced cyclic stress to decide whether liquefaction is occurred or not. Finally, based on this empirical-intelligent scheme, a fuzzy liquefaction index can be computed, the system is designed to give an additional definition through a linguistic label. The proposed simplified procedure is illustrated with the help of case studies. From the comparison results, it is found that the developed system may provide a very simple and accurate method with success rate as high as nearly 90% for assessing liquefaction potential.

Machine learning represents a powerful alternative in predicting the liquefaction potential and more accurate results than the conventional methods are obtained. No calibration and normalization with respect to the other parameters is needed. Also the relative importance of the effective parameters can be compared. Using normalized and calibrated parameters make the data set noisy and are not suitable for the training process. In other words, ML has the ability to find the relations between basic parameters of a multifactor problem, helping to achieve better results.

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