# Coupling Heat and Moisture Flow for the Computation of Actual Evaporation

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# ABSTRACT

Numerous analytical procedures have been proposed for the calculation of *Potential Evaporation, PE*, from a saturated soil surface with ample free water. However, it is the *Actual Evaporation, AE*, which is required for the design of nearground surface structures such as soil cover systems. *Actual Evaporation* is generally less than *Potential Evaporation* because the suction in the soil decreases the evaporation rate. Wilson et al. (1994) proposed a coupled heat and moisture flow in order to compute *Actual Evaporation*. (i.e., *Modified Wilson-Penman* equation). Later, other procedures were proposed for the calculation of *Actual Evaporation*. One method is known as the *Limiting Function* procedure for *Actual Evaporation*. Another procedure is known as the *Experimental Based* procedure for *Actual Evaporation*. Each of the above-mentioned procedures can be solved in a "coupled" or "uncoupled" manner. This paper compares the theory and the solutions for several procedures that can be used to calculate *Actual Evaporation*, *AE*.

## SOMMAIRE

Plusieurs procédés d'analyse ont été proposés pour le calcul du potentiel d'évaporation des sols saturés avec une abondante quantité d'eau. Mais, c'est l'évaporation actuelle qui est requise pour les plans structural à ras de terre comme les systems de protection du sol. En general, l'évaportion actuelle est moindre que le potentiel d'évaporation parce que la perméabilité du sol diminue le taux d'évaporation. Wilson et al (1994) propose l'utilisation de la circulation de la chaleur et de l'humidité simultanément pour le calcul d'évaporation actuelle (i.e., équation modifiée de Wilson-Penman). D'autres procédés incluant la fonction limitée pour l'evaporation actuelle et le procédé expérimental d'évaporation actuelle, ont été proposés plus récemment. Chacun des procédés cités peut être résolue simultanément ou séparément. Cette étude compare la théorie et les résultats pour les procédés utilisés pour calculer l'évaporation actuelle.

## 1 INTRODUCTION

Actual Evaporation from a soil surface can be considerably less than Potential Evaporation. The quantification of Actual Evaporation at a particular site is of interest for establishing the water balance at the ground surface for geotechnical engineering applications. The concepts behind the evaporation of water from the soil surface can be described as follows. Net radiation from the sun heats the ground surface and the air above the ground surface. Wind provides a mixing effect of the air near to the ground surface and a removal of water vapour from near ground surface. At the same time when the sun and the wind are removing water vapour from the ground surface, the soil is holding onto and storing water in the pores (i.e., soil suction). It is also hard to deliver water to the ground surface because the permeability of the unsaturated soil can become extremely low. The net result of the competition between the weather-related factors and suction in the soil is a reduction in evaporation from Potential Evaporation conditions to the net moisture flux or Actual Evaporation.

Vapour pressure gradients constitute the fundamental driving mechanism for vapour flow, (Edlefsen and Anderson 1943). Weather conditions above the ground surface create a relative humidity (or vapour pressure) condition in the air immediately above ground surface. The air in the soil at ground surface also has a relative humidity that is related to total suction through thermodynamic considerations. The difference in vapour pressure between the soil and the overlying air provides the vapour pressure gradient for *Actual Evaporation*. If the vapour pressure in the air above ground surface and the vapour pressure in the soil at ground surface are the same, then evaporation from the ground surface will cease since there is no longer a vapour pressure gradient. *Actual Evaporation* can be significantly influenced by the "total suction", (i.e., metric suction plus osmotic suction) in the soil at the ground surface.

Geotechnical engineers are interested in calculating *Actual Evaporation* for the determination of water fluxes and water balances at the ground surface. This paper focuses on three equations that have been proposed for the calculation of *Actual Evaporation*. All three equations are the outcome of research by Wilson (1990) who used evaporation from thin soil layers and sand column drying tests to test theoretical evaporation considerations. The results of one of the methods were used to extend the Penman (1948) equation. The extended method is

referred to as the Wilson-Penman equation for Actual Evaporation.

A second methodology, also based on thermodynamic considerations is referred to as the *Limiting Function* equation for *Actual Evaporation* (Wilson *et al.*, 1994). This equation calculates *Actual Evaporation* based on different assumptions related to temperature conditions above the soil surface and at the soil surface.

A third methodology was based on laboratory measurements of evaporation rates from sand, silt and clay soils. The method is referred to as the *Experimental-Based* equation, (Wilson et al. 1994).

One of the main differences in the above-mentioned methodologies is the assumption made regarding the air and soil temperatures. There are different procedures for handling the difference between the soil and air temperatures and this gives rise to "coupled" and "uncoupled" moisture/heat flow formulations. The computational times differ for each of the analyses. The temperature of the soil and the temperature of the air are generally different; however, the temperature effect on the difference between a "coupled" and "uncoupled" moisture/heat flow analysis is not clearly understood for various climatic conditions. A flowchart of the six procedures that can be used to solve the suggested methodologies is presented in Figure 1. It is also possible to use other proposed methods for the calculation of Potential Evaporation and then use the results in conjunction with the procedures proposed for the calculation of Actual Evaporation. However, these solution procedures are considered to be outside the scope of this paper.

#### 2 MOISTURE AND HEAT BALANCE EQUATIONS

The moisture mass balance at ground surface,  $q_g$ , is defined as,

$$q_g = P - AE - R \tag{1}$$

where  $q_g$  = moisture flow rate at the soil surface, *mm/day*, P = precipitation flux. *mm/day*, R = water runoff, *mm/day*, and AE = Actual Evaporation, *mm/day*.

The heat flux balance at ground surface,  $Q_G$ , is defined as,

$$Q_G = Q_N - Q_H - Q_L$$
 [2]

or

$$Q_G = Q_N - Q_H - L_v * AE/1000$$
[2a]

and

$$Q_H = L_v C_f \eta f(u) (T_s - T_a) / 1000$$
 [2b]

where  $Q_G$  = heat flux at the ground surface,  $J/(m^2-day)$ ,  $Q_N$  = net radiation,  $J/(m^2-day)$ ,  $Q_H$  = sensible heat flux,  $J/(m^2-day)$ , and  $Q_L$  = latent heat flux due to water evaporation,  $J/(m^2-day)$ ;  $L_v$  = volumetric latent heat of vaporization,  $J/m^3$ ;  $T_s$  = soil surface temperature,  $^{\circ}C$ ;  $T_a$  = air temperature,  $^{\circ}C$ ;  $\eta$  = psychrometric constant,  $kPa/^{\circ}C$ ; f(u) = function depending wind speed, mm/day-kPa;  $C_f$  = conversion factor, (i.e., 1 kPa = 0.00750 mHg).



Figure 1 Methodologies suggested for "coupled" and "uncoupled" solutions of moisture/heat flow equations for Actual Evaporation.

# 3 MODIFIED WILSON-PENMAN EQUATION FOR COMPUTING ACTUAL EVAPORATION

The modified Wilson-Penman (1990) equation took into consideration the difference in temperature and relative humidity (and therefore vapour pressure) between the soil surface and the overlying air. The difference in temperature conditions formed the basis for the development of the *Soil-Atmospheric Model* which was subsequently implemented into the *SoilCover* computer code (1994).

The partial differential equation governing moisture and vapour flow equation, (Wilson 1990; Gitirana 2005) is given by:

$$\frac{\partial}{\partial y} \left[ k_{11}^{y} \frac{\partial(u_{w})}{\partial y} + k_{y}^{w} + k_{12}^{y} \frac{\partial T}{\partial y} \right] + S_{sk} = \frac{\partial \theta_{u}}{\partial t} + \frac{\rho_{i}}{\rho_{w}} \frac{\partial \theta_{i}}{\partial t}$$
[3]

The partial differential heat flow beneath the soil surface is modified based on the equation given by Jame (1977), Jame and Norum (1980); Wilson et al., (1994); Gitirana (2005) to include soil freezing/thawing process:

$$\frac{\partial}{\partial y} \left[ k_{2i}^{y} \frac{\partial (u_{w})}{\partial y} + k_{2i}^{y} \frac{\partial T}{\partial y} \right] = (C + L_{f} m_{2}^{i}) \frac{\partial T}{\partial t}$$
[4]

where:  $\begin{aligned} k_{11}^{y} &= \frac{k_{y}^{w} + k^{vd}}{\gamma_{w}} \\ ; \\ k_{21}^{y} &= \frac{(L_{f}k_{y}^{w} + L_{v}k^{vd})}{\nu} \\ ; \\ k_{22}^{y} &= \lambda_{y} + L_{v}k^{vT} + L_{f}k_{y}^{LT} \end{aligned}$ 

where u = pore-water pressure which is part of hydraulic head, kPa; T = temperature,  $^{\circ}C$ ; t = time, s;  $k_{i}^{i}$  = hydraulic conductivity, m/s;  $k^{vd}$  = pore-water vapour conductivity by vapour diffusion within the air phase, m/s;  $k^{vT}$  = vapour diffusion due to temperature gradient, *m*/s;  $k_{u}^{LT}$  = hydraulic conductivity related to temperature gradient, *m/s;*  $\rho_{w}$  = water density, *kg/m*<sup>3</sup>;  $\rho_{i}$  = ice density,  $kg/m^3$ ;  $\gamma_w =$  unit weight of water,  $kN/m^3$ ;  $\theta_u =$  volumetric unfrozen water content,  $m^3/m^3$ ;  $\theta_i$  = volumetric ice content,  $m^3/m^3$ ;  $\lambda$  = thermal conductivity, J/s-m-°C; S<sub>sk</sub> = water sink or source;  $L_{u}$  = volumetric latent heat of vaporization or condensation,  $J/m^3$ ;  $L_v = 2.5 \times 10^9 J/m^3$  if  $T > T_{ef}$ , otherwise  $L_v = 0$ ;  $L_f$  = volumetric latent heat of water freezing or thawing,  $J/m^3$ ;  $L_f = 3.34 \times 10^8$   $J/m^3$  if  $T_{ef} >= T >= T_{ep}$ , otherwise  $L_f = 0$ ;  $T_{ef}$  = temperature at soil freezing point, °C; Tep = the temperature at which soil phase change is finished during freezing, °C; C = volumetric heat capacity of material,  $J/m^3$ ; and  $m_2^i$  = slope of the Soil-Freezing Characteristic Curve, SFCC,

(i.e., relationship between unfrozen water content and soil temperature).

## 3.1 Determining Vapour Pressure during a Day

The saturated vapour pressure for the air,  $u_{v^0}^{air}$  can be calculated based on air mean temperature, (Lowe 1977) and the water vapour pressure in the air,  $u_v^{air}$ , is defined as follows.

$$u_v^{air} = u_{v0}^{air} h_r$$
<sup>[5]</sup>

where  $h_r$  = relative humidity of the air above the soil surface.

The relative humidity,  $h_r$ , of the air above the soil surface is measured at a weather station. The relative humidity at the soil surface,  $h_s$ , and the water vapour pressure,  $u_v^{soil}$ , at the soil surface are calculated using the relative humidity equation proposed by Edlefsen and Anderson equation (1943):

$$h_{s} = \frac{u_{v}^{soil}}{u_{vo}^{soil}} = \exp\left(\frac{\psi g \,\omega_{v}}{\gamma_{v} R(273.15 + T_{s})}\right)$$
[6]

where  $u_{vo}^{soll}$  = saturated vapour pressure at the soil surface, kPa;  $\psi$  = total suction, kPa, (total suction is matric suction plus osmotic suction, (i.e.,  $\psi = (u_a - u_w) + \pi$ )),  $\omega_v$  = molecular weight of water, 0.018 *kg/mol*, *g* = gravity acceleration,  $m/s^2$ ;  $\gamma_w$  = unit weight of water,  $kN/m^3$ ; *R* = universal gas constant, 8.314 *J/(mol.K)*,  $T_s$  = soil surface temperature,  $^cC$ ;  $u_a$  = pore-air pressure, kPa,  $u_w$  = pore-water pressure, kPa, and  $\pi$  = osmotic suction, kPa.

The osmotic suction in a soil is related to the salt content. Osmotic suctions may range from less than 100 kPa up to 10,000 kPa or more. As a soil dries the salt content increases and may also accumulate as a salt crust on the surface of the soil, (Fredlund 1991). When the total suction in soil is less than about 3,000 kPa, the relative humidity at the soil surface stays near 100%. Under these conditions the Actual Evaporation is approximately equal to the Potential Evaporation, (i.e., AE = PE).

The vapour pressure in the soil,  $u_v^{soil}$ , used in the diffusive flow equation and the heat flow equation can be calculated using the following equation:

$$u_v^{soil} = u_{vo}^{soil} h_s = u_{vo}^{soil} \exp(\frac{\psi g \omega_v}{\gamma_w R(273.15 + T_s)})$$
[7]

The saturated vapour pressure,  $u_{vo}^{soil}$ , is a function of soil surface temperature, (Lowe 1977). The soil surface temperature,  $T_s$ , can either be calculated through use of a coupled heat and moisture flow analysis or through use of an approximation of the relationship between the air and soil temperatures. The manner in which the soil temperature is determined largely controls whether a coupled or uncoupled analysis is performed.

The total suction,  $\psi$ , is comprised of a negative porewater pressure (i.e., one component of matric suction) and the osmotic suction component,  $\pi$ . The negative pore-water pressure component is obtained from the solution of the partial differential equation governing moisture and vapour flow. However, the osmotic component is often ignored. This may not be a reasonable assumption when dealing with soil at the ground surface because salts tend to accumulate at the surface of the soil while water evaporates into the atmosphere.

Later, it will be shown that it was necessary to make an empirical assumption regarding the total suction at the soil surface in order to obtain a "match" between the theoretical formulation solution and experimental results. The negative pore-water pressure was arbitrarily changed by raising it to an exponential power (i.e., an adjustment factor,  $\delta$ ) as shown in the following equation.

$$u_{wa} = u_{w0} \times 10^{-\delta}$$
 [8]

where  $u_{wa}$  = adjusted negative pore-water pressure,  $u_{wo}$  = original pore-water pressure computed from the partial differential equation governing moisture and vapour flow, and  $\delta$  = empirical adjustment factor.

The adjustment of the negative pore-water pressures from the partial differential equation governing moisture and vapour flow must be made for all coupled and uncoupled solutions. The adjustment of the calculate pore-water pressures provided close simulations with the experimental results; however, it is acknowledged that there is need for a better way in which to accommodate the effects of osmotic suction and salt accumulation at the soil surface.

#### 3.2 Coupled Wilson-Penman Solution

The coupled moisture and heat flow formulation assumed the following conditions for above freezing conditions:

- 1.) Moisture and vapour flow beneath the soil surface were governed by hydraulic head gradients, vapour pressure gradients and temperature gradients.
- Heat transfer beneath the soil surface (i.e., ground thermal flux in equation) is mainly governed by thermal conduction. Heat transfer by thermal convection is generally negligible.
- 3.) Soil freezing/thawing processes can be considered when the soil temperature falls below the freezing point. In the frozen soil region of the soil profile, the hydraulic conductivity is reduced because of the existence of ice in the freezing zone.
- The latent heat due to evaporation and freezing/thawing phase changes can be taken into consideration.
- 5.) The soil temperature at soil surface, however, can be different from the air temperature. The heat exchanged between the air and soil surface is consistent with the convective heat flow law shown in the following equation.

When performing coupled modeling of heat and water flow, the soil temperature is solved using the heat flow equation with the application of appropriate boundary conditions. The boundary condition when solving the coupled model is the ground surface flux designated by Eq. 2a. The ground surface temperature is computed using the convective heat flow equation.

Alternatively, the ground surface temperature can be used as the temperature boundary condition. In this case, the ground surface temperature can be computed by the following equation which includes the ground thermal flux.

$$T_{s} = T_{a} + \frac{1}{C_{f} \eta f(u)} \left( Q_{n} - AE - \frac{1000Q_{G}}{L_{v}} \right)$$
[9]

where  $T_s$  = soil temperature at soil surface, C;  $T_a$  = air temperature, C;  $C_f$  = = conversion factor, (i.e., 1 *kPa* = 0.00750 *mHg*);  $\eta$  = psychrometric constant, 0.06733 *kPa*/C; f(u) = function depending wind speed, f(u) = 0.35 (1. + 0.146 *W<sub>w</sub>*), *mm/day-kPa*; *W<sub>w</sub>* = wind speed, *km/hr*;  $Q_n$  = net radiation, *mm/day*; *AE* = Actual Evaporation, *mm/day*,  $Q_G$  = heat flux balance at ground surface,  $J/(m^2-day)$ , with the factor, 1000 is the conversion from *m* to *mm*, and  $L_v$  = volumetric latent heat of vaporization,  $J/m^3$ . Equation 9 and 2a are equivalent but are presented in different mathematical formats.

Actual Evaporation, AE, in *mm/day* is calculated using the following Wilson-Penman (1994) equation.

$$AE = \frac{\Gamma Q_n + \eta E_a}{\Gamma + \eta / h_s}$$
[10]

where  $E_a$  = flux associated with "mixing", *mm/day*;  $\Gamma$  = slope of the saturation vapour pressure versus temperature curve,  $kPa/^{\circ}C$ ;  $Q_n$  = net radiation at the soil surface, *mm/day*; and  $\eta$  = psychrometric constant,  $kPa/^{\circ}C$ , equal to 0.06733  $kPa/^{\circ}C$ .

Net radiation,  $Q_n$ , in Eqs. 9 and 10 is expressed in terms of water flux, *mm/day*, while the net radiation,  $Q_{N}$ , in Eq. 2 is expressed as an energy flux,  $(J/m^2-day)$ . The two representation of net radiation can be converted from one format to the other format through use of the latent heat of water vaporization,  $L_{\nu}$ ,  $(J/m^3)$ , (i.e.,  $Q_n = 1000 Q_N/L_{\nu}$ ).

3.3 Uncoupled Modified Wilson-Penman Solution

The uncoupled solution of the modified Wilson-Penman (1997) equation can be achieved using the moisture flow partial differential equation along with the following assumptions.

- 1.) Moisture and vapour flow occurs through the unsaturated soil near to ground surface.
- 2.) Soil temperatures in the entire domain are the same, and are equal to the temperature at the soil surface. Therefore, ground thermal flux is neglected (i.e.,  $Q_G = 0$ ).

The ground surface temperature,  $T_{s}$ , can then be computed using following approximation suggested by Wilson (1990), and Gitirana (2005).

$$T_s = T_a + \frac{1}{C_f \eta f(u)} Q_n - AE$$
[11]

The ground surface temperature is approximated based on the assumption that  $Q_G = 0$ . Actual Evaporation, *AE*, in *mm/day* is calculated using the Wilson-Penman (1994) equation, (i.e., Eq. 10), and  $Q_n$  is net radiation expressed in *mm/day*.

## 4 LIMITING FUNCTION EQUATION FOR COMPUTING ACTUAL EVAPORATION

In 1997 a *"Limiting Function"* type relationship was proposed by Wilson, Fredlund and Barbour. The *"Limiting Function"* related *Actual Evaporation* and *Potential Evaporation* by scaling the vapour pressures associated with the relative humidity at ground surface and the relative humidity in the air above ground surface. Inherent in the derivation was the assumption that the air and soil temperatures were the same.

4.1 Coupled Limiting Function for Actual Evaporation

The *"Limiting Function"* solution is based on the following assumptions:

- 1.) The soil temperature at the ground surface is equal to the air temperature.
- The moisture flow and heat transfer beneath the ground surface are the same as in the fully coupled rigorous solution.
- 3.) Soil freezing/thawing processes can be considered when the soil temperature falls below the freezing point. The hydraulic conductivity is reduced because of the existence of ice in the freezing zone.
- 4.) The latent heat due to evaporation and freezing/thawing phase changes must be considered in the heat transfer beneath the ground surface.

Actual Evaporation, AE, in *mm/day*, is calculated using the following "*Limiting Function*" equation proposed by Wilson *et al.*, (1997):

$$AE = PE\left[\frac{u_v^{soil} - u_v^{air}}{u_{vo}^{soil} - u_v^{air}}\right]$$
[12]

where PE = Potential Evaporation, mm/day;  $u_v^{soil}$  = vapour pressure at the soil surface, kPa,  $u_{vo}^{soil}$  = saturated vapour pressure at ground temperature, kPa, (Lowe 1977), and  $u_v^{air}$  = vapour pressure in the air above ground surface, kPa. Potential evaporation, PE, can be computed using the Penman (1948) equation which was derived and verified by measurement of evaporation rate from the open-water pan as follows:

$$PE = \frac{\Gamma Q_n + \eta E_a}{\Gamma + \eta}$$
[13]

4.2 Uncoupled Limiting Function for Actual Evaporation

The "*Limiting-Function*" can be used in an uncoupled manner when solving for *Actual Evaporation*. In this case, the following assumptions are made.

- 1.) Liquid and vapour flow near through the soil are governed by hydraulic head gradients and vapour pressure gradients.
- 2.) The soil temperature in the entire domain is assumed to be the same and equal to the air temperature above the soil surface. (i.e., ground thermal flux is neglected,  $Q_c = 0$ ).

Actual Evaporation, AE, is calculated using the Limiting Function, (i.e., Eq. 12), proposed by Wilson et al., (1997).

# 5 EXPERIMENTAL-BASED EQUATION FOR COMPUTING ACTUAL EVAPORATION

Wilson (1990) also presented experimental results that showed a unique relationship between total suction at the soil surface and the ratio of *Actual Evaporation* to *Potential Evaporation*, *AE/PE*. In 1997, Wilson *et al.*, presented an equation that passed through the experimental data with reasonable fit. As a consequence, there was now another way to empirically relate *Actual Evaporation* and *Potential Evaporation*.

## 5.1 Coupled Experimental-Based Solution

The following assumptions are made for the coupled analysis:

- 1.) The temperature of the soil at ground surface is equal to the air temperature.
- 2.) The moisture flow and heat transfer beneath the ground surface are the same as in the fully coupled solution.
- 3.) The latent heat of evaporation is considered for heat transfer beneath the soil surface.

Actual Evaporation, AE, is based on the following empirical mathematical equation fit to the data shown in Fig. 2.

$$AE/PE = \exp(\frac{-\psi g \omega_v}{\zeta (1 - h_a) \gamma_w R(T_s + 273.15)})$$
[14]

where  $\psi$  = total suction, (i.e., matric suction plus osmotic suction), *kPa*;  $\omega_v$  = molecular weight of water, 0.018 *kg/mol*;  $\gamma_w$  = unit weight of water, 9.807 *kN/m*<sup>3</sup>; *g* = gravity acceleration, *m/s*<sup>2</sup>; *R* = universal gas constant, 8.314 *J/(mol.K)*; *T<sub>s</sub>* = soil surface temperature,  $C; \zeta$  = dimensionless empirical factor suggested to be 0.7; and *h<sub>a</sub>* = air relative humidity.

The curve on the right in Figure 2 is the Lord Kelvin equation where the ordinate is assumed to correspond to relative humidity. It is clear that the data points shown for sand, silt and clay soils lie to the left of the Lord Kelvin curve. The solid best-fit line through the data points has the same character as the Lord Kelvin equation; however, an empirical reduction factor has been added to the denominator of the equation. A value of 0.7 for the reduction factor results in a best-fit line through the experimental data points.

#### 5.2 Uncoupled Experimental-Based Solution

The uncoupled solution that utilizes the *Experimental-Based* Eq. 12, and requires the following assumptions:

- 1.) Moisture flow and vapour flow beneath soil surface are driven by hydraulic head gradients and vapour pressure gradients, respectively.
- 2.) The soil temperature is the same throughout the model domain and is assumed to be equal to the air temperature above the soil surface. In other words, the ground thermal flux can be neglected (i.e.,  $Q_G = 0$ ).

Actual Evaporation can be calculated using the empirical Experimental-Based equation (i.e., Eq. 14 proposed by Wilson *et al.*, (1997)



Figure 2: Experimentally measured ratios of *Actual Evaporation* to *Potential Evaporation* for sand, silt and clay.

# 6 CALCULATION OF ACTUAL EVAPORATION

Measurements of evaporation from a soil column were presented by Wilson (1990). The results can be used to compare the numerical calculations of *Actual Evaporation* obtained when using the SVFlux and SVHeat software. Figures 3 and 4 show the calculation of *Actual Evaporation* for the three uncoupled and coupled *Actual Evaporation* models; namely, the Wilson-Penman model, the *Limiting-Function* model, and the Experimental-Based model. The numerical result obtained by Wilson (1990) is also illustrated in Figures 3 and 4.

The thermal conductivity for Beaver Creek sand was calculated for the moisture and heat coupled model using the Johansen approach in which the thermal conductivity for the solid component was estimated to be 8.2 W/m-°C when the dry density was 1697 kg/m<sup>3</sup>. The calculated thermal conductivity for the Beaver Creek sand was 2.18 W/m-°C and the calculated heat capacity was 2.6 E+6 J/m<sup>3</sup>-°C.

It can be seen from Figures 3 and 4 that there is no significant difference in the prediction of *Actual Evaporation* between the uncoupled and coupled models. This may be due to the fact that the ambient air temperature does not undergo significant change during the test. The recorded temperatures varied from 37.4 °C to 38.1 °C during the experiment. Consequently, thermal transfer in the soil column was not a dominant term in the coupled modeling of *SVFlux* and *SVHeat*.

The Actual Evaporation calculated using the Wilson-Penman equation, and the Limiting-Function closely matches the experimental data. The Experimental-Based equation resulted in slightly under-predicted values for the measured Actual Evaporation. Figures 3 and 4 show that in the first 5 days the predicted evaporation was close to the Potential Evaporation, but the predicted values are slightly larger than the observed values.



Figure 3 Comparison of the measured evaporation with the simulated uncoupled result using *SVFlux*.

The over-prediction of *Actual Evaporation* in the first 5 days comes from the over-prediction of relative humidity at the soil surface. It can be seen from the Figure 2 that there is no significant decrease in relative humidity until the soil suction is larger than about  $3,000 \ kPa$ . This means that the soil should evaporate water at the potential rate of evaporation up to a suction value of  $3,000 \ kPa$ . However, this is not the case with a sand soil because the soil will go past residual water content conditions when the matric suction is well below  $3,000 \ kPa$ .



Figure 4 Comparison of measured evaporation with the simulated coupled model of *SVFlux* and *SVHeat*.

The prediction of relative humidity at the soil surface for sand appears to produce a surface resistance to evaporation, (Alvenas and Jansson 1997; Lee and Pielke 1992; Kondo and Saigusa 1992). To improve stability in the numerical solution and provide closer simulations of the experimental results, *SVFlux* introduced an adjustment factor to compensate for the vapour pressure calculations at ground surface. The adjustment factor appears to range from 0 to -2. An adjustment factor of -1.8 was used in this study. It is anticipated that the adjustment factor will vary for different soils with the most negative values applicable for coarse-grained soils.

Figure 5 presents the measured air temperature, and the temperature at the soil surface that is observed during experiment, (Wilson 1990), and is calculated in the simulation as well. The predicted temperature at the soil surface shows reasonable agreement with the measured values for the coupled models.

For uncoupled model using the Wilson-Penman formulation, there is a significant difference between the predicted and measurement values in the first 5 days. For uncoupling *Limiting-Function* and *Experimental-Based* models, the temperature at the soil surface is equal to the air temperature.

During the evaporation from the soil surface, a large amount of heat is absorbed. This is the latent heat of evaporation which has a value of about 2.5 x 109 J/m3. As a result, there is a significant drop in temperature at the soil surface in the first 5 days. With the decrease in the evaporation, the temperature at soil surface increases and becomes close to the air temperature when the evaporation is reduced.



Figure 5 Comparison of the predicted soil surface temperature with the measured values; the surface temperatures for the coupled model are equal to the air temperatures.

# 7 CONCLUSIONS

It has been demonstrated that there are a number of different solution procedures that can be used for the calculation of *Actual Evaporation* for a soil surface with soil suction. A total of six calculation procedures were described in this manuscript. The solution procedures were first separated into "coupled" and "uncoupled" solutions. The "uncoupled" procedures require considerably less computational effort than the "coupled" procedures. Then in each category, there were three procedures that arose out of the research by Wilson (1990).

All six calculation procedures gave quite similar results. It is difficult to say whether "uncoupled" solutions are satisfactory in all situations; however, there is sufficient evidence to support further comparative studies of all six procedures.

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