Numerical simulation of soil settlement in liquefiable grounds

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ABSTRACT
In the presented study, the effect of soil permeability coefficient on numerical modeling of soil settlement is investigated; accordingly, the capability of two different approaches (i.e. constant initial permeability and variable permeability) to simulate both ground settlement and pore pressure is studied using a fully coupled, dynamic, inelastic (*u-P*) formulation. In addition to this, two different versions of a critical state two-surface plasticity model, which provides a unified approaches for modeling the stress-strain response of sands for a wide range of confining stresses and soil densities at the pre-failure and post failure regimes, are used to provide realistic simulation of soil skeleton. To implement the mentioned procedures, two different finite element programs, OpenSees and PISA are employed. The accuracy of the numerical models is evaluated by the recorded results of a centrifuge test. Comparison of numerical and experimental results indicates that variation of permeability should be necessarily taken into account in the numerical models in order to achieve an acceptable simulation of both pore pressure and ground settlement.

RÉSUMÉ
Dans la présente étude, l'effet du coefficient de perméabilité du sol sur la modélisation numérique du tassement du sol est étudié ; en conséquence, les capacités de deux approches différentes (la perméabilité initiale constante et perméabilité variable) pour simuler le tassement de la terre et la pression intersticielle sont étudiées utilisant une formulation entièrement combinée, dynamique et non élastique. De plus, deux versions différentes d'un modèle de plasticité de deux surfaces d'état critique, qui fournit des approches unifiées pour modeler la réponse de contrainte-tension des sables pour un éventail d'efforts et de densités d'emprisonnement de sol aux régimes de pré-échec et d'échec de poteau, sont employées pour fournir une simulation réaliste du squelette de sol. Pour mettre en application les procédures mentionnées, deux différents programmes d'éléments finis, OpenSees et PISA sont utilisés. L'exactitude des modèles numériques est évaluée par les résultats enregistrés d'un essai fait avec centrifugeuse. La comparaison des résultats numériques et expérimentaux indique que la variation de la perméabilité devrait être nécessairement prise en considération dans les modèles numériques afin de réaliser une simulation acceptable de pression interstitielle et de tassement du sol.

1 INTRODUCTION

Liquefaction is usually accompanied with large amount of lateral and vertical ground deformations because excessive generation of pore pressure in the soil deposit leads to material softening and soil shear strength lost, and this results in permanent deformations of buildings and structures. These failures have been observed during large earthquakes such as Niigata 1964, Alaska 1964, San-Fernando 1971, Loma-Prieta 1989, Hyogoken-Nambu 1995.

Due to the significance of liquefaction phenomenon, research activities in two categories of physical and numerical investigation have been of interest.

A wide range of centrifuge and shaking table tests have been employed. The laboratory tests conducted on Nevada sand by the Earth Technology Corporation in the course of Verification of Liquefaction Analysis by Centrifuge Studies (VELACS) project is one of the comprehensive and well-known experimental studies in this field. Some of the researchers who have simulated liquefaction phenomenon by centrifuge test are listed for reference: Prevost et al. (1981); Heidari et al. (1982); Hushmand et al. (1988); Arulanandan and Zeng (1993); Stadler et al. (1993); Dobry and Tabaoda (1995); Dobry and Sharp (1999).

In addition to this, various numerical methods have been employed to simulate liquefaction phenomenon. Here, some numerical studies on simulating VELACS model No.1 are briefly explained (The focus of attention is on the applied constitutive model and proposed permeability coefficient (i.e. constant or variable)): Chan et al. (1993) used constant permeability and Pastor-Zienkiewicz model to simulate this phenomenon. Although the computed pore pressures were acceptable, the computed vertical displacements were too small (At the end of excitation the computed settlement was 5 cm; however, the measured value was 20 cm during the centrifuge test). Iai et al. (1993) employed a plasticity model defined in strain space accompanied with a fully undrained analysis to simulate liquefaction so dissipation of excess pore water pressure were not considered in their simulation. Also, Ishihara et al. (1993) concluded the same results using a hypo-plasticity model and a fully undrained analysis. Lacy (1993) applied a multi-yield surface plasticity model and predicted excess pore pressure well, but lateral displacement did not match with the experimental records (The computed values for lateral displacement were considerably larger than the measure values). Manzari and Arulanandan (1993) used variable permeability and a bounding surface constitutive model in their numerical simulation. In this study, excess pore pressure and settlement were predicted satisfyingly but lateral displacements were not simulated reasonably well. Elgamal et al. (2002) employed constant permeability and a multi-surface plasticity constitutive model. In this study, pore pressures and horizontal displacement were simulated accurately, but computed settlement was excessively smaller than experimental results. Taiebat et al. (2007) simulated excess pore pressure and settlement more acceptably compared to the previous studies in which soil permeability was assumed to remain constant (the initial value). This was because Taiebat et al. (2007) used an increased permeability equal to 4 times of the initial value together with an advanced critical state two-surface plasticity model.

In general, according to the previous numerical studies, the constitutive model for soil mass and permeability coefficient highly affect the obtained results of the numerical analysis. And, it is important to note that effects of soil permeability variation on the performance of liquefying soil have not been studied adequately, so far. Therefore, in this paper it is intended to investigate the effect of permeability coefficient on numerical simulation of liquefaction by using two different versions of an advanced critical state two-surface plasticity model.

2 NUMERICAL FORMULATION

A fully coupled formulation, namely *u-P* formulation, where pore pressures and displacements are computed simultaneously at each time step, is used to model the behavior of a two-phase porous medium of saturated soil. In this formulation balance for the soil–fluid mixture, momentum balance for the fluid phase, and finally mass balance for the whole system of soil and fluid are satisfied. The primary unknowns are displacement of solid phase (*u*) and pore fluid pressure (*P*). The *u-P* formulation is applicable for dynamic problems in which high-frequency oscillations are not important, such as soil deposit under earthquake loading. Using the finite element method for spatial discretization, the *u–P* formulation is as follows (Zienkiewicz and Shiomi 1984):

$$M\ddot{U} + \int_V B^T \sigma' \, dV - QP - f^{(s)} = 0 \qquad [1]$$

$$Q^T \dot{U} + HP + S\dot{P} - f^{(p)} = 0 \qquad [2]$$

Where M is the mass matrix, U is the solid displacement vector, B is the strain-displacement matrix, σ' is the effective stress tensor, Q indicates the discrete gradient operator coupling the motion and flow equations, P is the pore pressure vector, S is the compressibility matrix, and H is the permeability matrix. The vectors $f^{(s)}$ and $f^{(p)}$ include the effects of body forces, external loads and fluid fluxes.

Numerical integration of the above-mentioned equations and their finite element formulation are carried out in OpenSees and PISA finite element programs which are briefly described here:

PISA: The first version of this program was developed at the University of Alberta, known as SAGE. Later, a commercial version of this program was released with the name of PISA. Pak (1997) and shahir (2001) further increased the capabilities of this program by completing the formulation to simulate THM (Thermal Hydro-Mechanical) and dynamic problems. In this study, the finite element PISA program was modified to include the capability to consider variation of permeability coefficient during simulation of liquefaction.

OpenSees: This program is an object-oriented program for finite element analysis. The Open System for Earthquake Engineering Simulation (OpenSees) is an open-source finite element framework which has been developed at PEER (Pacific Earthquake Engineering Research Center). It is a comprehensive and continually developing software that is used for simulation of seismic response of structural and geotechnical systems.

It is to be noted that in this study, The numerical procedures are implemented employing two different Finite Element Software firstly to demonstrate the capability of two finite element programs in simulation of liquefaction and secondly to demonstrate that the final conclusions do not change by employing different Finite Element programs. Note that in this study the focus of attention is not on the comparison of capabilities of the employed constitutive models.

3 CONSTITUTIVE MODELING OF SAND BEHAVIOUR

The formulation of the model is based on two different versions of a soil plasticity constitutive model developed by Manzari and Dafalias (1997), Dafalias and Manzari (2004). The theory of bounding surface plasticity and general two-surface plasticity are the origins of the formulation of the model. The most striking feature of this model is its capability to utilize a single set of material parameters for a wide range of void ratios and initial stress states for the same soil. It is to be noted that the early version of the constitutive model (i.e. Manzari and Dafalias 1997) has been used in PISA program and the later (i.e. Dafalias and Manzari 2004) has been used in OpenSees program. Differences between the two models can be found in the paper by Dafalias and Manzari (2004).

The early version, developed by Manzari and Dafalias (1997), has 17 parameters divided into 5 categories based on their functions. The calibrated parameters of this model are shown in Table 1 for Nevada sand. However, the later version possesses 15 parameters divided into 6 categories based on their functions. These parameters are calibrated for Nevada sand by Shahir (2009) using tests performed by Earth Technology Corporation in the course of the VELACS project. The calibrated parameters are listed in Table 2.

4 NUMERICAL MODELING PROCEDURES

Results from centrifuge model test No. 1 from VELACS project, conducted by Taboada and Dobry (1995) at Rensselaer Polytechnic Institute (RPI), are used to demonstrate the capability of the numerical models for reliable analysis of dynamic response of a saturated soil layer.

Description of the centrifuge test: The soil profile consists of a uniform horizontal layer of Nevada sand with approximately 40% relative density. The soil layer is placed in a laminar box and is fully saturated with water, and its height is 10 meters in the prototype scale. A sketch of the laminar box and the instruments for the foregoing model test are shown in Figure 1. The laminar box is spun at a centrifugal acceleration of 50g resulting in prototype soil permeability equal to 50 times greater than the permeability of soil specimen; the model is simultaneously excited horizontally at the base with the target prototype accelerogram shown in Figure 2.

Table 1. Material parameters used for Manzari-Dafalias Model (Manzari and Dafalias 1997)

Parameter Function	Parameter Index	Value
Elasticity		
	$G_0(kPa)$	31400
	$k_0(kPa)$	31400
	a	0.6
Critical State		
	M_c	1.14
	M_e	1.14
	λ	0.025
	$e_{c\,ref}$	0.8
Dilatancy		
	A_0	0.6
	C_f	100
	F_{max}	100
Hardening		
	h_0	800
	m	0.05
	C_m	0
State Parameters		
	k_c^b	3.975
	k_e^b	2
	k_c^d	4.2
	k_e^d	0.07

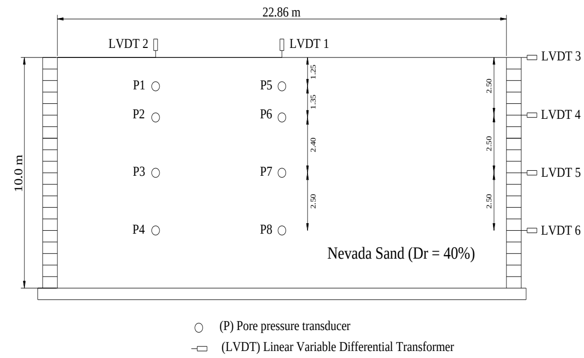

Figure 1. Schematic view of VELACS centrifuge model test No. 1 (Prototype scale).

Table 2. Material parameters used for Dafalias-Manzari Model (Dafalias and Manzari 2004)

Parameter Function	Parameter Index	Value
Elasticity		
	G_0	150
	ν	0.05
Critical State		
	M	1.14
	c	0.78
	λ_c	0.027
	e_0	0.83
	ξ	0.45
Yield surface		
	m	0.02
Plastic Modulus		
	h_0	9.7
	c_h	1.02
	n^b	2.56
Dilatancy		
	A_0	0.81
	n^d	1.05
Fabric-Dilatancy		
	z_{max}	5
	c_z	800

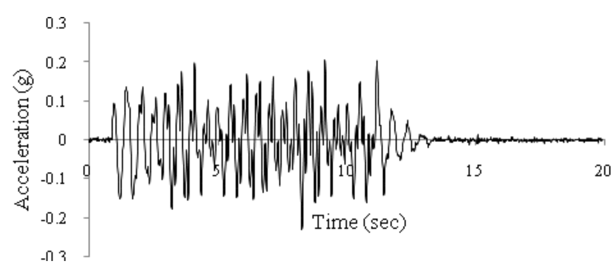

Figure 2. Horizontal input acceleration at the base of the laminar box (Prototype scale).

Description of numerical models: In this research, soil layer is modeled by rectangular 8-noded elements with *u-P* formulation in which each node has three degrees of freedom: two for soil skeleton displacements and one for pore water pressure. Properties of Nevada Sand used in the numerical model are presented in Table 3. According to the study of Gonzalez et al. (2002), the recorded pore pressure time histories at the same elevations are essentially identical, and this indicates the one-dimensional behavior of the model. Therefore, in this study, a one-dimensional finite element mesh with 8 rectangular elements is used (Figure 3) in which boundary conditions are set in the following way:

- Base of the mesh is fully fixed in all directions.
- Nodes at equal depths are constrained to have equal displacements in x and y direction to simulate the laminar box.
- Pore water pressures are free to develop for all nodes except the ones at the ground surface.

Table 3. Material parameters for Nevada sand (Taiebat et al. 2007)

Parameter	Unit	Value
Porosity (n)		0.42
Saturated Unit weight	KN/m^3	20.05
Permeability Coef.	m/s	6.6×10^{-5}
Permeability Coef. in Prototype scale (k_{st})	m/s	3.3×10^{-3}

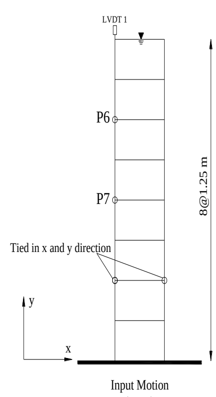

Figure 3. Finite element discretization and boundary conditions.

Simulations are carried out in two loading stages. In the first stage of loading, self-weight, including both the soil skeleton and the pore water weight, are applied on soil elements. In this stage the initial stress state, void ratio and soil fabric evolve. These values are used as initial values for the next stage of loading. Then, at the second stage, an acceleration time history (shown in Figure 2) is applied to the model base as an input motion, and dynamic analysis is performed.

5 EFFECT OF PERMEABILITY ON NUMERICAL SIMULATION OF LIQUEFIED SOIL SETTLEMENT

To investigate the influence of permeability coefficient on the numerical simulation of liquefaction phenomenon, results of the centrifuge test are compared with results the of numerical models obtained by the finite element programs: OpenSees and PISA. Two cases are taken into account in each simulation: considering constant initial permeability and considering variable permeability during liquefaction analysis. In the following sections the obtained results are discussed.

5.1 Numerical simulation using constant initial permeability, k_{st}

In the first step, the permeability coefficient is assumed to remain constant ($k_{st}=3.3\times10^{-3}$ m/s) over time at which the ground is being liquefied. Figs. 4 and 5 display the computed and recorded time histories of excess pore water pressures and settlement, respectively. A number of observations can be made about these results. As shown in Figure 4, both of the numerical models are able to predict the generation and dissipation of pore pressure over time. The r_u line indicates whether the generated pore pressure reaches the condition of zero effective stress (i.e. liquefaction) or not (In this paper, excess pore water pressure ratio (r_u) is defined as the ratio of the difference between current pore pressure and hydrostatic pore pressure over the initial effective vertical stress $(r_u = \Delta u / \sigma'_{v0})$. It is observed that liquefaction occurs at the depths of 2.5 and 5 m, and it has been acceptably predicted by numerical models. It should be noted that the observed differences between two numerical models is due to application of different constitutive models and different numerical algorithms.

However, as it is shown in Figure 5, the computed ground surface settlement is about half of its actual value measured in the centrifuge test. This is due to the fact that permeability coefficient significantly increases during liquefaction phenomenon because of structural change in soil skeleton. This is demonstrated in the previous experimental studies such as Arulanandan and Sybico (1992) and Jafarzadeh and Yanagisawa (1995). At the onset of liquefaction, soil particles lose full contact with each other, and this change creates additional pathways for water. The creation of such new, larger flow pathways reduces the pore shape factor and tortuosity parameters, and consequently leads to a significant increase in permeability coefficient. Accordingly, it can be concluded that using the constant initial permeability of soil in the numerical modeling of liquefaction leads to inaccurate results for ground settlement; however, it leads to accurate results for excess pore water pressure.

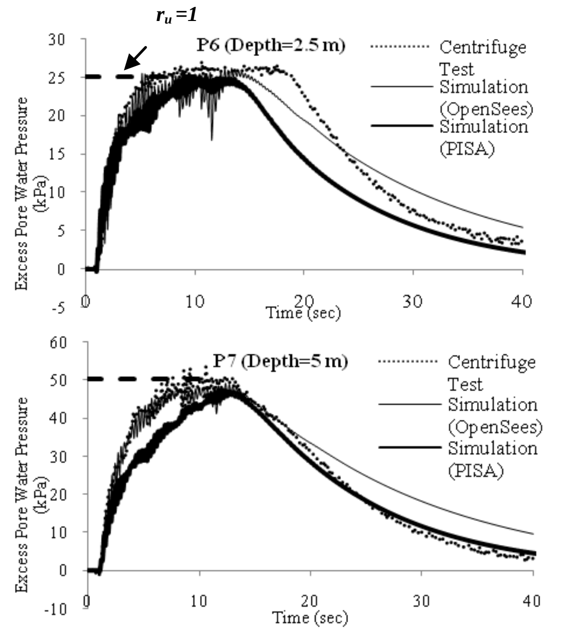

Figure 4. Measured and computed excess pore pressure time histories using constant initial permeability.

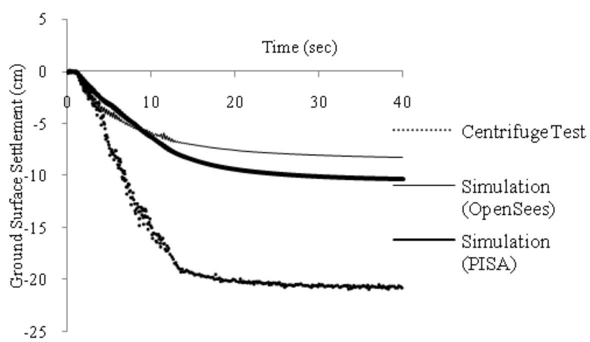

Figure 5. Measured and computed settlement time histories at the ground surface (LVDT 1) using constant initial permeability.

5.2 Numerical simulation using variable permeability during liquefaction

The above discussions indicate that an accurate simulation of pore pressure generation and dissipation and consequent settlement during liquefaction requires incorporating the actual variation of permeability in the analysis. Nevertheless, numerical studies in which a variation in permeability has been considered are rare. Some of the recent papers that considered the variation of permeability are listed for reference: Manzari and Arulanandan (1993), Kontantinos et al. (2010) and Shahir (2009).

In the presented study, the formulation for the variation of permeability suggested by Shahir (2009) is employed in both numerical models. In this formulation, a direct relationship between the permeability coefficient and excess pore water pressure ratio (ru) was proposed. This relationship is as follows:

$$\frac{k_b}{k_i} = 1 + (\alpha - 1) r_u^{\beta_1} \qquad \text{During PWP build up phase } (r_u < 1)$$

$$\frac{k_b}{k_i} = \alpha \qquad \text{During liquefied state } (r_u = 1) \qquad [3]$$

$$\frac{k_b}{k_i} = 1 + (\alpha - 1) r_u^{\beta_2} \qquad \text{During consolidation phase } (r_u < 1)$$

Where k_i is initial permeability coefficient, k_b is permeability coefficient during excitation, r_u (excess pore water pressure ratio) is defined as the ratio of the difference of current pore pressure and hydrostatic pore pressure over the initial effective vertical stress ($r_u = \Delta u / \sigma'_{v0}$), α, β_1, β_2 are positive material constant; Shahir (2009), suggested the values of 20, 1.0 and 8.9, respectively, for Nevada Sand. As mentioned earlier, the permeability gradually increases due to generation of excess pore pressure so in the first stage, the permeability should not increase significantly. It is clear that using the value of 1.0 provides a linear relationship between permeability coefficient and excess pore water pressure ratio which results in a considerable increase of permeability before liquefaction (i.e. PWP build up phase). Therefore, in this study the value of 6.0 is used for β_1 because this leads to a gradual increase in permeability coefficient and excess pore water pressure ratio, and also this value of β_1 decreases the rather sharp rate of permeability coefficient during the build up phase. Schematic views of permeability function for Nevada sand proposed by Shahir (2009) and by the authors are illustrated in Figure 6.

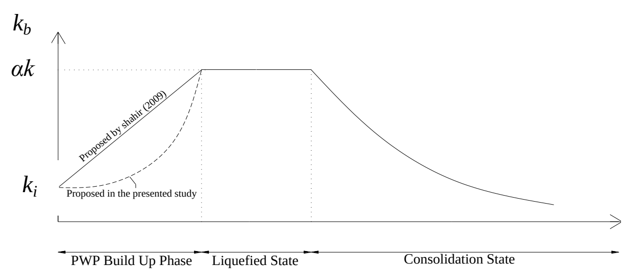

Figure 6. Schematic view of permeability function suggested by Shahir (2009) and by the authors for Nevada sand.

Figs. 7 and 8 show the computed and recorded time histories of excess pore water pressures and settlement, respectively. It is generally observed that application of Eq. (3) in numerical formulations satisfyingly captures all features of the soil response in liquefaction modeling. As shown in Figure 7, both of the numerical models can predict the generation and dissipation of excess pore water pressure by the maximum error of about 20%. In addition to this, ground surface settlement is computed by the maximum error of 25%, as shown in Figure 8. This error is approximately 52% using the constant initial permeability method (see Figure 5). In general, it can be concluded that considering permeability variation in the numerical modeling of liquefaction phenomenon leads to acceptable simulation of generation and dissipation of pore pressure and also soil settlement.

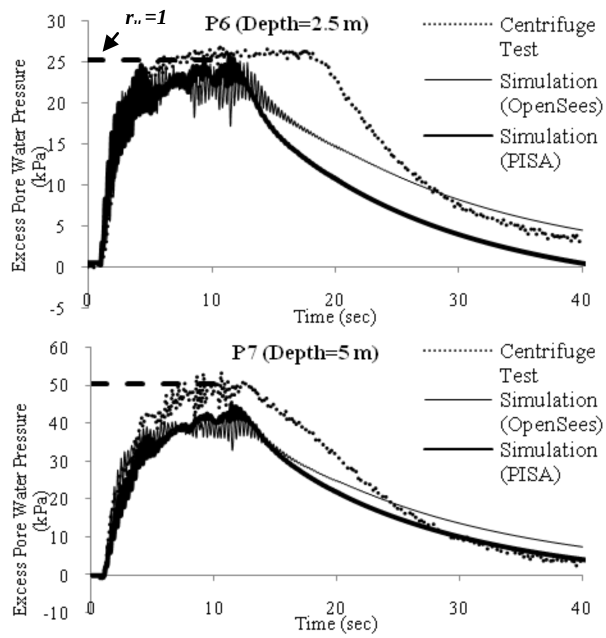

Figure 7. Measured and computed excess pore pressure time histories using variable permeability.

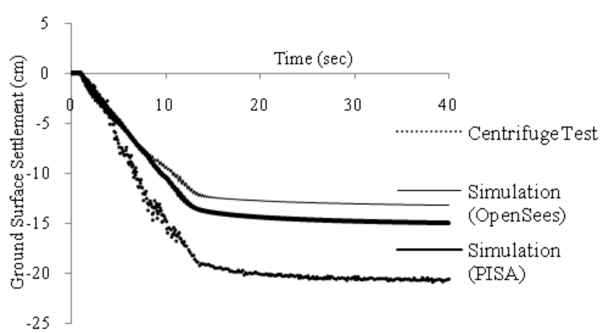

Figure 8. Measured and computed settlement time histories at the ground surface (LVDT 1) using variable permeability.

It is worth mentioning that soil lateral displacement have been studied as well, and it is concluded that using variable permeability during liquefaction, much more accurate values can be obtained for lateral movement of ground. The evaluation of these results can be found elsewhere (Rahmani et al. 2011).

6 CONCLUSIONS

In this paper the efficiency of two available approaches (i.e. constant initial permeability and variable permeability) are studied by employing two different finite element programs, OpenSees and PISA, which apply different versions of a bounding surface critical state elastic-plastic model. The computed results are compared with the measured values of a centrifuge test to demonstrate the accuracy of the numerical models. The following results are reached:

1) Using constant initial permeability in numerical analysis results in satisfying prediction of pore pressure generation and dissipation while results obtained for settlement are significantly different from the measured values.

2) Considering the variation of soil permeability, the predicted values for pore pressure and settlement show good match with the experimental measurements so considering the variation of permeability during liquefaction process can capture all features of the soil response in liquefaction simulation.

3) The Finite Element Programs, PISA and OpenSees, are both capable of accurate simulation of liquefaction phenomenon using the bounding surface critical state elastic-plastic model together with using the proposed formulation for the variation of permeability.

7 REFRENCES

Arulanandan K., and Sybico J.Jr., (1992), Post liquefaction settlement of sand, Proceeding of the Wroth Memorial Symposium, Oxford university, England.

Arulanandan K., Zeng X., (1993), Experimental results of Model No 1, in Verification of Numerical procedures for the analysis of soil liquefaction problems, Arulanandan, K. and Scott, R.F.(eds.), A.A. balkema, Rotterdam, pp.19-24.

Balakrishnan A., (2000), Liquefaction remediation at a bridge site, PhD Dissertation, University of California, Davis.

Chan A.H.C., Famiyesin O.O., Wood D.M., (1993),Numerical prediction for Model No 1, in Verification of Numerical procedures for the analysis of soil liquefaction problems, Arulanandan, K. and Scott, R.F.(eds.), A.A. balkema, Rotterdam, pp.87-108.

Chan D., Morgenstern N., (1988), SAGE : a finite element program for stress analysis in geotechnical engineering. Program user manual. Geotechnical Group. The University of Alberta.

Cheng Zh., Jeremic B., (2009), Numerical modeling and simulation of pile in liquefiable soil, Soil Dynamics and Earthquake Engineering ; 29: 1404-1416.

Dafalias Y.F., (1993), Overview of constitutive model used in VELACS, Verification of numerical procedures for the analysis of soil liquefaction problems, Vol. II, Arulanandan and Scott(eds.), Rotterdam, Balkema.

Dafalias Y.F., Manzari M.T., (2004), Simple plasticity sand model accounting for fabric change effects, Journal of Engineering Mechanics, 130(6), pp. 622-634.

Elgamal A., Yang Z., Parra E., (2002), Computational modeling of cyclic mobility and post-liquefaction site response, Soil Dynamics and Earthquake Engineering, 22, pp.259-271.

Gozalez L., Abdoun T., Sharp M.K., (2002), Modeling of seismically induced liquefaction under high confining stress, International Journal of Physical Modeling in Geotechnics, 2(3), pp. 1-15.

Heidari M., James R.G., (1982), Centrifuge modeling of earthquake induced liquefaction in a column of sand, Proceeding of the Conferences on Soil Dynamics and Earthquake Engineering, Volume 1, A.A. Balkema, Rotterdam, Netherlands.

Hushmand B., Scott R.F., Crouse C.B., (1988), Centrifuge liquefaction tests in a laminar box, Geotechnique, 38(2), pp. 253-262.

Iai S., Kameoka T., MatsunagaY., (1993), Numerical (Class A) prediction for Model No 1, in Verification of Numerical procedures for the analysis of soil liquefaction problems, Arulanandan, K. and Scott, R.F.(eds.), A.A. balkema, Rotterdam, pp.109-128.

Ishihara K., Cubrinovski M., Tsujino S., Yoshida N., (1993), Numerical prediction for Model No 1, in Verification of Numerical procedures for the analysis of soil liquefaction problems, Arulanandan, K. and Scott, R.F.(eds.), A.A. balkema, Rotterdam, pp.129-140.

Jafarzadeh F., Yanagisawa E., (1995), Settlement of sand models under unidirectional shaking, First international conference on earthquake geotechnical engineering, Ishihara K. (ed.), IS-Tokyo, pp.693-698.

Konstantinos I.A., Achilleas G.P., George D.B., (2010), Bounding surface plasticity model for the seismic liquefaction analysis of geostructures, Soil Dynamics and Earthquake Engineering, 30(10), pp.895-911.

Lacy S.J., Numerical prediction for Model No 1, in Verification of Numerical procedures for the analysis of soil liquefaction problems, Arulanandan, K. and Scott, R.F.(eds.), A.A. balkema, Rotterdam, 1993, pp.153-168.

Manzari M.T., Arulanandan K., (1993), Numerical predictions for Model No. 1, Verification of Numerical Procedures for the Analysis of Soil Liquefaction Problems, Arulanandan K. and Scott R.F. (eds.), A.A. Balkema, Rotterdam, pp.179-185.

Manzari M.T., Dafalias Y.F., (1997), A critical state two-surface plasticity model for sands, Geotechnique, 47(2), pp.255-272.

Mazzoni S., McKenna F., Fenves G.L., (1999), Open System for Earthquake Engineering Simulation User Manual, Pacific Earthquake Engineering Research Center, University of California, Berkeley. (WebSite: http://OpenSees.berkeley.edu/).

Pak A., (1997), Numerical modelling of hydraulic fracturing, PhD thesis, Department of Civil and Environmental Engineering, University of Alberta.

Prevost J.H., Romano J.d., Abdel Ghaffar A.M., Rowaland, R., (1981), Dynamic response of laterally loaded piles in a centrifuge, Proceeding of ASCE/EMD specialty Conferences on Dynamic Response of Structures , Atlanta, pp. 386-400.

Rahmani A, Ghsemifare O., Pak A., (2011), Investigation on the influence of permeability coefficient on numerical modeling of liquefaction phenomenon, Submitted to Scientia Iranica (International Journal of Science and Technology).

Shahir H., (2001), Dynamic analysis of saturated porous media for numerical simulation of liquefaction. MSc thesis, Department of Civil Engineering, Sharif University of Technology.

Shahir H., (2009), A performance-based approach for design of ground densification for mitigation of liquefaction, Ph.D. Dissertation, Sharif University of Technology.

Sharp M.K., Dobry R., (1999), Development of centrifuge based prediction charts for liquefaction and lateral spreading from cone penetration testing, PhD Dissertation, Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute, Troy, New York.

Stadler A.T., Ko H.Y., Sture S., (1993), Experimental results of Model No 1, in Verification of Numerical procedures for the analysis of soil liquefaction problems, Arulanandan, K. and Scott, R.F.(eds.), A.A. balkema, Rotterdam, pp.25-44.

Taboada V.M., Dobry R., (1995), Centrifuge modeling of earthquake-induced lateral spreading in sand using a laminar box, PhD Thesis, Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute, Troy, New York.

Taboada VM, Dobry R., (1993), Experimental results of Model No. p. 1 at RPI. In: Arulanandan K, Scott RF, editors. Verification of numerical procedures for the analysis of soil liquefaction problems. Rotterdam: AA Balkema; pp. 3–18.

Taiebat M., Shahir H., Pak A., (2007), Study of pore pressure variation during liquefaction using two constitutive models for sand, Soil Dynamics and Earthquake Engineering, 27(1), pp.60-72.