Bearing capacity and settlement behaviour of footings in an unsaturated sand

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ABSTRACT
A comprehensive experimental program was undertaken to study the bearing capacity and settlement behaviour of a sandy soil using model footings in specially designed equipment. The focus of the study is to understand the influence of the capillary stresses (i.e., matric suction), overburden stress (i.e., confinement) and dilation on the variation of the bearing capacity and settlement behaviour of surface and embedded footings in unsaturated sand. The results of the study show that the bearing capacity and settlement behaviour of unsaturated sandy soils are significantly influenced by the matric suction, overburden stress and dilation. Comparisons are provided between the measured bearing capacity and settlement behaviour under both saturated and unsaturated conditions using the modified Terzaghi's (1943) bearing capacity equation proposed by Vanapalli and Mohamed (2007) and the modified Schmertmann's method based on the cone penetration test data proposed by Mohamed et al. (2011), respectively. There is a good comparison between the measured and estimated values of the bearing capacity and settlement behaviour using the proposed modified equations. The framework for estimating both the bearing capacity and settlement behaviour is simple and promising and can be extended in geotechnical engineering practice for the design of shallow of foundations using the mechanics of unsaturated soils.

RÉSUMÉ
Un programme expérimental complet a été entrepris afin d'étudier la capacité portante et le tassement d'un sol sableux en utilisant des modèles de semelles à échelle réduite dans une enceinte spécialement conçue. L'étude a mis l'emphase sur l'influence des contraintes capillaires (c.-à-d. la succion matricielle), la surcharge (c.-à-d. le confinement) et la dilatation sur la variation de la capacité portante et du tassement de semelles de surface et enfouies dans un sable non-saturé. Les résultats de l'étude démontrent que la capacité portante et le tassement sont influencés de façon marquée par la succion matricielle, la surcharge, et la dilatation. Des comparaisons sont fournies entre les valeurs mesurées de la capacité portante et le tassement sous des conditions saturées ainsi que non-saturées en utilisant l'equation modifiée de Terzaghi (1943) proposée par Vanapalli et Mohamed (2007) et la méthode modifiée de Scmertmann basée sur les résultats de pénétromètre au cône proposée para Mohamed et al. (2011), respectivement. Il existe une bonne comparaison entre les valeures mesurées et estimées de la capacité portante et du tassement en utilisant les équations modifiées proposées. Le cadre théorique pour l'estimation de la capacité portante et du tassement est simple et prometteur et peut être appliqué dans la pratique géotechnique pour la conception de fondations à faible profondeur en faisant usage de la mécanique des sols non-saturés.

1 INTRODUCTION

The two key properties required in the design of shallow foundations are the bearing capacity (i.e., q_u) and settlement (i.e., δ) behaviour of soils. Structures such as silos, antenna towers, bridges, power plants, and housing subdivisions can be constructed on shallow foundations in sandy soils (e.g., spread footings near the ground surface) assuring a safe and economical design. The shallow footings are typically designed to transfer the loads safely from the superstructure to the supporting soil such that the settlements are in acceptable limits as per the design and construction codes. The bearing capacity of shallow foundations is estimated using the approaches originally presented by Terzaghi (1943) and Meyerhof (1951) assuming the soil is in a state of saturation condition. Typically, shallow foundations are placed above the ground water table and the variation of stress with respect to depth associated with the loads from the superstructure is distributed through the substructure (i.e., shallow foundations) in sandy soils that are in a state of unsaturated condition. This is true in many regions and especially in semi-arid and arid regions. A framework for interpreting the bearing capacity and settlement behaviour of sands from experimental and modelling studies for unsaturated soils is recently evolving (Vanapalli and Mohamed 2007, Vanapalli 2009, Oh and Vanapalli 2011a, Oh and Vanapalli 2011b, and Mohamed et al. 2011).

Comprehensive data for interpreting the bearing capacity and settlement behaviour of footings in unsaturated sands taking account of influence of the capillary stresses (i.e., matric suction), overburden stress (i.e., confinement) and dilation is however not available in the literature. Due to these reasons, an experimental program is undertaken to study the bearing capacity and settlement behaviour of a sandy soil using model footings in specially designed equipment. In addition, comparisons are provided between the measured and estimated values of the bearing capacity and settlement behaviour of model footings respectively using the approaches provided by

Vanapalli and Mohamed (2007) and Mohamed et al. 2011 in a sandy soil modifying the original contributions of Terzaghi (1943) and Schmertmann et al. (1978) respectively. The study shows that there is a good comparison between the measured and estimated values of the bearing capacity and settlement behaviour using the proposed modified equations.

2 BACKGROUND

2.1 Bearing capacity of soils

Terzaghi (1943), Meyerhof (1951), and De Beer (1965) studies were directed towards understanding the bearing capacity of shallow foundations in saturated or dry conditions using conventional soil mechanics. However, soils are typically found in a state of unsaturated condition in semi-arid and arid regions. Due to this reason, estimation of the bearing capacity of shallow foundations using conventional soil mechanics for these regions may underestimate the bearing capacity values and lead to conservative and costly foundation designs.

Several researchers carried out investigations to study the bearing capacity of unsaturated soils (Broms 1963, Steensen-Bach et al. 1987, Fredlund and Rahardjo 1993, Oloo 1997, Costa et al. 2003, Vanapalli and Mohamed 2007, Vanapalli and Oh, 2010). Mohamed and Vanapalli (2006) designed special equipment and conducted studies to understand the bearing capacity of surface model footing in a sandy soil. These studies have shown that the matric suction values in the range of 2 to 6 kPa contribute to 5 to 7 times bearing capacity values in comparison to saturated condition. Vanapalli and Mohamed (2007) provided a framework to predict the variation of bearing capacity of a soil with respect to matric suction using the saturated shear strength parameters (c' and ϕ') and the Soil-Water Characteristic Curve (SWCC).

2.2 Settlement of shallow footings

The shallow footings are typically designed in sandy soils such that the settlement is less than 25 mm in addition to safely carrying the loads from the superstructure to the soil below with a factor of safety recommended by the design and construction codes. Elastic or immediate settlements in sandy soils may be assumed to occur instantaneously when static loads are applied. A number of empirical equations are proposed in the literature that can be used in the estimation of the settlement of footings in sands based on cone penetration tests (CPT) results (Meyerhof 1956, DeBeer 1965, Schmertmann et al. 1978). The presently available methods in the literature overestimate the settlements leading to an overly conservative footing design (Das and Sivakugan 2007, and Mohamed et al. 2011). This can be attributed to ignoring the influence of matric suction below the foundations. In many scenarios such an assumption is not valid, particularly for soils in the arid and semi-arid regions where the soils typically are in a state of unsaturated condition.

Mohamed et al. (2011) proposed simple relationships by modifying the Schmertmann et al. (1978) method that is conventionally used in practice for settlement estimations from the CPT results. The modified method was successfully used in the estimation of the settlement behaviour of model footing tests and full-scale footings tested in-situ under both saturated and unsaturated conditions in sandy soils.

The focus of the present study is to understand the influence of the capillary stresses (i.e., matric suction), overburden stress (i.e., confinement) and dilation on the variation of the bearing capacity and settlement behaviour of both surface and embedded footings in unsaturated sand. The sand used by Vanapalli and Mohamed (2007) is used in the present research program.

3 PROPERTIES OF THE TESTED SAND

3.1 Soil properties

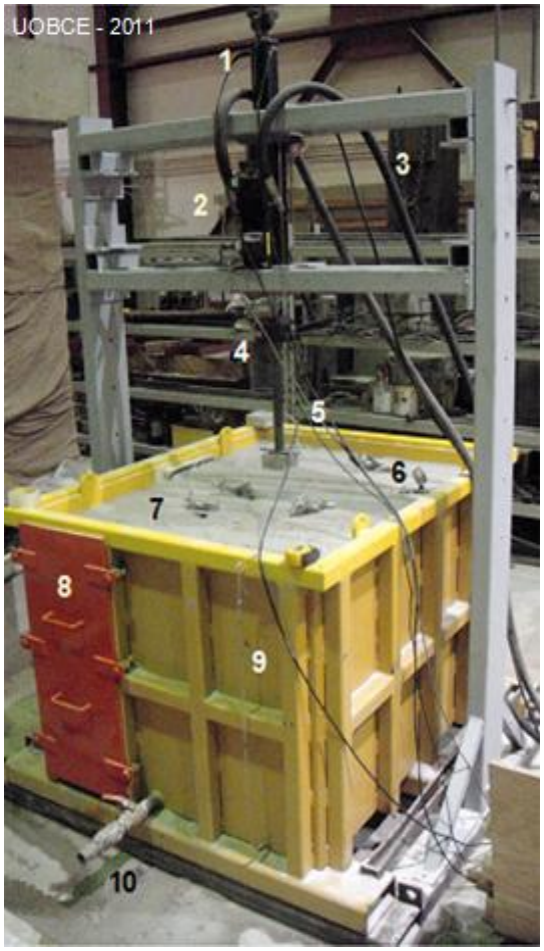
Table 1 summarizes the properties of the sand used in this study. The soil can be classified according to the USCS as poorly graded coarse-grained sand. The average initial void ratio and the dry unit weight in the test box were 0.63 and16.02 kN/m^3, respectively.

Table 1. Properties of the tested soil

Soil properties	Value
Coefficient of uniformity, C_u	1.83
Coefficient of curvature, C_c	1.23
Specific gravity, G_s	2.65
Average dry unit weight, γ_d , kN/m³	16.02
Min. dry unit weight, $\gamma_{d\,(min)}$, kN/m³	14.23
Max. dry unit weight, $\gamma_{d\,(max)}$, kN/m³	17.25
Average relative density, %	65.0
Optimum water content, o.w.c, %	14.6
Void ratio, e (after compaction)	0.62 – 0.64
Effective cohesion, c' , kN/m^2	0.6
Effective friction angle, $\phi'(^o)$	35.3

3.2 Influence of dilation in sandy soils

The effective friction angle, ϕ' was 35.3^o measured from the direct shear test (DST) results. Several studies suggest the overburden effective stress (i.e., confinement) and soil density influence the dilatancy behaviour of sands (DeBeer 1965, Bolton 1986, and Sfriso 2009). Vermeer and DeBorst (1984) studies show that the dilatancy angle, Ψ is always less than the effective friction angle, ϕ'. The dilation behaviour of sand can be attributed to the soil particles rolling on top of each other without crushing during the shearing stage. Bishop (1972) conducted experiments using steel shots that do not break down during shearing and has shown that increasing of confinement leads to a decrease in the dilatancy angle, Ψ. More recent studies by Chakraborty and Salgado (2010) show that the dilation of sand decreases with an increase in the effective overburden stress. Therefore, in the analysis of surface model footing results of the present study, bearing capacity and settlement behaviour of shallow footings were interpreted taking into account of the influence of dilation angle, Ψ on the effective friction angle, ϕ' of the sand.



1-	MTS actuator	6-	Tensiometer
2-	Loading frame	7-	Sand
3-	Hydraulic hose	8-	Test box
4-	Load cell	9-	Piezometer
5-	Model footing	10-	Drainage

Figure 1. University of Ottawa bearing capacity equipment (UOBCE-2011)

4 EQUIPMENT AND METHODOLOGY

Figure 1 shows the details of the University of Ottawa Bearing Capacity Equipment (UOBCE-2011) designed to determine the variation of bearing capacity and settlement of sands with respect to matric suction using model footings which are interpreted similar to plate load tests (i.e., PLTs). In the remainder of the paper, model footing tests are referred to as model PLTs for brevity. The equipment setup consists of a rigid-steel frame made of rectangular section pipes with thickness of 6 mm and a box of 1500 mm (length) × 1200 mm (width) × 1060 mm (depth). The test box can hold up to 3 tons of soil and the capacity of the loading machine (i.e., Model 244 Hydraulic Actuator with stroke length of 250 mm) is 28.5 kN. The model PLTs were performed using different strain rates of 1.2 mm/min and 2.5 mm/min. The results suggest that the load carrying capacity is not influenced by the different strain rates used in the present study.

The equipment used in the present study (see Fig. 1) in terms of test box size and its loading capacity is twice in comparison to the UOBCE-2006 used by Mohamed and Vanapalli (2006). The equipment in the present study has special provisions to achieve different degrees of saturation conditions below the model footings similar to the UOBCE-2006. The variation of matric suction with respect to depth in the unsaturated zone of the test box can be measured using commercial tensiometers.

5 LABORATORY PLT AND CPT TESTS

Several tests were conducted to determine the bearing capacity of the sandy soil with different values of matric suction using surface PLTs (i.e., model footing depth, D_f = 0 mm) or embedded PLTs (i.e., D_f =150 mm) and CPTs (i.e., cone penetration tests). A minimum of three tests were conducted and average values are reported in this paper.

5.1 Surface Plate Load Tests (PLTs)

Model PLTs of 150 mm × 150 mm (i.e., surface footings) were conducted by Mohamed and Vanapalli (2006) using UOBCE-2006 which consists of a rigid-steel tank of 900 mm (length) × 900 mm (width) × 750 mm (depth). Applied stress versus settlement relationships for surface model footing of 150 mm × 150 mm from that study are summarized in Figure 2.

5.2 Embedded Plate Load Tests (PLTs)

The model footings embedded to a certain depth are analyzed considering the influence of average matric suction value in the proximity of the stress bulb zone which is equal to depth 1.5B. Figure 3 provides details of the procedure used in the estimation the average matric suction value. The depth 1.5B considered is the zone in which stresses are predominant due to the loading of shallow square footings with $D_f/B \leq 1.0$ (Davis and Poulos 1968, Vanapalli and Mohamed 2007, and Oh and Vanapalli 2011).

In this series of tests, the model footing of 150 mm × 150 mm size is placed at a depth of 150 mm below the soil surface to investigate the effect of the overburden stress. The tests were conducted with different average matric suction below the footing (i.e., 0 kPa, 2 kPa and 6 kPa). Equilibrium conditions with respect to matric suction in the test tank were typically achieved in a period of 48 hrs in the test box shown in Figure 1. Table 2 summarizes typical set of results in which the average matric suction in the vicinity of the footing base is 6 kPa (see Figure 3).

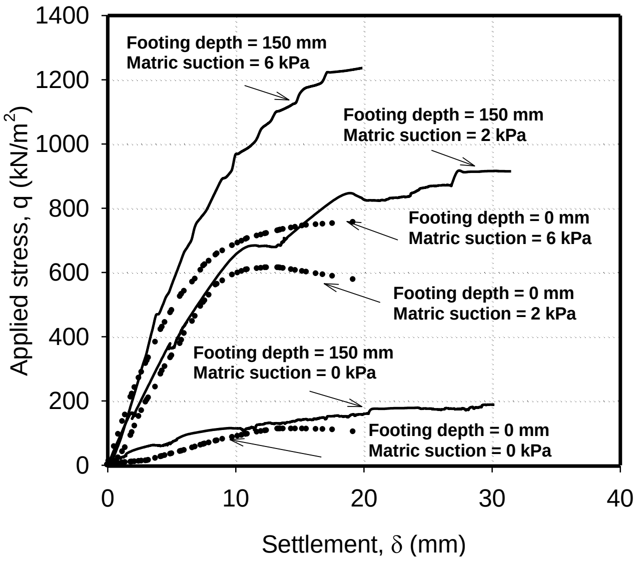


Figure 2. Relationship between the applied stress versus settlement behaviour of surface and embedded model footing tests (PLTs) of 150 mm × 150 mm

Table 2. Typical data from the test box for AVR matric suction of 6 kPa in the stress bulb zone (i.e., 1.5B)

D^1 (mm)	γ_t (kN/m^3)	γ_d (kN/m^3)	w (%)	S (%)	$(u_a - u_w)_{AVR}$ (kPa)
12	18.16	16.20	12.10	53	8.0
150	19.00	16.24	17.00	75	7.0
355	19.20	16.13	19.00	82	5.0
500	19.50	16.12	21.00	91	2.0
700	19.74	16.03	23.11	98	1.0
800	19.75	15.95	23.81	100	0.0

[1] Depth of a Tensiometer from the soil surface

The measured water content and matric suction values from the test box are similar to the corresponding water content and matric suction values in both measured and predicted SWCC of the tested sand. This observation provides credence to measurements using the tensiometers. The air-entry value for the sand was found to be between 2.5 kPa and 3 kPa. Details of the SWCC for the tested sand are available in Mohamed and Vanapalli (2006).

Based on the experimental results, the measured bearing capacity values of the compacted unsaturated sand for both surface and embedded footings were in the range of 5 to 7 times higher than the saturated bearing capacity.

5.3 Cone Penetration Tests (CPT)

Mohamed and Vanapalli (2009) conducted several CPTs in a laboratory environment in a compacted sand (D_r = 65%) in the UOBCE-2006 under both saturated and unsaturated conditions (i.e., 0 kPa, 1 kPa, 2 kPa and 6 kPa). The first series of tests were carried out under saturated condition and the second series of tests were conducted under unsaturated conditions (i.e. using different average matric suction values of 1 kPa, 2 kPa and 6 kPa). The experimental results and analyses of the variation of cone resistance, q_c with penetration depth were presented in Mohamed and Vanapalli (2009).

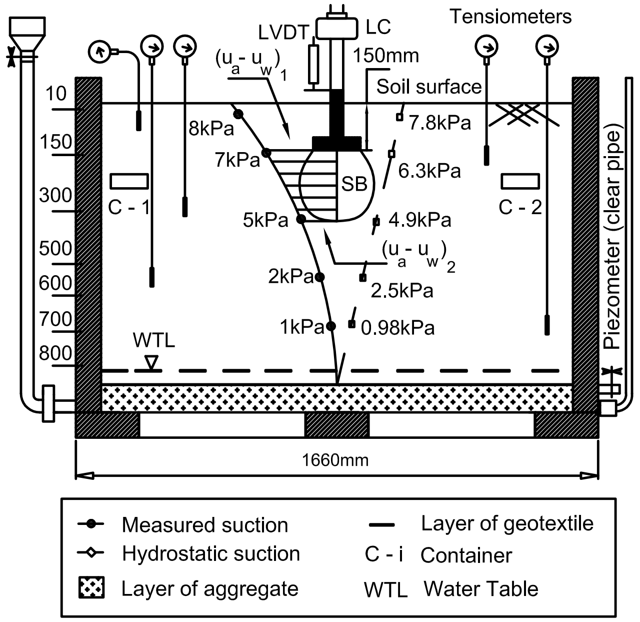


Figure 3. Schematic to illustrate the test setup and the procedure used for estimating the average matric suction of 6 kPa within the stress bulb (SB) zone of the embedded footing

The measured settlement results of the studies described in section 4.3 are used to check the validity of the proposed simple relationships based on the CPTs results in a later section to estimate the settlement of shallow footings in sand under both saturated and unsaturated conditions.

6 BEARING CAPACITY OF UNSATURATED SOILS

Terzaghi (1943) suggested Eq. 1 to estimate the ultimate bearing capacity, q_u of saturated soils assuming general shear failure:

$$q_u = c'N_c + \gamma D_f N_q + 0.5\gamma B N_\gamma \qquad [1]$$

where:

q_u = ultimate bearing capacity, kN/m^2
c' = effective cohesion, kPa
γ = unit weight, kN/m^3
D_f = footing base level, m
B = footing width, m

N_c , N_q , N_γ = bearing capacity factors which are function of effective friction angle, ϕ'.

Vanapalli and Mohamed (2007) suggested a semi-empirical equation (i.e., Eq. 2) based on surface model footing tests to predict the variation of bearing capacity with respect to matric suction for surface footings on unsaturated soils using the effective shear strength parameters (i.e. c' and ϕ') and the SWCC as below:

$$q_u = [c' + (u_a - u_w)_b(\tan\phi' - S^{\Psi}\tan\phi') + (u_a - u_w)_{AVR} S^{\Psi}\tan\phi'] N_c\zeta_c + 0.5\gamma B N_\gamma \zeta_\gamma \qquad [2]$$

where:

$(u_a - u_w)_b$ = air entry value from SWCC, kPa

$(u_a - u_w)_{AVR}$ = average air-entry value, kPa (see Fig. 3)

ϕ' = effective friction angle, °

S = degree of saturation, %

Ψ = B.C. fitting parameter

ζ_c , ζ_q , ζ_γ = shape factors

There is a smooth transition between the bearing capacity equation proposed by Vanapalli and Mohamed (2007) for unsaturated soils and the conventional Terzaghi's (1943) bearing capacity equation for saturated soils. In other words, Vanapalli and Mohamed (2007) equation will be the same as Terzaghi's bearing capacity equation when the matric suction value is set equal to zero.

The general form equation to estimate the bearing capacity of unsaturated soils is shown below as Eq. 3. This equation takes into account of the influence of overburden stress and the shape factors.

$$q_u = [c' + (u_a - u_w)_b(\tan\phi' - S^{\Psi}\tan\phi') + (u_a - u_w)_{AVR} S^{\Psi}\tan\phi')] N_c\zeta_c F_c + \gamma D_f N_q \zeta_q F_q + 0.5\gamma B N_\gamma \zeta_\gamma F_\gamma \qquad [3]$$

where:

F_c , F_q , F_γ = depth factors

The bearing capacity fitting parameter, ψ along with the effective shear strength properties (c' and ϕ') and the SWCC are required for predicting the variation of bearing capacity with respect to matric suction assuming drained loading conditions. The bearing capacity fitting parameter, ψ can be estimated from relationship provided by Vanapalli and Mohamed (2007) in Eq. 4 given below:

$$\psi = 1.0 + 0.34(I_p) - 0.0031(I_p^2) \qquad [4]$$

Several investigators provided bearing capacity factors for cohesion, N_c; surcharge, N_q and unit weight, N_γ (Terzaghi 1943, Meyerhof 1951, Vesić 1973, and Kumbhojkar 1993). The values for bearing capacity factors of N_c and N_q provided various investigators are approximately the same. For this reason, the bearing capacity factors, N_c and N_q originally proposed by Terzaghi (1943) were used in the analysis. The N_γ values suggested by Kumbhojkar (1993) have been more widely used and accepted in recent years. For this reason, the bearing capacity factor, N_γ values proposed by Kumbhojkar (1993) are used in this study.

7 COMPARISON BETWEEN MEASURED AND PREDICTED BEARING CAPACITY

7.1 Measured and predicted B.C. for surface PLTs

The bearing capacity of surface model footings of 150 mm × 150 mm were measured using the UOBCE-2006 by Mohamed and Vanapalli (2006) under both saturated and unsaturated sandy soil conditions (see section 5.1 for more details). Both the bearing capacity under saturated and unsaturated conditions was interpreted taking into account of influence on the dilatancy angle, Ψ for sand. The dilatancy angle, Ψ was not measured in this study but was approximated for a typical sand based on the information reported in the literature.

Table 3. B.C. factors, shape factors and depth factors used in the analysis for the surface PLT

Effective friction angle, ϕ' = 35.3 °

Estimated dilatancy angle, Ψ = 3.53 °

Modified friction angle, ϕ'_m = (ϕ' + Ψ) ≈ 39 °

B.C. Factors[1]			Shape Factors[2]			Depth Factors[3]		
N_c	N_q	N_γ	ζ_c	ζ_q	ζ_γ	F_c	F_q	F_γ
86	70	95	1.8	1.8	0.6	1.0	1.0	1.0

[1] from Terzaghi (1943); [2] from Vesic (1973); [3] from Hansen (1970)

The dilatancy angle, Ψ value assumed to be equal to 10% of effective friction angle, ϕ' of 35.3 is equal to 3.53 ° following Danish Code (DS 415. 1984). In other words, the modified friction angle, ϕ'_m is 38.53 or approximately 39 °. Summary of the values of the bearing capacity factors, shape factors and depth factors for the surface model footing with a modified friction angle of ϕ'_m = 39 ° are presented in Table 3.

The same approach has been extended for analyzing surface model footing results of another three sands tested by Steensen-Bach et al. (1987) Similar to the test results of sand used in the present study, good comparison between the measured and predicted bearing capacity values for these three sands considering a dilatancy angle value equal to 10% of effective friction angle. Summary of these discussions are available in Mohamed (2006). More recently, Oh and Vanapalli (2011a) have undertaken model studies to predict the variation of bearing capacity with respect to matric suction. These modeling results also show that reasonably good comparisons were possible between the measured and modeled bearing capacity values using a dilatancy angle, Ψ which is equal to 10% of effective friction angle, ϕ'.

7.2 Measured and predicted B.C. for embedded PLTs

The bearing capacity of 150 mm × 150 mm embedded model footing (in both saturated and saturated sandy soil conditions) was measured using the UOBCE-2011 (see Figure 1). The model footing is located at a depth, D_f of 150 mm below the soil surface simulating an overburden stress which also acts as a confinement all around the footing.

Equation 3 is used in the interpretation of the bearing capacity results of embedded footings in saturated and unsaturated sandy soils taking account of the influence of the overburden stress and the shape factors. However, the influence of dilatancy angle, Ψ was not considered.

Table 4. B.C. factors, shape factors and depth factors used in the analysis for the embedded PLT

Effective friction angle, $\phi' = 35.3^{\circ}$
Estimated dilatancy angle, $\Psi = 0^{\circ}$

B.C. Factors[1]			Shape Factors[2]			Depth Factors[3]		
N_c	N_q	N_γ	ζ_c	ζ_q	ζ_γ	F_c	F_q	F_γ
58	41	45	1.7	1.7	0.6	1.4	1.2	1.0

[1] from Terzaghi (1943); [2] from Vesic (1973); [3] from Hansen (1970)

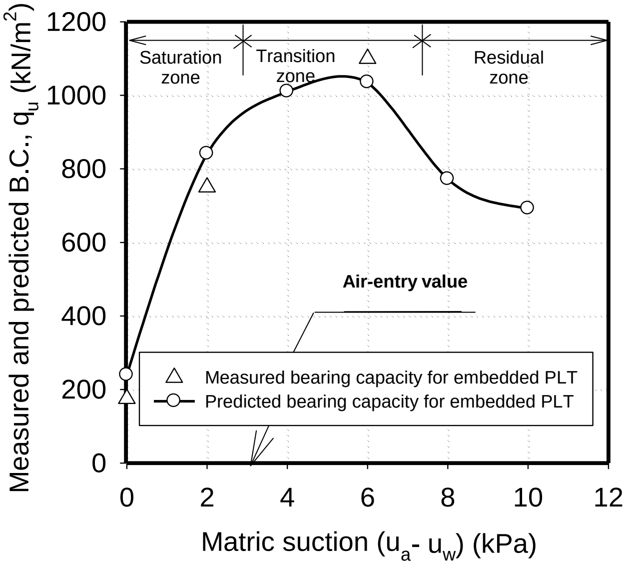


Figure 4. Measured and predicted B.C. for embedded model footing (PLT) of 150 mm × 150 mm

In other words, the bearing capacity factors, shape factors and depth factors were obtained using the effective friction angle, $\phi' = 35.3^{\circ}$ (see Table 4). There is a good comparison between the measured and estimated bearing capacity values for interpreting the embedded model footing results without taking account of the influence of dilatancy angle, Ψ. Such a behavior can be attributed to the influence of the confinement with a depth which is equal to the width, B of the foundation (see Figure 4).

These results are also consistent with the studies of several investigators who have shown that the influence dilation in the sand decreases with an increase in overburden effective stress or confinement (Bolton 1986, Liao 2003, and Chakraborty and Salgado 2010).

8 COMPARISON BETWEEN MEASURED AND ESTIMATED SETTLEMENT

The modulus of elasticity, E_s typically increases with depth in sandy soils and the stresses associated with the applied load decrease with an increase in depth. In other words, settlement will be less in deeper layers in comparison to shallow layers. Schmertmann et al. (1978) suggested an equation (i.e., Eq. 5) extending this philosophy for the estimation of footing settlement in sands using average cone penetration resistance, q_{ci} over a depth of 2B from the bottom of the footing.

$$\delta = C_1 C_2 (q_{app} - \sigma'_{z,d}) \sum_{0}^{2B} \left[\frac{I_{zi} \Delta_{zi}}{E_s} \right] \qquad [5]$$

where:

$$C_1 = 1 - 0.5 \left[\frac{\sigma'_{z,d}}{q_{app} - \sigma'_{z,d}} \right] \; ; \; C_2 = 1 - 0.21 \log \left[\frac{t}{0.1} \right]$$

δ = settlement, C_1 = depth factor, C_2 = time factor, q_{app} = footing pressure, $\sigma'_{z,d}$ = vertical effective stress at footing base level, $E_s = f \times q_{ci}$ elastic modulus of soil, I_{zi} = influence factor, B = footing width, q_{ci} = resistance of each layer, f = coefficient, t = time, and Δ_z = thickness of each layer.

This method is widely used in geotechnical engineering practice. One of the key limitations of this method is that it does not take into account the influence of capillary stress or matric suction and is used for sands both in saturated and unsaturated conditions.

Mohamed et al. (2011) suggested two empirical relations that can be used in the Schmertmann et al. (1978) equation to estimate the settlements in sands. These empirical relations are useful in estimating the modulus of elasticity, E_s of sands in saturated and unsaturated conditions. The relationships are proposed based on the analysis from the PLT and CPT results.

Eq. 6 is suggested to estimate the modulus of elasticity, E_s for saturated sands (i.e. $(u_a - u_w) = 0$ kPa) as given below:

$$E_{s(sat)} = f_1 \left[q_{c(sat)} \right] \qquad [6]$$

where: $E_{s\ (sat)}$ = modulus of elasticity for saturated homogenous sand, $f_1 = 1.5 \times (D_r^2 + 3)$ (i.e. f_1 is a correlation factor), $q_{c\ sat}$ = average cone resistance under saturated sands condition (e.g. within an influence zone, IZ equal to 1.5B from the footing base level) and B = footing width.

Eq. 7 is suggested to estimate the modulus of elasticity, E_s for unsaturated sands (i.e. $(u_a - u_w) > 0$ kPa):

$$E_{s(unsat)} = f_2 \left[q_{c(unsat)} \right] \qquad [7]$$

where: $E_{s\ (unsat)}$ = modulus of elasticity for unsaturated homogenous sands, $f_2 = 1.2 \times (D_r^2 + 3.75)$ for sands with $D_r < 50$ or $f_2 = 1.7 \times (D_r^2 + 3.75)$ for sands with $D_r \geq 50\%$, (i.e. f_2 is a correlation factor), $q_{c\ unsat}$ = average cone resistance under unsaturated sands conditions (e.g. within influence zone, IZ equal to 1.5B from the footing base level) and B = footing width.

The modulus of elasticity, E_s from Eq. 6 or Eq. 7 can be substituted into Schmertmann et al. (1978) equation (i.e., Eq. 5) to estimate the immediate settlement. Figure 5 provides comparisons between the estimated and measured settlement values for embedded model PLTs of 150 mm × 150 mm in the tested sand with different average matric suction values (i.e., 0 kPa, 2 kPa and 6 kPa) using the proposed relationships into the Schmertmann et al. (1978) equation. The footing settlements decrease with an increase in the matric suction and the overburden stress.

More details of the analysis and comparisons between the measured and estimated settlement values using the proposed relationships in Eq. 5 and measured settlement values for both saturated and unsaturated sands using two model PLTs and six in-situ footing load tests (FLTs) are summarized in Mohamed et al. (2011).

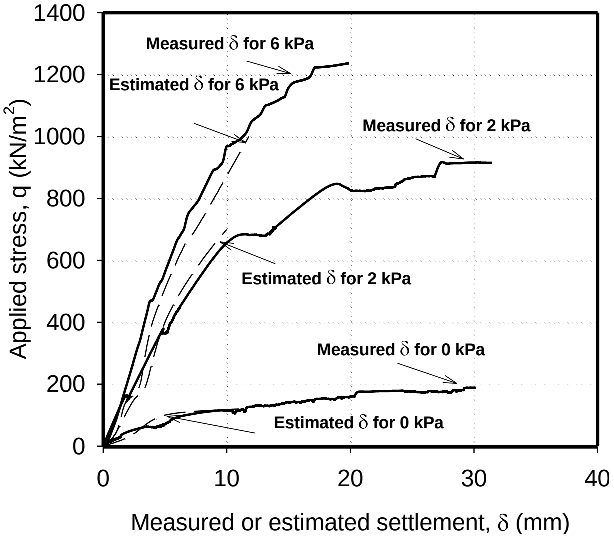


Figure 5. Comparison between the measured and estimated settlements using the modified Schmertmann et al. (1978) relationships

9 SUMMARY AND CONCLUSIONS

The bearing capacity and settlement behaviour of sandy soil under saturated and unsaturated conditions using surface and embedded model footings tests are studied in this research program. The bearing capacity values are underestimated for surface model footings (i.e., the depth of the model footing is equal to zero) when calculations are based on effective friction angle, $\phi' = 35.3\,^{o}$ for the tested sand both in saturated and unsaturated conditions. Reasonably good comparisons were observed between measured bearing capacity and predicted values using Eq. 2 by taking account of the influence of the dilatancy angle, Ψ. Typical value of dilatancy angle for sands is equal to 10% of effective friction angle, ϕ' (see Table 3).

There is no need to increase the effective friction angle, ϕ' by 10% to obtain reasonable comparison between the measured and estimated bearing capacity values (see Table 4) for embedded footings. In other words, the dilatancy angle, $\Psi = 0\,^{o}$ for shallow foundations whose $D_f/B = 1$. Such a contrasting behavior between surface and embedded footings may be attributed to the contribution of the overburden stress which eliminates the influence of dilation. These observations are consistent with the conclusions drawn by (Vesić and Clough 1968, Bolton 1986, Oh and Vanapalli 2008, and Chakraborty and Salgado 2010) with respect to dilation effects in sandy soils.

The bearing capacity of unsaturated sands increases with matric suction in a linear fashion up to the air-entry value (saturation zone). There is a non linear increase in the bearing capacity in the transition zone (i.e., air-entry value to the residual suction). The bearing capacity however decreases in residual zone of unsaturation. Figure 4 shows typical behaviour of variation of bearing capacity with respect to matric suction. The behaviour of bearing capacity of unsaturated soils is consistent with the shear strength behaviour of unsaturated sands (Vanapalli et al. 1996, Vanapalli and Lacasse 2009).

Schmertmann et al. (1978) equation (i.e., Eq. 5) with proposed relationships for modulus of elasticity, Es for saturated and unsaturated conditions (i.e., Eqs. 6 and 7) provide reasonably good comparisons between measured and estimated settlements for both model PLTs and in-situ FLTs (see Figure 5). More details with respect to the test results and analysis are available in Mohamed et al. (2011).

Both the procedures for predicting the bearing capacity and settlement behavior of sandy soils in both saturated and unsaturated conditions are promising and can be used by practising engineers.

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