# Rational approach for the design of retaining structures using the mechanics of unsaturated soils

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2011 Pan-Am CGS Geotechnical Conference

ABSTRACT
The earth pressures acting on the soil retaining structures are conventionally estimated using the mechanics of saturated soils using conventional soil properties such as the saturated shear strength parameters and soil density values. However, the backfill material behind the retaining walls is typically in a state of unsaturated condition. Thus, the influence of the capillary stress above the ground water table in the backfill is conventionally not taken into account; due to this reason the resultant earth pressure is likely to be conservative. Rigorous analyses of the design of retaining walls in semi-arid and arid regions where the backfill materials are typically in a state of unsaturated condition should be based on the mechanics of unsaturated soils taking account of the influence of capillary stress (i.e., matric suction). In this paper, several examples are presented to highlight the differences in the earth pressures calculated using conventional soil mechanics and using the mechanics of unsaturated soils.

RÉSUMÉ
La pression des terres agissant sur les structures de rétention des sols est conventionnellement estimées en utilisant la mécanique des sols saturés, faisant usage des paramètres de sol conventionnels tels que la résistance au cisaillement et la densité d'un sol saturé. Cependant, le matériel de remblayage derrière un mur de rétention se trouve généralement dans une condition non-saturée. On ne tient alors généralement pas compte de l'influence des contraintes capillaires au-dessus du niveau phréatique dans le remblais, ce qui se traduit par des résultats conservateurs. Une analyse rigoureuse de la conception de murs de rétention dans des zones semi-arides et arides où les matériaux de remblais sont typiquement dans des conditions non-saturées devrait être basés sur la mécanique des sols non-saturés, tenant compte de l'effet des contraintes capillaires (c.-à-d. la succion matricielle). Dans le présent article, plusieurs exemples sont résolus afin de mettre en évidence les différences dans la pression des terres calculée en utilisant la mécanique des sols conventionnelle et la mécanique des sols non-saturés.

## 1 INTRODUCTION

Retaining walls are structures that are designed and constructed following the safety codes to withstand pressures associated with soil, water and other loading conditions to restrain the backfill soil movement. They are also used in several situations to stabilize slopes. The earth pressures acting on such structures depend mainly on the type and mechanical properties of backfill soils, wall geometry and its material properties. The frictional resistance that arises between the retaining wall and backfill soil can also be a key parameter in the stability analysis of retaining structures in certain scenarios. The lateral earth pressures associated with these structures are commonly estimated using either Rankine or Coulomb's approaches using the mechanics of saturated soils in practice. Such an approach is simple; however, the estimated earth pressures are typically conservative for backfill soils that are in unsaturated conditions. This is especially true for retaining walls in arid and semi-arid regions which typically have unsaturated soils as backfill. In many cases, the natural ground water table is at a greater depth in these regions. About 33% of the earth's surface constitutes of arid and semi-arid regions (Dregne 1976). More recently, geocomposites and geotextiles are being used in the construction of retaining structures to retain the capillary stress or matric suction in the backfill soils to take advantage of the engineering behaviour of unsaturated soils (McCartney et al. 2008). Therefore, it is more rational in such scenarios to estimate the resultant earth pressure extending the mechanics of unsaturated soils rather than using the conventional approaches.

In this paper, a brief background of effective and total shear strength of unsaturated soils is provided along with the details of how they can be extended in the estimation of resultant earth pressures on the retaining walls using some practical examples.

## 2 BACKGROUND

Limited studies are available in the literature with respect to the estimation of earth pressures or critical height of vertical cuts in unsaturated fine-grained (hereafter referred to as UFG) soils extending the mechanics of unsaturated soils (Pufahl et al. 1983, Vanapalli et al. 2009 and Zhang et al. 2010). Pufahl et al. (1982) estimated the lateral earth pressure on retaining structures taking account of the influence of matric suction. They suggested that the most efficient way to visualize the influence of matric suction (i.e., negative pore-water pressure) on the lateral earth pressure is to include its contribution to the cohesion parameter. These investigators considered the shear strength contribution of matric suction, $\phi^b$ as a constant value.

More recently, Vanapalli et al. (2009) revisited published results of a full scale instrumented test trench to estimate the critical height of an unsupported vertical trench in UFG soils. Similar to Pufahl et al. (1983), the contribution of matric suction, $\phi^b$ was considered as a parameter that can be included to the cohesion term in the analysis of the stability of unsaturated vertical trench. However, the $\phi^b$ was considered as a non-linear variable with respect to matric suction using the approach presented in Vanapalli et al. (1996a).

A simplistic approach can be extended in the rational design of retaining structures by assuming hydrostatic variation of matric suction above the ground water table. Typically, the variation of matric suction above the ground water table, in most scenarios, is non-linear in nature. These values of matric suction are also higher than the hydrostatic condition. The assumption of hydrostatic condition is therefore reasonable and yet conservative approach. In this study, Rankine approach is extended in the determination of earth pressures using both the mechanics of saturated and unsaturated soils. Different scenarios were used in the calculation of resultant pressure; (i) neglecting the influence of matric suction, (ii) including the influence of matric suction assuming hydrostatic conditions above the ground water table.

The discussion based on the studies summarized in this paper would be of interest to the practicing engineer to understand how conservative the presently used approaches are in the estimation of resultant pressures when they are extended for unsaturated soils.

## 3 SHEAR STRENGTH OF UNSATURATED SOILS USING THE EEFECTIVE AND THE TOTAL STRESS APPROACHES

The earth pressures generated on retaining structures can best be explained in terms of the changes arising in the principal stresses which can be addressed using the shear strength behavior of soils extending Rankine's theory. A well established approach is available in the literature using the shear strength behavior of saturated soils. However, this approach has to be modified using the concepts of mechanics of unsaturated soils for estimating the earth pressures for backfill soils that are in a state of unsaturated condition. The mechanics of unsaturated soils has been put forward taking account of the influence of stresses that arise in the three phases of a soil (i.e., solid, liquid, gas). The rational approach of understanding the engineering behavior of unsaturated soils is to determine the relation of one phase (i.e., solid phase or soil particles) to the other two phases (liquid and gas which are typically water and air). The mechanical properties of unsaturated soils can be rationally interpreted in terms of two independent stress states; net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$ (Fredlund and Rahardjo 1993).

### 3.1 Effective stress analysis approach (ESA)

Bishop (1959) proposed an equation to estimate the shear strength of unsaturated soils (hereafter referred to as SSUS) using the effective shear strength parameters, c', and $\phi$' along with the soil parameter, $\chi$ that is a function of the degree of saturation.

$$\tau = c' + \left[ (\sigma_n - u_a) + \chi (u_a - u_w) \right] \tan\phi' \qquad [1]$$

where c', $\phi$' = effective cohesion and internal friction angle respectively, $\chi$ = soil parameter that is a function of degree of saturation, $(\sigma_n - u_a)$ = net normal stress and $(u_a - u_w)$ = matric suction.

Fredlund et al. (1978) proposed the SUSS in terms of stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$ which is consistent with the continuum mechanics (Eq. [2]). The form of Eq. [2] indicates that the effective internal friction angle, $\phi$' is not influenced by matric suction. Due to this reason, the term effective cohesion can be combined with the shear strength contribution due to matric suction as apparent or total cohesion, $c_t$ (Eq. [3]). The rate of increase in the shear strength with respect to matric suction, $\tan\phi^b$ was originally assumed to be constant value. Later studies have shown that the shear strength contribution due to matric suction is non-linear (Escario and Saez 1987, Gan and Fredlund 1988). Typical variation of shear strength with respect to the stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$ is shown in Figure 1.

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad [2]$$

$$c_t = c' + (u_a - u_w)\tan\phi^b \qquad [3]$$

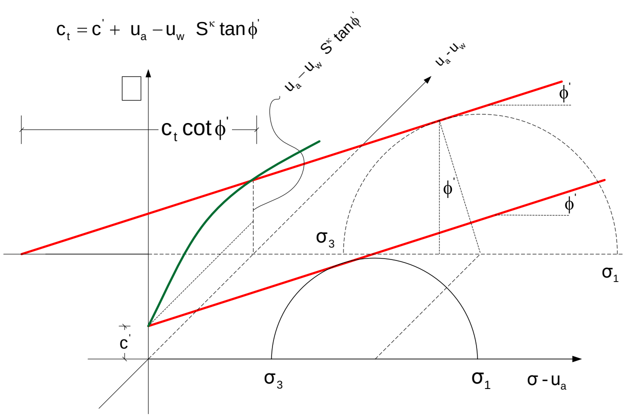



Figure 1. The variation of shear strength with respect to the net normal stress and matric suction.

Figure 2 shows the variation of shear strength with respect to matric suction under different net normal stresses for Indian Head till (Vanapalli et al. 1996a). The air-entry value of the soil increases with an increase in the net normal stress. This is because of the influence of stress state which contributes to a decrease in void ratio and the coefficient of permeability. The results in Figure 2 indicate that the variation of shear strength is non-linear when it is measured over a large matric suction range.

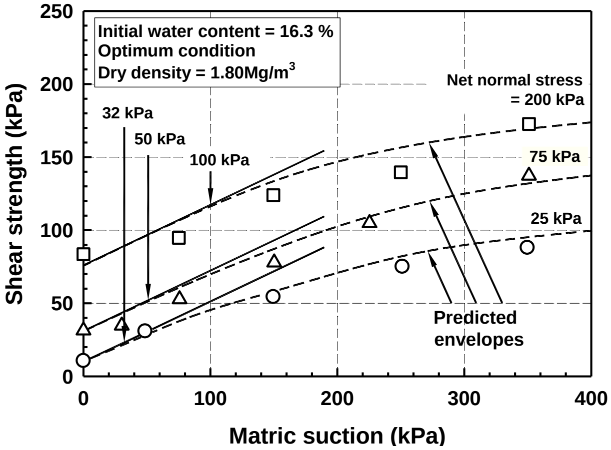



Figure 2. Variation of shear strength with respect to matric suction under different net normal stresses (modified after Vanapalli et al. 1996a).

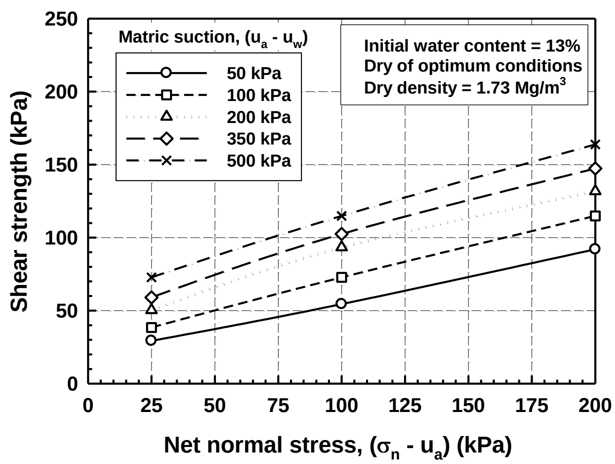



Figure 3. Variation of shear strength with respect to different net normal stresses (modified after Vanapalli et al. 1996b).

Figure 3 shows the variation of shear strength with net normal stress under different matric suction values (Vanapalli et al. 1996b). The relationship between shear strength and the slope of net normal stress (i.e., tanϕ') is constant. In other words, tanϕ' is independent of matric suction. Such a relationship suggests that the Soil-Water Characteristic Curve (SWCC) can be used as a tool along with the effective shear strength parameters (i.e., c' and ϕ') to approximately predict the SSUS with respect to matric suction.

The SWCC shows the relationship between the degree of saturation and soil suction, which is established conventionally using the data measured with the pressure plate and the vapor pressure technique in low and high suction range, respectively (Fredlund and Rahardjo 1993, Vanapalli et al. 2004). As the condition of a soil changes from a saturated state to drier state; the distribution of soil, water, and air phase also changes.

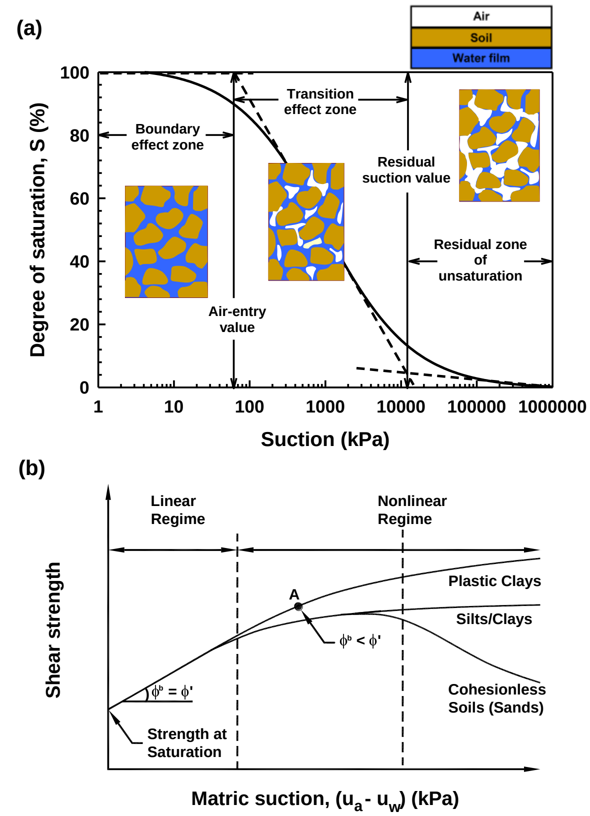



Figure 4. (a) SWCC showing different zones and (b) the variation of shear strength of unsaturated soils in various zones of unsaturation for different soils (from Vanapalli 2009).

The SWCC typically consists of three zones; i) boundary effect, ii) transition, and iii) residual zone of desaturation) as shown in Figure 4(a) (Vanapalli et al. 1996a). During the process of desaturation, the wetted area of contacts between the soil particles decreases. The suction as a stress state contributes to shear strength along the wetted area of contact of soil particles. In other words, there is a relationship between the rate at which shear strength changes with respect to matric suction and the wetted area of soil particles or aggregates. This implies that the variation of shear strength with respect to matric suction can be reasonably estimated using the SWCC.

Figure 4 (a and b) provide the relationship between the SWCC and the SUSS. More detailed explanations of these relationships are available in Fredlund et al. (1996), Vanapalli et al. (1996a) and Vanapalli (2009).

## 4 GENERAL FORMULATIONS FOR EXTENDING EFFECTIVE AND TOTAL STRESS APPROACH FOR ESTIMATING THE EARTH PRESSUES

### 4.1 Effective stress approach (ESA)

Fredlund et al. (1996) and Vanapalli et al. (1996a) suggested that the variation of $\tan\phi^b$ with respect to matric suction can be estimated using the SWCC and fitting parameter, $\kappa$ (Eq. [4]).

$$\tan\phi^b = S^\kappa \tan\phi' \qquad [4]$$

Using the concept in Eq. [4], Eq. [2] can be rewritten as Eq. [5].

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)(S^\kappa \tan\phi') \qquad [5]$$

The total cohesion, $c_t$ can be expressed as Eq. [6]

$$c_t = c' + (u_a - u_w)(S^\kappa \tan\phi') \qquad [6]$$

Eq. [6] can be used for predicting the shear strength behavior of unsaturated soils over the entire range of suction (i.e., from fully saturated to dry condition). Garven and Vanapalli (2006) suggested that the fitting parameter, $\kappa$ is a function of plasticity index, $I_p$ after analyzing the shear strength test results for various compacted soils (Eq. [7]).

$$\kappa = -0.0016(I_p)^2 + 0.0975(I_p) + 1 \qquad [7]$$

In case of active state, the vertical pressure ($\sigma_v$) is constant as the wall moves away from the backfill soil while the horizontal pressure ($\sigma_h$) decreases (i.e., the Mohr circle size increases and approaches the failure envelope). Therefore, $\sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_h$ for active state. In case of passive state, the vertical pressure ($\sigma_v$) is constant as the wall moves into the backfill soil while the horizontal pressure ($\sigma_h$) increases until the Mohr circle touches the failure envelope. Therefore, $\sigma_1 = \sigma_h$ and $\sigma_3 = \sigma_v$ for the passive state.

The concept of the Mohr circle in Figure 1 can be expressed using the equations as below.

$$\sin\phi' = \frac{\tfrac{1}{2}(\sigma_1 - \sigma_3)}{\tfrac{1}{2}(\sigma_1 + \sigma_3 + 2c_t\cot\phi')} \qquad [8]$$

$$\sigma_1\sin\phi' + \sigma_3\sin\phi' + 2c_t\cos\phi' - \sigma_1 + \sigma_3 = 0 \qquad [9]$$

Combining Eq. [6] and Eq. [9] yields the following relationships.

$$\sigma_3 = \sigma_1\left(\frac{1-\sin\phi'}{1+\sin\phi'}\right) - 2\left[c' + (u_a - u_w)S^\kappa\tan\phi'\right]\sqrt{\left(\frac{1-\sin\phi'}{1+\sin\phi'}\right)} \qquad [10]$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi'}{1-\sin\phi'}\right) + 2\left[c' + (u_a - u_w)S^\kappa\tan\phi'\right]\sqrt{\left(\frac{1+\sin\phi'}{1-\sin\phi'}\right)} \qquad [11]$$

From the above derived mathematical relationships (i.e., Eq. [10] and Eq. [11]), the active and passive earth pressure of unsaturated soils can be estimated using Eq. [12] and Eq. [13], respectively.

$$(\sigma_h - u_a) = (\sigma_v - u_a)\left(\frac{1-\sin\phi'}{1+\sin\phi'}\right) - 2\left[c' + (u_a - u_w)S^\kappa\tan\phi'\right]\sqrt{\left(\frac{1-\sin\phi'}{1+\sin\phi'}\right)} \qquad [12]$$

$$(\sigma_v - u_a) = (\sigma_h - u_a)\left(\frac{1+\sin\phi'}{1-\sin\phi'}\right) + 2\left[c' + (u_a - u_w)S^\kappa\tan\phi'\right]\sqrt{\left(\frac{1+\sin\phi'}{1-\sin\phi'}\right)} \qquad [13]$$

The coefficient of active and passive earth pressure, $k_a$ and $k_p$ respectively for the unsaturated soil conditions are the same as saturated state as the angle of internal friction, $\phi$' is not influenced by matric suction.

Eq. [12] and Eq. [13] can be simplified as Eq. [14] and Eq. [15], respectively.

$$\sigma_a = \gamma z K_a - 2\left[c' + (u_a - u_w)S^\kappa\tan\phi'\right]\sqrt{K_a} \qquad [14]$$

$$\sigma_p = \gamma z K_p + 2\left[c' + (u_a - u_w)S^\kappa\tan\phi'\right]\sqrt{K_p} \qquad [15]$$

### 4.2 Total stress approach (TSA)

The contribution of matric suction towards shear strength of UFG soils (i.e., $\phi^b$) can be most reliably estimated using the constant water content (CW) test with measurement of suction during shearing stages. However, obtaining the equilibrium condition with respect to volume and suction in the unsaturated soil specimens is time consuming and the measurement of pore-air and pore-water during shearing stages requires elaborate testing equipment. Due to this reason, Oh and Vanapalli (2009) suggested the use of total stress approach (TSA) to interpret the mechanical behavior of unsaturated soils instead of the ESA. The main concept of the TSA is that the cohesion parameter, $c_{CW}$ in terms of the total stress includes the contribution of matric suction towards shear strength. The shear strength parameters obtained from unconfined compression test or isotropic confinement undrained test (IU) obtained for unsaturated soils without suction measurement during loading stages can also be used instead of $c_{CW}$ due to the similarities in the drainage conditions (i.e., pore-air is drained condition and pore-water is undrained condition) and its simplicity (Oh et al. 2008).

Oh and Vanapalli and (2009) proposed an equation to estimate the variation of undrained shear strength of unsaturated soils with respect to matric suction as below.

$$c_{u(unsat)} = c_{u(sat)}\left[1+\frac{u_a-u_w}{P_a/100}\ S^{\nu}\ /\mu\right] \qquad [16]$$

where $c_{u(sat)}$, $c_{u(unsat)}$ = shear strength under saturated and unsaturated condition, respectively, $P_a$ = atmosphere pressure (i.e., 101.3 kPa), S = degree of saturation and $\nu$, $\mu$ = fitting parameters. The fitting parameter $\nu$ = 2 is required for UFG soils and $\mu$ = 9 for soils with plasticity index values between 8% and 15.5%.

As explained earlier, when deformation takes place in UFG soils, the pore-air pressure is atmospheric pressure (i.e., drained condition) while the pore-water in under undrained condition. This drainage condition can be most reliably represented using the CW test. Extending this concept, the earth pressure of unsaturated soils can be estimated as below.

$$\sigma_a = \gamma z K_a - 2c_{CW}\sqrt{K_a} \qquad [17]$$

## 5 SUCTION DISTRIBUTION

The variation of matric suction with depth above the ground water table (GWT) is typically non-linear. However, the distribution of matric suction above the GWT can be assumed to be hydrostatic. Such an approach is simple, practical and conservative to address geotechnical engineering problems. This is because the measured matric suction values are greater than those from the assumed hydrostatic matric suction distribution diagram (Figure 5).

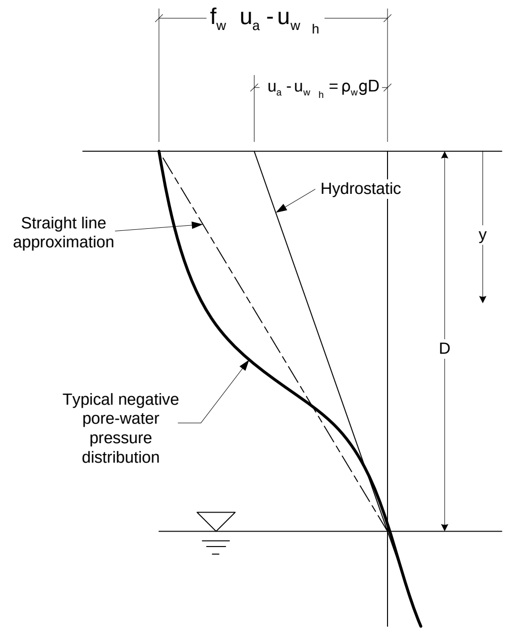



Figure 5. Different scenarios of variation of matric suction with respect to depth.

Figure 6 shows the assumed hydrostatic matric suction distribution profile and the depth of matric suction induced tension crack. The tension crack in unsaturated soils propagates from the surface to the depth where the horizontal active pressure becomes zero. From Figure 6, the matric suction value of the soil in equilibrium condition with respect to GWT can be calculated as $(u_a - u_w) = \gamma_w(D - y_c)$. By replacing these factors in active pressure equation (i.e., Eq. [14]), the depth of matric suction induced tension crack for unsaturated coarse-grained soils (i.e., c' = 0) can be estimated using Eq. [18].

$$y_c = \frac{\gamma_w D S^{\kappa}\tan\phi'}{\gamma_w S^{\kappa}\tan\phi' - 0.5\gamma_{unsat}\sqrt{K_a}} \qquad [18]$$

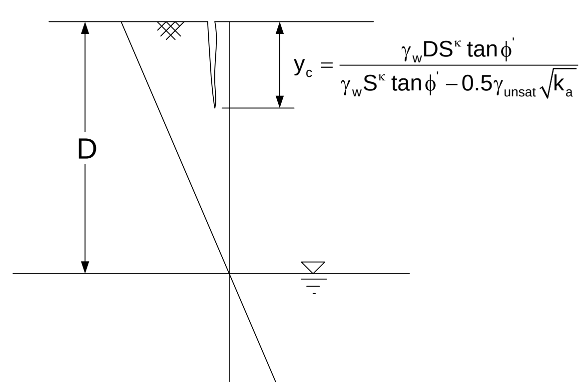
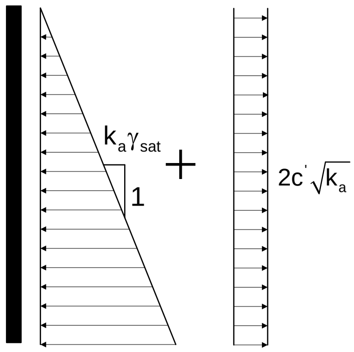



Figure 6. Estimation of the depth of crack in unsaturated backfill soil

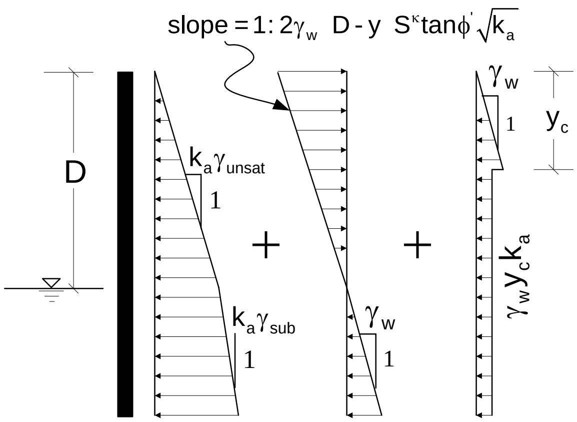
## 6 EARTH PRESSURE CALCULATIONS FOR A NON-COHESIVE BACKFILL MATERIAL

Figure 7 shows the earth pressure distribution diagram for non-cohesive backfill assuming saturated conditions using conventional soil mechanics.




$$P_a = k_a\gamma_{sat}z - 2c'\sqrt{k_a}$$

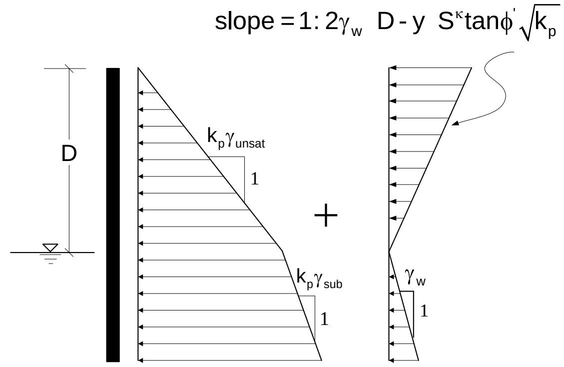
Figure 7. Active earth pressure distribution diagram for a coarse-grained soil under saturated condition (i.e., for non-cohesive back fill, c' = 0).




$$P_a = \;\; \gamma z \;\; k_a - 2 \;\; c' + \;\; u_a - u_w \;\; S^{\kappa}\tan\phi' \;\; \sqrt{k_a} \qquad c' = 0$$

Figure 8. Earth pressure distribution diagram taking account of matric suction and considering the influence of the tension crack.

The earth pressure distribution diagram taking account of matric suction and the matric suction induced tension crack can be represented as shown in Figure 8. The passive earth pressure diagram extending the mechanics of unsaturated soils can be estimated as shown in Figure 9.




$$P_p = \;\; \gamma z \;\; k_p + 2 \;\; c' + \;\; u_a - u_w \;\; S^{\kappa}\tan\phi' \;\; \sqrt{k_p}$$

Figure 9. Passive earth pressure distribution behind the retaining structure for granular material (non-cohesive) taking account of matric suction.

## 7 FACTOR OF SAFETY FOR AN EXAMPLE PROBLEM USING THE CONVENTIONAL AND THE PROPOSED APPROACH

An example problem is presented in this section for calculating lateral active pressures on a retaining wall along with the factor of safety values using both the conventional and the proposed unsaturated soil mechanics approaches using two different soils (i.e., Botkin silt and Indian Head till). The geometry of the retaining wall and the properties of the backfill soils are presented in Figure 10. The ground water table is assumed to be at the depth of 14m below the backfill surface. The surface of the wall is assumed vertical and smooth with no friction developing between backfill soil and the wall. Figure 11 (a) and (b) show the SWCCs (for Botkin silt and Indian Head till) and the variation of shear strength with respect to matric suction obtained using Eq. 5, respectively.

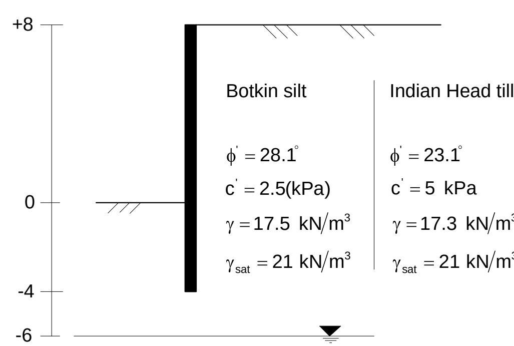



Figure 10. Geometry of the wall and the properties of the backfill.

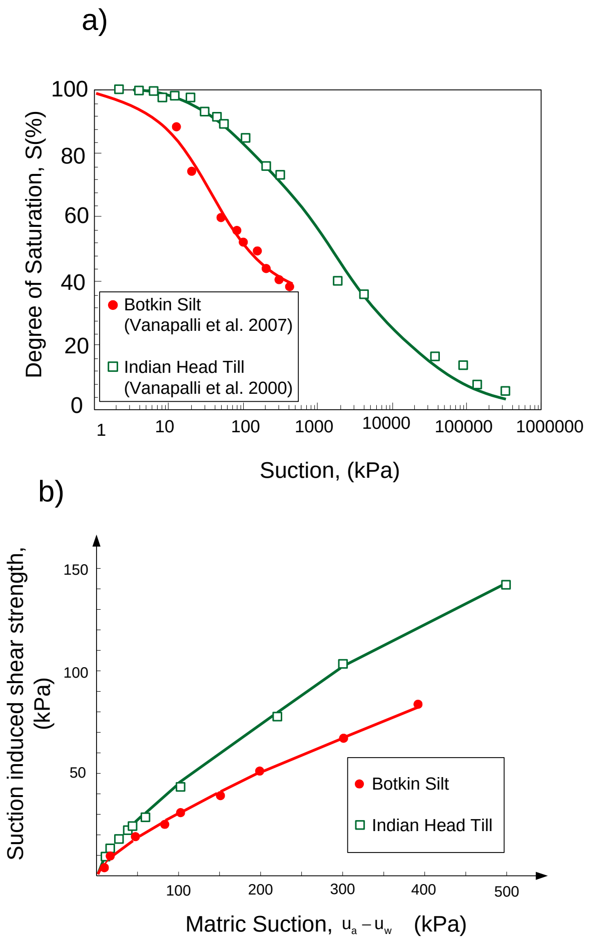



Figure 11. (a) Soil-Water Characteristic Curves of Botkin silt and Indian Head till (b) Variation of shear strength with respect to matric suction.

As discussed earlier, ϕ' value is independent from matric suction (Vanapalli 2009), therefore, the coefficients of active and passive pressure are constant for both saturated and unsaturated conditions.

Figure 12 (a) and (b) show the active pressure distribution diagram for saturated and unsaturated conditions for Botkin silt, respectively using the ESA. In this method, the distribution of matric suction above the ground water table is assumed to be hydrostatic in nature (i.e., the pore-water pressure increases linearly above the GWT). The corresponding water content value with respect to different matric suction values can be estimated from the measured SWCC (see Figure 11). The active earth pressure variation with respect to the depth can be calculated from the unsaturated shear strength values estimated for different matric suction values using Eq. [5]. The factor of safety using conventional soil mechanics indicates that the retaining wall is in a state of unstable condition. On the contrary, the analysis results taking account of the influence of matric suction on the active earth pressure shows that the retaining wall is stable. This example encourages using Innovative construction techniques to maintain unsaturated conditions in the backfill such that cost effective and economical retaining walls can be constructed in engineering practice. More recently, such techniques are being proposed with geocomposites and geotextiles (McCartney et al. 2008).

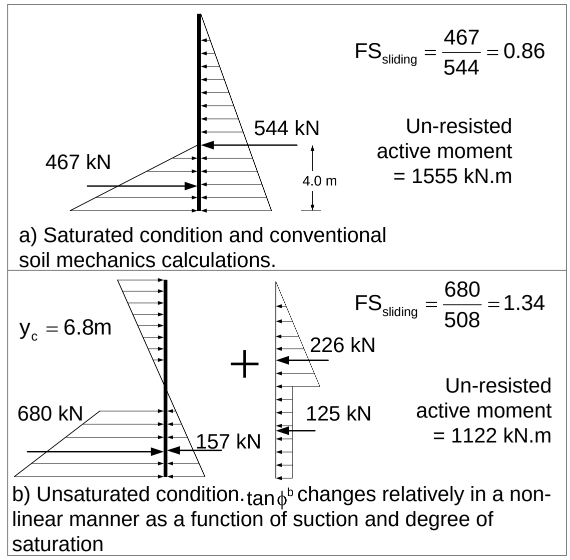



Figure 12. Earth pressure distribution extending effective stress approach (a) conventional approach and (b) proposed approach using the mechanics of unsaturated soils.

The TSA approach for unsaturated fine grained (UFG) soils is extended using properties of the Indian Head till for the native fill material. The same wall geometry shown in Figure 10 is used for comparison purposes. The earth-pressure coefficient for backfill material is assumed to be 1.0 for engineering design (i.e., $k_a = 1$) following Canadian Foundation Engineering Manual (4th edition) provisions for “silt and clayey silts”. This coefficient is used for both soils both in saturated and unsaturated conditions.

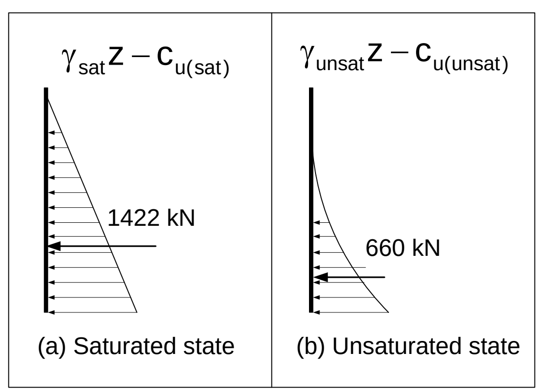



Fig. 13. Earth pressure distribution extending effective stress approach (a) conventional approach and (b) proposed approach using the mechanics of unsaturated soils.

Figure 13 shows the earth pressure distribution extending the conventional TSA approach for saturated soils and the proposed TSA for unsaturated soils presented in this paper. The variation of $c_{u(unsat)}$ with respect to matric suction was estimated using Eqn, [16]. The required information for extending the TSA approach for unsaturated soils include shear strength of saturated soil, $c_{u(sat)}$ which is 11.2 kPa and the SWCC (see Figure 11a). The comparison shows the significant differences in results using the conventional and the proposed approach. The resultant earth pressure for unsaturated conditions is approximately 50% in comparison to saturated conditions for the Indian Head till.

## 8 SUMMARY

This paper provides a background of how mechanics of unsaturated soils can be extended in the estimation of earth pressures on retaining structures using the mechanics of unsaturated soils. Some examples are provided extending both effective and total stress approaches for explaining the differences in the estimation of earth pressures using conventional approach and employing the mechanics of unsaturated soils. These examples highlight how conservative the conventional approach is in the estimation of earth pressures for backfills that are in a state of unsaturated condition.

## 9 ACKNOWLEDGEMENT

Authors would acknowledge the help received from Dr. Won Taek Oh in editing and formatting of this paper.

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