FINITE ELEMENTS AND WINKLER MODEL APPLIED TO RETAINING WALLS DESIGN

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ABSTRACT
This work presents a numerical study of the structural behavior presented by concrete retaining walls in frictional soils, as well as results from a comparison among two computer programs based on complementary approaches: Finite Element Method (FEM) and the Winkler Spring Model (WSM). The simulation using a comercial FEM program is the reference for estimating maximum displacements and bending moments, which are used to validate the analysis with the WSM. The works also aim at calibrating stiffness coefficients for the Winkler springs, enabling this approach to be adopted as a simple and expeditious way to estimate moments and displacements of retaining structures.

RÉSUMÉ
Ce travail présente une étude décomposé en numéros du comportement structurel présenté par les rideaux de béton face à des sols de frottement, aussi que, une comparaison des résultats entre deux programmes informatiques utilisés pour chaque méthode basées en deux approches complémentaires: La méthode des Éléments Finis (MEF) et le modèle des Ressorts de Winkler (MRW). La simulation à l'aide d'un programme FEM est la référence pour l'estimation des déplacements et des moments de flexion maximales, ces valeurs sont utilisées dans la validation de l'analyse en utilisant le MRW. Ce projet envisage aussi calibrer les coefficients de rigidité du modèle Winkler utilisés sur MATLAB, permettant à cette approche d'être utilisé comme une forme simple et rapide d'estimer les moments et des déplacements de structures de soutènement.

1 INTRODUCTION

Conventional approaches to geotechnical design of retaining structures consider soil stress fields that are analytically calculated and provide the basis for a limit state equilibrium analysis. More sophisticated approaches are nowadays available that are resourced with numerical techniques, like the *Finite Elements Method* (FEM) to assess the soil stresses loading retaining structures, as well as the stresses in the structural elements. These methods are suitable for geometrically well defined problems, but sometimes lack practicality in a pre-desing stage, where elements re-meshing may be too often required. Optionally, the *Winkler Spring Model* (WSM) may be accurate enough, and hence useful, for initial estimates of wall moments and displacements, even though the entire stress distribution behind the wall may not be represented.

This paper also seeks to evaluating the soil pressures and the wall structural response by means of a computer program developed on the basis of the WSM. The springs are calibrated to enforce compatibility between WSM and the FEM, and some practical rules are derived to ensure a consistent definition of their stiffnesses. The wall geometry adopted for these calibrations is depicted in Figure 1(a), which dimensions are varied in a parametrical study.

2 METHODOLOGIES

2.1 FEM with Plaxis®

The comercial program Plaxis® is based on the finite element method and is a reliable tool for assessing deformations and stability in geotechnical design. Simulations can use different constitutive models, from a simple linear stress-strain relationships to any modelled rheological phenomena.

In the present paper, Plaxis was used to evaluate the stress distribution acting against a retaining wall illustrated in Figure 1(a), considering a Mohr-Coulomb model for sands with a dry weight volume of $16KN/m^3$. A total of 24 cases were simulated in Plaxis, twelve to a bending stiffness wall of $1.6\times10^5 kN/m^2/m$ corresponding to a 0.4m thickness, and twelve for a stiffness of $5.4\times10^5 kNm^2/m$, corresponding to a wall thickness of 0.6m. In a parametric approach, the friction angle (Φ) ranged from 30˚ to 42˚, with soil Young's Modulus (E_s) of 4MPa, 20MPa and 40MPa. The concrete Young's Modulus (E_c) was kept constant as 30GPa.

2.2 WSM program

The WSM approach was computationally implemented as a Matlab script. The calculation process is iterative and the soil is represented by one-dimensional elastic-plastic spring elements, as shown in Figure 1(b). The yielding point of the elastic- perfectly plastic springs are related with the active and passive limit soil pressures. The wall is represented by linear beam elements, with two degress of freedom per node, horizontal displacement and in-plane rotation. Deformations by axial compression are not taken into consideration. The discretization length is 5cm, ensuring an adequate accuracy level for the problem a floating concrete wall behavior that serves as a restraint for excavations.

The calculations for the wall deformation process are iteractive and summarized below.

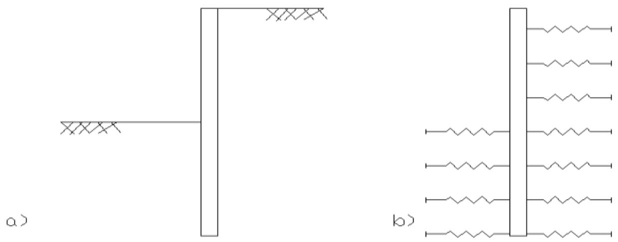

Figure 1. Represention of a retaining wall an excavation: a) real situation, b) Winkler model

Firstly, the soil pressures along the wall are estimated, as well as the resulting horizontal forces and the active and passive limits. Pore pressures are not considered. The soil stiffness, represented by the Winkler springs, is added to the diagonal of the wall stiffness matrix and the horizontal loads added to corresponding degree of freedom in the load vector. The system is solved and the wall displacements are calculated. Once these displacements are known, the elastic-plastic constitutive relations of the springs are checked out. If the plastic limit is reached, the spring stiffness is decreased in order to keep the soill pressure within the active or passive limits. Hence the system in solved again, iteractivelly, until convergence is reached. If the convergence does not occur, equilibrium is not possible and the system is unstable.

The main input parameters for a parametrical study are the wall length and thickness, the soil parameters, the elevation of the bottom of each soil layer, excavation depth, specific weight, friction angle, cohesion intercept, soil elastic stiffness per area, K_{Es}, and the dimensionless coefficients K_{wA} (Winkler active coefficient) and K_{wP} (Winkler passive coefficient).

3 RESULTS

Horizontal displacements at the top of the retaining wall obtained with FEM are analysed in conjunction with maximum bending moments. Calculated values are hence compared with those estimated by the WSM.

Plaxis results from the parametrical analysis are presented as diagrams describing axial forces, shear forces and bending moments. An example of Plaxis output for the case of sand friction angle of 30˚, soil Young Modulus 4GPa, 5m excavation, and wall thickness 40cm, is presented in Figure 2. The equivalent results from the Matlab script are presented in Figure 3.

Having Plaxis as reference, horizontal displacements and maximum bending moments for the top of the retaining wall are compared with those from Matlab script such that at each run the parameters K_{wA}, K_{wP} and K_{Es} are adjusted to enforce compatibility between both methods (FEM and WSM). Physically, the coefficients K_{wA}, K_{wP} for the Winkler model are the same as the active and passive coefficients for the Mohr-Coulomb model, because with these two factors the program calculates acting pressures on the retaining structure.

Figure 2. Example of Plaxis FEM output

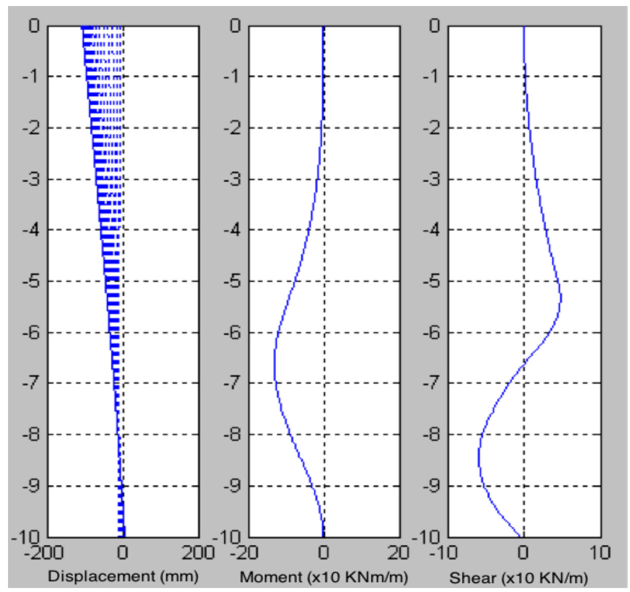

Figure 3. Matlab script for WSM output

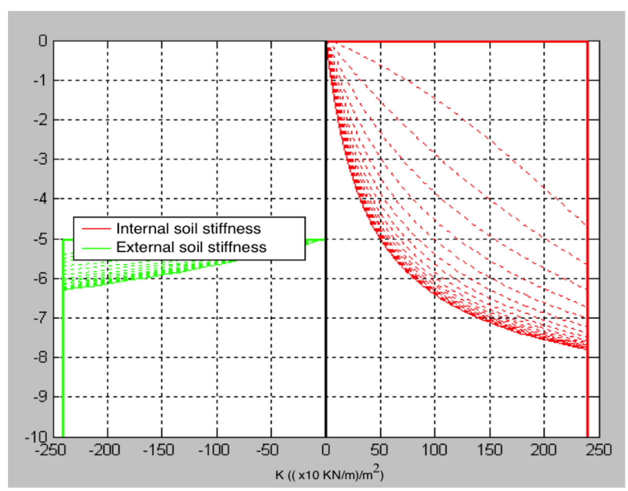

Figure 4. Soil stiffness decay during iteration, represented along the concrete wall, passive and active sides

Figure 4 shows the soil stiffness evolution and its variation with depth. The fast convergence of the iteractive process can be observed by the dotted lines reaching a well defined limit at both passive and active sides of the wall. The convergence process may also be observed in Figure 3, in the dotter lines showing the wall displacement evolution as the soil stiffness is decreased by plastification. Usually, it has been observed that less than 20 iteractions are requided for convergence, and the computational time is really not a matter of concern.

4 INTERPRETATION OF THE RESULTS

Interpretation of the results is focused on the observed relationship between the limit state of active and passive pressures calculated in FEM using a Mohr-Coulomb model and the coefficients adopted in the WSM.

From the analysis, it is possible to observe that the Winkler active coefficient (K_{wA}) depends on soil friction angle, as the Rankine active pressure coefficient (K_A) does. Figure 5 shows that K_{wA} also depends on the ratio between excavation depth (H_{esc}) and total wall length (H_{total}). It is once again emphasized that K_{wA} was obtained after fitting Matlab to Plaxis data.

The Rankine passive pressure coefficient is shown to depend solely on the friction angle. The Winkler passive coefficient seems to depend only on the ratio between the excavation depth and the total wall length, as shown in Figure 6.

Database from Figure 5 and Matlab data output was used to establish a set of equations to guide the choice of coefficients in the Winker model. Coefficient K_{wA} was expressed as a function of the friction angle and the "H_{esc} / H_{total}" ratio with the equation:

$$\mathbf{K_{wA}} = \tan\left(\frac{\pi}{4} - \frac{1,2 * \Phi}{2}\right)^2 \left(\frac{\mathbf{H_{esc}}}{\mathbf{H_{total}}} + 0,35\right) \qquad [1]$$

Figure 7 makes clear that, for different sands and different excavation levels, the ratio "K_{wA} (adjusted) / K_{wA}(equation)" exhibits little variation and is close to unity.

Similarly, an expression for the *Winkler's* passive coefficient, K_{wP}, was proposed from the observation of Figure 6 as:

$$\mathbf{K_{wP}} = 10\left(\frac{\mathbf{H_{esc}}}{\mathbf{H_{total}}}\right) - 1 \qquad [2]$$

Figure 8 also indicates that the relationship "K_{wP}(adjusted) / K_{wP}(equation)" is close to unity, giving a perfect setting for the Winkler passive coefficient, which is not dependent on the friction angle.

From the sole observation of fitted data, it was found that K_{Es} is of the same order of magnitude as the Young's soil modulus (E_s) and is affected by the excavation depth. As a result, the following equation was proposed:

$$\mathbf{K_{Es}} = 3\frac{\mathbf{E_s}}{\mathbf{H_{esc}}} \qquad [3]$$

With this set of parameters, further cases were analyzed in Matlab, allowing comparisons for sand with different friction angles to predict horizontal displacements and bending moments.

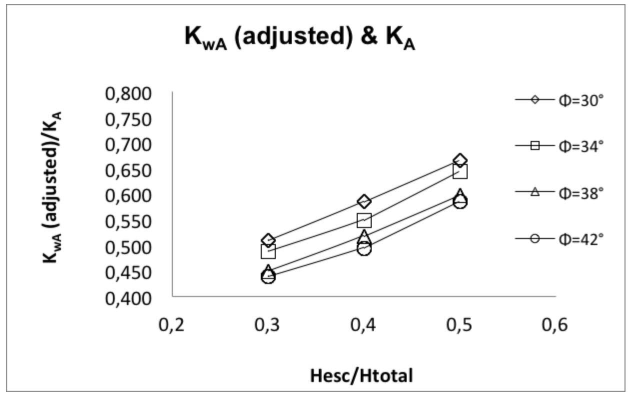

Figure 5. K_{wA} and K_A coefficients

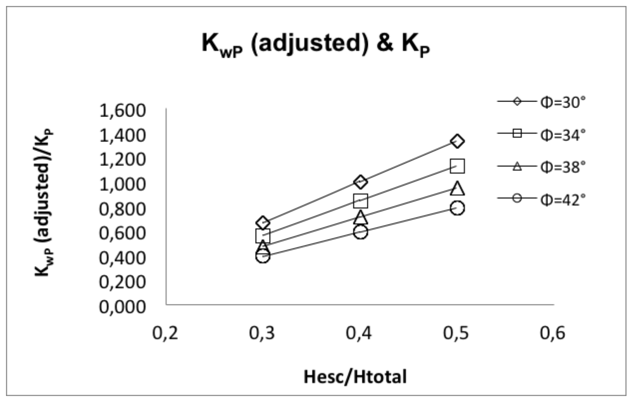

Figure 6. K_{wP} and K_P coefficients

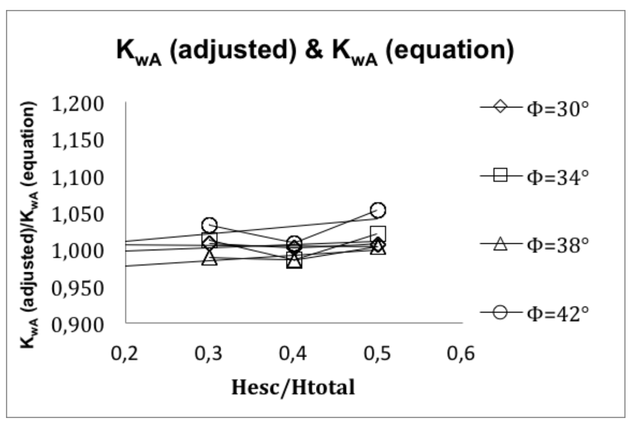

Figure 7. K_{wA}(adjusted) and by K_{wA}(equation) coefficients

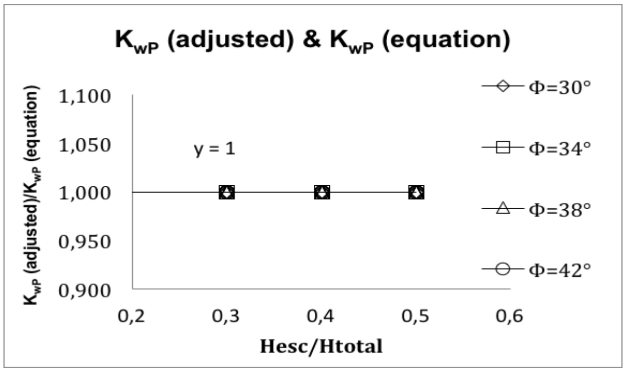

Figure 8. K_{wP}(adjusted) and K_{wP}(equation) coefficients

Figures 9 and 10 show a sound aggrement between both approaches, corroborating the suggested equations for parameters K_{wA}, K_{wP}, e K_{Es}. From this comparative analysis, it is possible to suggest that results expressed in terms of horizontal displacements and bending moments are similar in *WSM* and *FEM* analysis, in spite of the simplifications adopted in the former. The lines of linear approximation of each graph gives R^2 in the range between 0.987 and 0.998, confirming the accuracy of comparison between the two methods.

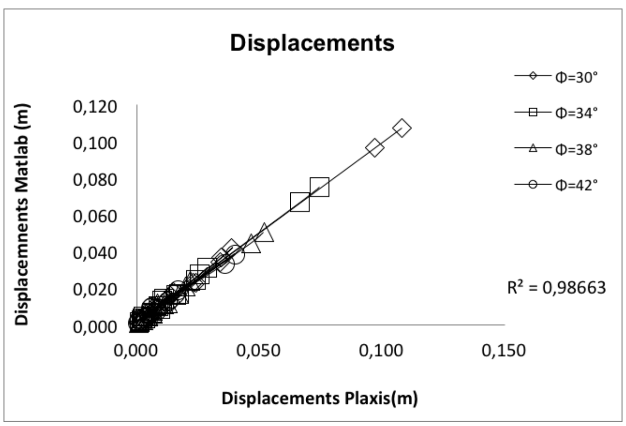

Figure 9. Displacements calculated for all excavation phases with the two wall stiffnesses and friction angles of 30 °, 34 °, 38 ° and 42 °

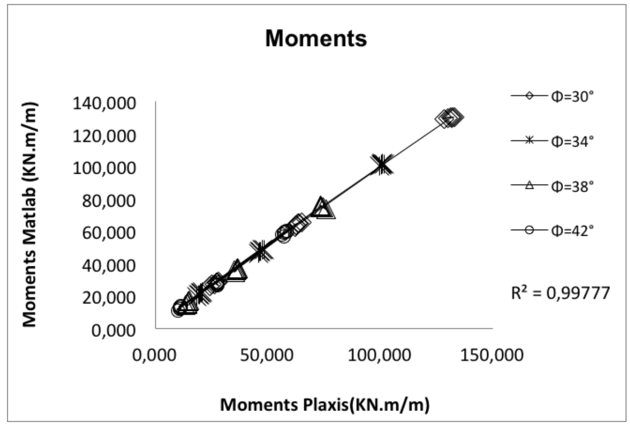

Figure 10. Bending moments calculated for all excavation phases with the two wall stiffnesses and friction angles of 30 °, 34 °, 38 ° and 42 °

5 CONCLUSIONS

Geotechnical and structural engineers often adopt different approaches to the design of retaining structures. Whereas the *Finite Element Method* became a routine in geotechnical practice, mainly due to the complexity of soil stress fields, from the structural engineer point of view the *Winkler Spring Model* remains an appealing alternative to predict displacements, shear and bending moments.

In this context, the present paper attempts to evaluate the compatibility between these two design approaches. Throughout a set of calibrations it has been demonstrated that a correct choice of Winkler coefficients might produce results which are in agreement to finite element calculations, provided that soil shear strength and stiffness are properly taken into account.

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