Analysis of laterally loaded pile using the combination of SW model and the imbedding method

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ABSTRACT
Beam on elastic foundation (BEF) theory provides an efficient solution for problem of laterally loaded pile. The accuracy of such solution depends upon the characterization of soil-pile interaction. The strain wedge (SW) model provides a theoretical link between the one-dimensional BEF and the corresponding three-dimensional soil-pile interaction. However, the implementation of SW model requires an iterative solution of BEF. In this paper the SW analysis of laterally loaded pile is simplified by the application of imbedding method. The imbedding method is an efficient approach for the analysis of BEF. In the imbedding method the beam is divided into n nodes and the information on shear and moments at adjacent nodes are related through reactions and six imbedding coefficients. Results of study show that the combination of the SW model and the imbedding method provides an efficient approach for the analysis of laterally loaded piles.

RÉSUMÉ
Poutre sur fondation élastique (BEF) la théorie fournit une solution efficace pour le problème de la pile chargés latéralement. La précision de cette solution dépend de la caractérisation de l'interaction sol-pieu. Le coin souche (SW) modèle fournit un lien théorique entre les unidimensionnel BEF et le correspondant interaction en trois dimensions sol-pieu. Toutefois, la mise en œuvre du modèle SW exige une solution itérative de BEF. Dans cet article l'analyse de SW pieu chargé latéralement est simplifié par l'application de la méthode de plongement. La méthode de plongement est une approche efficace pour l'analyse de BEF. Dans la méthode de plongement le faisceau est divisé en n nœuds et les renseignements sur le cisaillement et des moments au niveau des nœuds adjacents sont liés par des réactions et six coefficients plongement. Résultats d'études montrent que la combinaison du modèle SW et la méthode de plongement fournit une approche efficace pour l'analyse des pieux chargés latéralement.

1 INTRODUCTION

The response of piles to horizontal loads is an important foundation design problem. The pile response to lateral loads is generally nonlinear and depends on the properties of soil and pile as well as their interface condition, and hence represents a complex soil-structure interaction problem. Different approaches are used to evaluate the lateral capacity and response of piles, including: Brom's method (Brom 1965), finite element (FE) formulations (e.g. Wolf and Arx, 1978), semi-analytical and boundary element (BE) formulations (Douglas & Davis 1964, Poulos, 1971, 1972), the p-y curve method (Reese & Matlock 1956, Matlock & Reese 1960, Reese 1977) and its generalized form known as the beam-on-a-nonlinear Winkler foundation (BNWF) method (Gazetas and Mylonakis, 1998).
Even though it is empirically based, the p-y method has gained popularity because of its simplicity and its ability to simulate the nonlinear behaviour of pile under lateral loads. However, the p-y curves used in this method do not account for some important factors such as the pile properties (mechanical and geometrical) or soil continuity. Some of these deficiencies are accounted for using advanced BNWF that incorporate the concept of p-y curves to describe the force-deformation relationship for the soil (e.g. Allotey and El Naggr, 2008). Alternatively, a procedure entitled strain wedge method is developed for the analysis of flexible piles under lateral loading (Norris 1986). The strain wedge model (SWM) combines features of the one dimensional beam on elastic foundation (BEF) and the corresponding three-dimensional soil-pile interaction. In this model, the lateral displacement pattern of the pile is related to the soil strain throughout the developing passive wedge in front of the pile. However, the implementation of SWM requires an iterative solution of the BEF.
In this paper, a simplified SWM is proposed by applying the imbedding procedure. In this simplified method, the beam is divided into n nodes and the shear and moments at adjacent nodes are related through reactions and six imbedding coefficients. A computer code has been developed based on the proposed simplified SWM and has been verified by solving some benchmark examples.

2 STRAIN WEDGE MODEL:

A brief description of strain wedge model (SWM) is presented in this section. The SWM provides an approach to predict the response of a flexible pile under the lateral loading. In this model a three-dimensional passive wedge of soil is developed in front of the deflected pile (Figure 1). The stress-strain-strength behaviour of the layered soil in the 3D wedge model is related to the one dimensional BEF parameters. Thereafter, the response of the pile under lateral loading is obtained by solving the BEF equation

The deflection pattern of the pile is assumed to be linear over the controlling depth of the soil near the pile top, resulting in a linearized deflection angle, δ (Figure 2). Any changes in the uniform strain in the developing wedge result in the changes in the shape and depth of the passive wedge, and in the state of loading and pile deflection. The mobilized geometry of the passive wedge depends on the properties of the soil and the global equilibrium between the soil and the loaded pile. This 3D soil-pile interaction is accounted into consideration by calculating the subgrade reaction modulus, E_s, of soil at any level during pile loading or soil strain. The developed p-y curve based on this concept incorporates both soil and pile properties [9].An iterative process is performed to satisfy the equilibrium between the geometry of the soil passive wedge and the deflection pattern of pile for any level of loading. More details about the strain wedge model are available in the literature (Ashour et al. 1998).

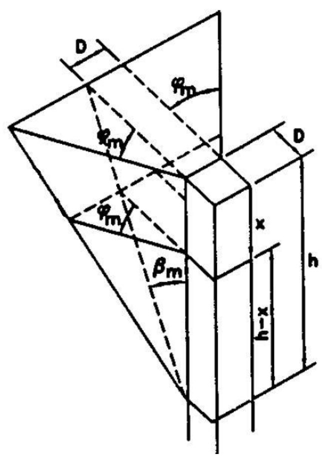


Figure 1. Passive Wedge

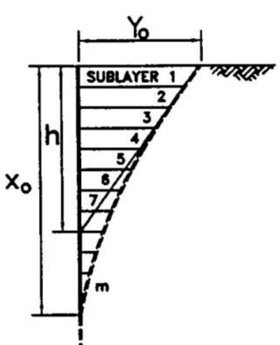


Figure 2. Linearized deflection

3 IMBEDDING METHOD

The imbedding method is a simple and powerful method to solve the equation of beam on Winkler foundation (Kadivar & Ghahramani 1990). In this paper, the imbedding method is reformulated to account for the variation of subgrade modulus and subsequently the nonlinear behaviour of soil.

As shown in figure 3, the beam on nonlinear Winkler foundation is divided into (n-1) equal length sections. The loads and moments are assumed to act only at the nodes. It should be noted that in the present representation, the nodal moments are neglected. The relation between the pressure transferred to soil foundation, p_s , and the related deflection, Y , is

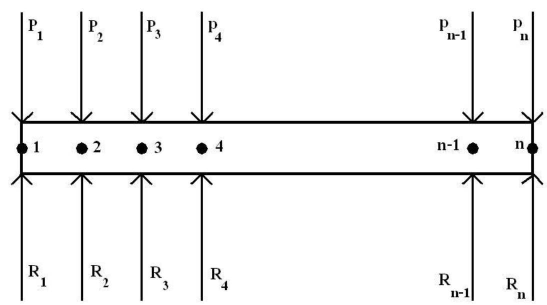


Figure 3. Beam on Winkler foundation divided to n-1 segment

$$p_s = KY \qquad (1)$$

where K is the modulus of subgrade reaction. Assuming $R_1, R_2, ..., R_n$ as nodal reactions, the value of nodal reaction at each node is obtained by

$$R_i = K_i Y_i \Delta L B \qquad (2)$$

where ΔL is the length of each section and B is the width of the beam. According to figure 4, it is apparent that the shear and moment at the node j+1 can be expressed as follow

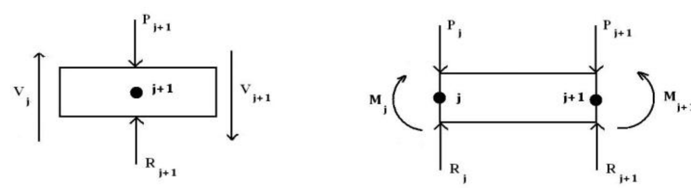


Figure 4. Shear forces and moments on an element

$$V_{j+1} = V_j + R_{j+1} - P_{j+1} \qquad (3)$$

$$\frac{M_{j+1}}{\Delta L} = \frac{M_j}{\Delta L} + V_j \qquad (4)$$

Applying the imbedding method, it is assumed that

$$V_j = A_j R_j + B_j R_{j-1} + C_j \quad (5)$$

$$\frac{M_j}{\Delta L} = D_j R_j + E_j R_{j-1} + F_j \quad (6)$$

where A_j to F_j are the basic coefficients which would be found later.

Considering the beam equation as $M = -EI \dfrac{\partial^2 Y}{\partial X^2}$ and using the finite difference method gives

$$M_j = -EI\left(\frac{Y_{j+1} + Y_{j-1} - 2Y_j}{\Delta L^2}\right) \quad (7)$$

where E is the young modulus of beam's material and I is the moment of inertia of beam's cross section. Substituting Eq. (2) into (7) and rearranging the formulation, the following relation can be obtained

$$R_{j+1} = \zeta_1 \frac{M_j}{\Delta L} + \zeta_2 R_{j-1} + \zeta_3 R_j \quad (8)$$

Where

$$\zeta_1 = -\left(\frac{K_{j+1} \Delta L^4 B}{EI}\right)$$

$$\zeta_2 = -\left(\frac{K_{j+1}}{K_{j-1}}\right) \quad (9)$$

$$\zeta_3 = \frac{2K_{j+1}}{K_j}$$

Substituting equations (5), (6) and (8) into equations (3) and (4), and rearranging the derived equations, leads to the following equations

$$\begin{aligned}&\left[\zeta_1 D_j A_{j+1} + \zeta_3 A_{j+1} + B_{j+1} - A_j - \zeta_1 D_j - \zeta_3\right] R_j + \\ &\left[\zeta_1 E_j A_{j+1} + \zeta_2 A_{j+1} - B_j - \zeta_1 E_j - \zeta_2\right] R_{j-1} + \\ &\left[\zeta_1 F_j A_{j+1} + C_{j+1} - C_j - \zeta_1 F_j + P_{j+1}\right] = 0\end{aligned} \quad (10)$$

$$\begin{aligned}&\left[\zeta_1 D_j D_{j+1} + \zeta_3 D_{j+1} + E_{j+1} - A_j - D_j\right] R_j + \\ &\left[\zeta_1 E_j D_{j+1} + \zeta_2 D_{j+1} - B_j - E_j\right] R_{j-1} + \\ &\left[\zeta_1 F_j D_{j+1} + F_{j+1} - C_j - F_j\right] = 0\end{aligned} \quad (11)$$

The above relations are true for any value of R_j and R_{j-1}. Therefore the coefficients of R and the constant values have to be zero. By applying these conditions

$$A_{j+1} = \frac{B_j + \zeta_1 E_j + \zeta_2}{\zeta_1 E_j + \zeta_2}$$

$$B_{j+1} = A_j + \zeta_1 D_j + \zeta_3 - \zeta_1 D_j A_{j+1} - \zeta_3 A_{j+1}$$

$$C_{j+1} = C_j + \zeta_1 F_j - P_{j+1} - \zeta_1 F_j A_{j+1} \quad (12)$$

$$D_{j+1} = \frac{B_j + E_j}{\zeta_1 E_j + \zeta_2}$$

$$E_{j+1} = D_j + A_j - \zeta_1 D_j D_{j+1} - \zeta_3 D_{j+1}$$

$$F_{j+1} = C_j + F_j - \zeta_1 D_{j+1} F_j$$

Thus the coefficient at node j+1 can be evaluated from node j. At node 1

$M_1 = m$, $V_1 = R_1 - P_1$

where m is the value of moment at initial point. By considering equations (5) and (6)

$A_1 = 1 \quad B_1 = 0 \quad C_1 = -P_1 \quad D_1 = 0 \quad E_1 = 0 \quad F_1 = m$

The coefficient at node (2) can be evaluated from the coefficient at node (1) by equations (12) and the coefficients would be found at each node successively. At the last node

$$M_n = 0\ ,\ V_n = 0 \quad (13)$$

Considering the system of equations of (5) and (6) together with equation (13), R_{n-1} and R_n can be calculated as below

$$R_n = \frac{\begin{vmatrix} -C_n & B_n \\ -F_n & E_n \end{vmatrix}}{\begin{vmatrix} A_n & B_n \\ D_n & E_n \end{vmatrix}} \quad (14\text{-}1)$$

$$R_{n-1} = \frac{\begin{vmatrix} A_n & -C_n \\ D_n & -F_n \end{vmatrix}}{\begin{vmatrix} A_n & B_n \\ D_n & E_n \end{vmatrix}} \quad (14\text{-}2)$$

After determination of R_{n-1} and R_n from (14-1) and (14-2), the nodes are renumbered and the imbedding method would be repeated from bottom to top. By knowing the imbedding coefficients and the values of R_1 and R_2 (i.e. R_n and R_{n-1}), the values of shear, moment and reaction would be determined at each node of the beam.

4 NUMERICAL STUDY

As mentioned before, the analysis of laterally loaded pile by SWM requires the solution of BEF equation iteratively. The efficiency of imbedding method is investigated in the present study to solve the BEF equation and to determine the deflection of laterally loaded pile.

4.1 Elastic analysis of laterally loaded pile

Reese and Matlock (Reese & Matlock 1956) established a series of curves for cohesionless soils for which the elastic modulus of the soil is assumed to increase from zero at the ground surface in direct proportion to the depth. To investigate the accuracy of the proposed method for the analysis of BEF, a laterally loaded steel pipe pile is considered. The pile is 13.0 m long, 0.5 m in diameter. As illustrated in Figure 5, the pile is installed in a soil layer with elastic modulus that increases linearly with depth. The variation of the subgrade modulus with depth is shown in Figure 5.

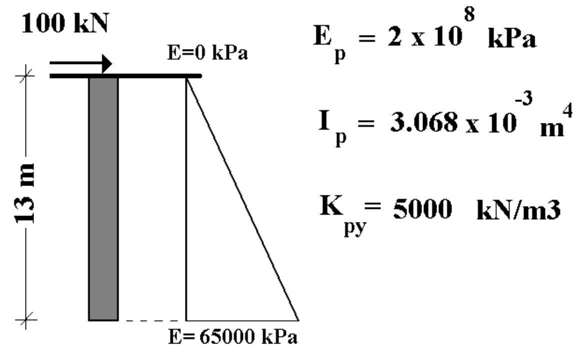
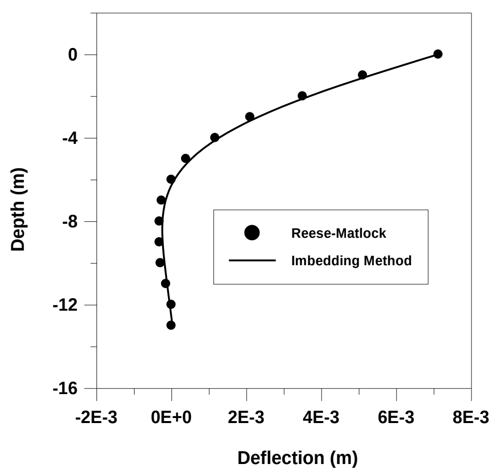
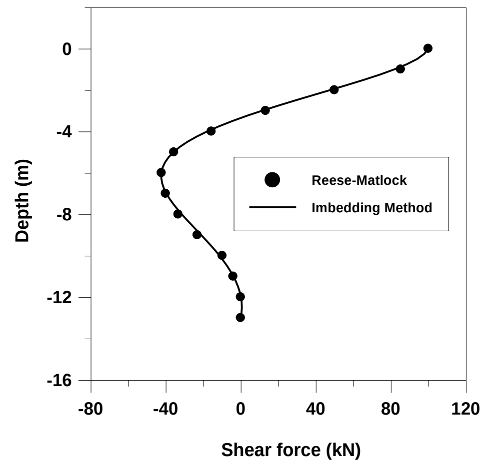


Figure 5. Laterally loaded pile



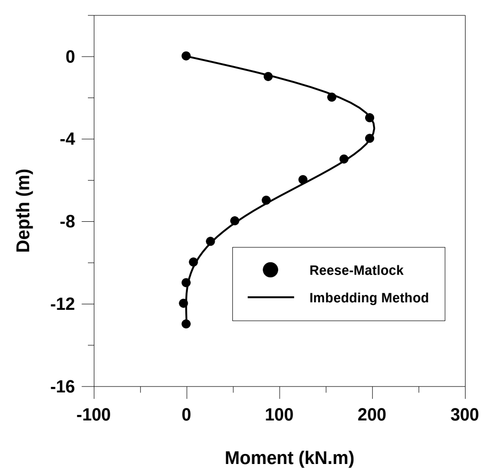
(a)



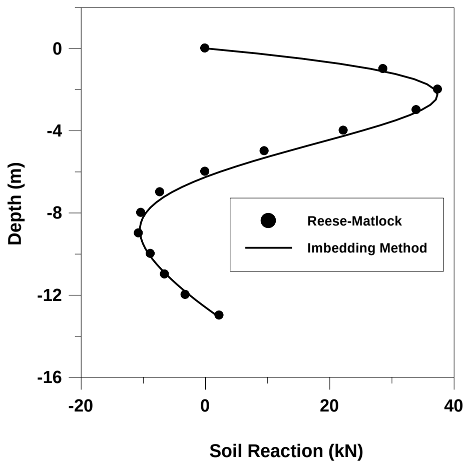
(b)

Figure 6. (a) deformed shape of pile (b) pattern of shear force distribution

The deflected shape of the pile is calculated using the proposed approach and the results are compared with those obtained by Reese and Matlock and the comparison is presented in Figure 6 for deflected shape and moment along the pile, while Figure 7 compares the results from the two methods in terms of shear force and soil reactions along the pile. Figures 6 and 7 show the results agree well.



(a)



(b)

Figure 7. The pattern of (a) moment (b) soil reaction distribution along the pile

4.2 Comparison with measured data

To further examine that ability of the proposed method to predict the performance of piles subjected to lateral loads, it is necessary to compare its predictions with measured field data. For this purpose, the field study conducted by Reese et al. in Mustang Island is considered herein. Reese et al. investigated the behavior of a laterally loaded pile driven in medium dense sand. The pile is subjected to a lateral load applied at the free head at an elevation of 0.3 m above the ground surface. The pile has the flexural rigidity of 1.67E5 kN-m2, the length of 21.3 m and the diameter of 0.61 m. The dense sand has the unit weight of 18 kN/m3 and the internal friction angle of 38°.

The proposed method was used to analyze the pile response to the lateral loads as applied in the experimental program. The results obtained from the analysis are shown in Figure 8 and are compared with the measured values. It is noted from Figure 8a that the agreement between the calculated and measured deflected shape is excellent, while Figure 8b shows reasonable agreement between the calculated and measured bending moment. It should be noted that it is more difficult to measure the pile moment with the same level of accuracy of the deflected shape. It is worth mentioning that the implementation of the imbedding method noticeably increased the efficiency and speed of the computational effort compared to the finite element methods. The simplicity of the mentioned method makes it appropriate for the hand calculation analysis of the laterally loaded pile.

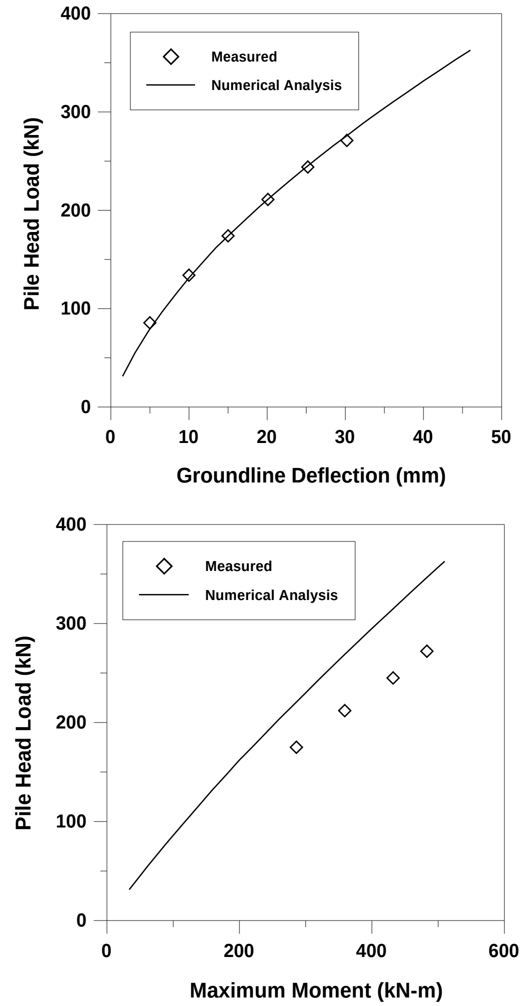


Figure 8: Measured and predicted response of laterally loaded pile

5 CONCLUSION

A simplified SWM, which incorporates the imbedding method, is proposed in order to solve the equation of BEF for the analysis of laterally loaded piles. The imbedding method simplifies the analysis of the BEF problem and eliminates the need for matrix inversion in the problem solution. This feature is especially important in dynamic problems which comprise nonlinear analyses as well as iterative solution of BEF. The results of analyses and comparisons with theoretical and experimental field studies confirmed the ability of the simplified SWM to evaluate the response of laterally loaded piles with sufficient accuracy.

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