Active earth pressure with numerically evaluated seepage effect

Pérsio Leister de Almeida Barros
School of Civil Engineering, Architecture and Urbanism, University of Campinas, SP, Brazil
Petrucio José dos Santos Jr.
Maccferri do Brasil Ltda., Jundiai, SP, Brazil

ABSTRACT
A method for the evaluation of the active earth pressure by a soil mass including the effect of water seepage with the determination of the seepage effect by the Boundary Element Method (BEM) followed by the trial wedge limit equilibrium procedure is presented. This process can be applied either for the case where the whole soil mass is subjected to the water seepage, or for the case where the seepage occurs only in a portion of the soil mass, which is delimited by a phreatic surface. For the last case, the BEM should be applied in an iteractive procedure to find the position of the phreatic surface. The numerical results obtained with the presented method shows that the seepage may significantly increase the active earth pressure against the retaining structure.

RÉSUMÉ
Une méthode est présentée pour la détermination de la poussée de terre en tenant compte de l'effet de la infiltration de l'eau dans le massif avec mur de soutènement. On utilise la méthode des éléments de frontière (BEM) pour déterminer l'effet de percolation, suivie d'une procédure d'équilibrage limite de coins du sol. Ce procédé peut être appliqué soit dans le cas ou toute le massif est soumis à la infiltration, soit dans le cas où la infiltration ne se produit que dans une partie du massif, délimitée par une nappe phréatique. Dans ce dernier cas, le BEM doit être appliquée dans une procédure itérative pour déterminer la position de la nappe phréatique. Les résultats numériques obtenus avec la méthode présentés ont montrée que la infiltration peut augmenter de manière significative la poussée active sur la structure de soutènement.

1 INTRODUCTION

The effect of seepage forces on the lateral active earth pressures on retaining structures has long been recognized by the geotechnical engineers (Terzaghi, 1943). Some methods have been developed along the years to cope with this effect. The most classical of those methods includes the drawing of a flow net through the earth mass and determining the water pressure along the potential failure surface from that flow net (Lambe, 1969). This approach, however, is both inaccurate and time consuming due to the hand-drawing process involved.

As an alternative to the above procedure, analytical or numerical methods could be used in the determination of the water pressure. Analytical methods have been developed to cope with this kind of problem, but the available solutions are restricted to very special cases with simple geometry and boundary conditions (Barros, 2006).

Numerical methods based on the Finite Element Method have also been used in this kind of analysis, but its availability is restricted, in general, to expensive software that can perform coupled analyses of the soil-water system. In the cases which involve phreatic surfaces, the need of specialized software is even greater because non-linear unsaturated seepage has to be considered.

This work proposes the use of the Boundary Element Method in a linear analysis of seepage (Santos Junior, 2010). The result of this analysis is then used in the determination of the active earth pressure, through the trial wedge method, a limit equilibrium procedure derived from the Coulomb's method (Coulomb, 1776).

Two main cases are analysed. The first one (case 1) is a fully saturated soil mass supported by a retaining wall with a drainage layer along the soil-wall interface, as shown in Figure 1.

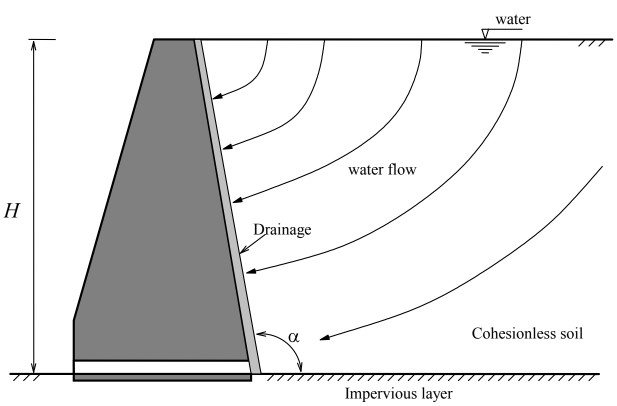


Figure 1: Fully saturated earth pressure against a drained wall (case 1)

The situation shown in Figure 1 may occur during heavy rainstorms. In this case the earth pressure against the retaining wall is believed to reach its peak value.

The second case (case2) is a soil mass with a phreatic surface delimiting a region where the water pressure is greater or equal to zero, supported by a wall equal to the first case. Figure 2 shows a schematic picture for this second case.

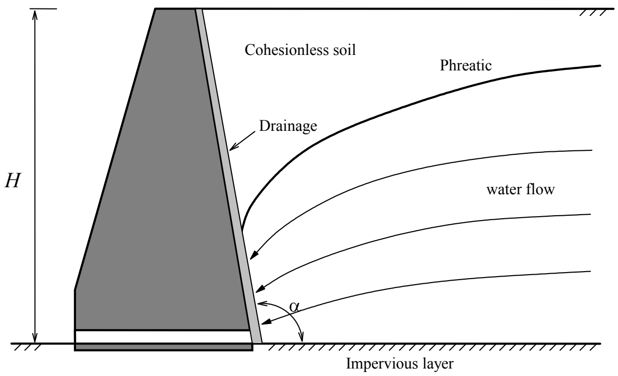


Figure 2: Earth pressure exerted by a soil mass with a phreatic surface (case 2)

The second case may arise for retaining walls built in partially submerged areas, such as retaining and regularization of water stream banks.

In both cases shown above, the water flow toward the wall face increase the earth pressure against the retaining structure. It is important to take this effect in account when evaluating the earth pressure.

Two-dimensional state for both the soil strain and the water flow is assumed. Also, the retained soil is considered cohesionless and homogeneous.

2 SEEPAGE SOLUTION

In the present study, the water seepage through the retained soil is assumed to be independent of the wall displacement necessary to develop an active state within the soil mass. A steady state flux regimen is assumed in both cases described above.

The mathematical model used to obtain the solution for a steady state seepage through a saturated porous medium is characterized by the differential equation (Harr, 1962):

$$\nabla^2 h(x,z) = \frac{\partial^2 h(x,z)}{\partial x} + \frac{\partial^2 h(x,z)}{\partial z} = 0 \qquad [1]$$

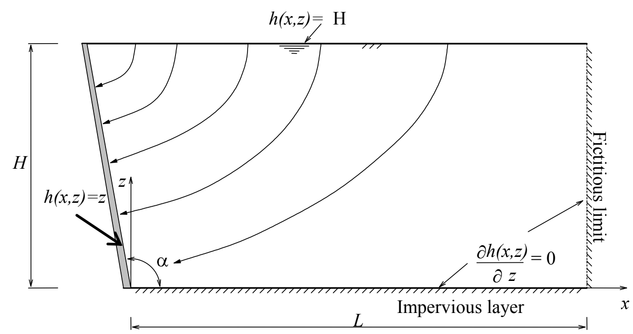
where $h(x,z)$ is the water total head or potential, which is given by:

$$h(x,z) = z + \frac{\mu(x,z)}{\gamma_w} \qquad [2]$$

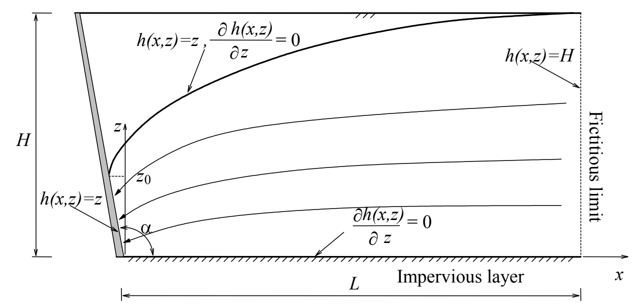
where $\mu(x,z)$ is the water pore pressure and γ_w is the unit weight of water.

The seepage models for cases 1 and 2, which are used in the numerical solution, are shown in Figure 3.

For the seepage problem in case 1, a unitary wall height $H = 1$ may be assumed. Later, the solution can be scaled to the real wall height. The boundary conditions imposed to the mathematical model are shown in Figure 3(a). Along the ground surface the water head is constant and equal to the unity; along the soil-wall draining interface the water pressure vanishes and the water head is equal to the elevation head z; and along the impervious face at the bottom the vertical component of the water flux vanishes.



(a) Case 1



(b) Case 2

Figure 3: Seepage models used in the numerical solution for cases 1 and 2

A fictitious vertical impervious face is placed at a distance L from the wall, in order to close the seepage model. The distance L should be large enough so it has a negligible effect on the results. Numerical experiments conducted on the model showed that for $L = 5H$ this condition is satisfied.

For the seepage problem corresponding to case 2, a unitary wall height $H = 1$ and a fictitious vertical face placed at $L = 5H$ are also used in the numerical solution. But here, the fictitious face is pervious and a constant water head $h(x,z) = H$ is assumed along it. Therefore, this fictitious face is the source of all water that flows through the soil mass.

Two boundary conditions are imposed along the phreatic surface in case 2. The water head along the phreatic surface should be equal to its elevation head z, and there should be no water flux crossing it. This later condition makes the phreatic surface impervious. The phreatic surface position is initially unknown. The determination of this position is done by the numerical procedure, using those two boundary conditions (Menezes and Pulino, 1984).

The solution for the described mathematical models comprises the determination of the potentials or water heads and hydraulic gradients or fluxes within the soil and along its boundaries.

Numerical methods like the Finite Difference Method (FDM), the Finite Element Method (FEM) and the Boundary Element Method (BEM) can be used for the seepage solution. In the present work the BEM is employed due to some of its advantages over the other methods. The BEM requires discretization only along the boundaries of the problem, which reduces considerably the size of the system of equations to solve. The boundary-only discretization also implies that the numerical approximations are assumed only along the problem boundary, which leads to a higher accuracy of the results. Additionally, after the numerical solution along the boundaries are obtained, the magnitude of the potential at any point within the problem domain can be accurately obtained with ease.

In order to apply the BEM, first the differential equation 1 is transformed to an integral equation (Brebbia and Dominguez, 1992):

$$c(x,z)h(x,z) = \int_\Gamma \left[\begin{array}{l} G(x,z;x^*z^*)\dfrac{\partial h}{\partial n}(x^*,z^*) - \\ F(x,z;x^*z^*)h(x^*,z^*) \end{array} \right] d\Gamma(x^*,z^*) \quad [3]$$

where $c(x,z)=1$ for points inside the problem domain. For points on the problem boundary, $c(x,z)$ is a function of the boundary geometry at the point. Also, Γ is the boundary of the problem and n is a unit vector outward normal to this boundary.

In equation 3, $G(x,z;x^*z^*)$ and $F(x,z;x^*z^*)$ are the fundamental solutions for the potential and flux, respectively, at the *field point* (x^*,y^*), due to a unit flux source concentrated at the *source or collocation point* (x,y), in a unbounded medium. Those fundamental solutions are given by:

$$G(x,z;x^*,z^*) = \frac{1}{2\pi}\log\left(\frac{1}{r}\right) \quad [4]$$

$$F(x,z;x^*,z^*) = -\frac{1}{2\pi r}\left[\frac{\partial r}{\partial x^*}n_x(x^*,z^*) + \frac{\partial r}{\partial z^*}n_z(x^*,z^*)\right] \quad [5]$$

In the above equations, $r=\sqrt{(x-x^*)^2+(z-z^*)^2}$ is the distance from the point (x,z) to the point (x^*,z^*) and (n_x,n_z) are the components of vector n.

The domain boundary is divided in *boundary elements*, with three *nodes* in each of them. Along each element, the potential h and the flux $q=\partial h/\partial n$ normal to the boundary is approximated by a quadratic polynomial.

The integral equation 3 is then discretized by the boundary element division and by h and q quadratic polynomials. A linear system of equations having as unknowns the flux normal to boundary surfaces where the water head is imposed, and potentials along the impervious surfaces, is obtained by successively setting the collocation point at the nodes on the boundary. Detailed description of this process can be found in Brebbia and Dominguez (1992).

The solution of the system of equations along with the given potential and flux values along the problem boundary may be used later to evaluate the potential value at any point inside the problem domain, by means of the same approximate, discretized version of the integral equation 3.

For the determination of the phreatic surface position in case 2, an initial guess for the elevation of each node along the surface is adopted. In the present method, the phreatic initial position is set at the ground surface. The impervious boundary condition is imposed to the nodes along this surface and then an initial numerical solution is obtained. This solution furnishes the water head along those nodes, which are then compared with their elevation. Then, the elevation values of those nodes are corrected and new numerical solutions are obtained. This iterative process is repeated until the elevation z set to the nodes along the phreatic surface are close enough to the head h obtained. The final solution provides the position of the phreatic surface, including the elevation z_0 of its exit point, at the wall face.

3 DETERMINATION OF EARTH PRESSURE

The evaluation of the earth thrust, which is the resultant of earth pressures along the soil-wall interface is performed through the equilibrium analysis of the soil wedge delimited by the wall face and a trial failure surface. When the wall movement is enough to mobilize all the shear strength inside the soil mass (active state), a failure surface, which is assumed to be planar, will form. The resulting soil wedge is treated as a rigid body and the forces acting along its boundaries are shown in Figure 4.

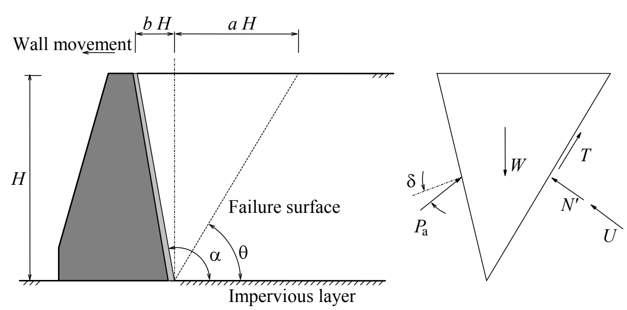


Figure 4: Forces on the soil wedge

The weight W of the soil wedge is given by:

$$W = \frac{1}{2}\gamma H^2 (a+b) \qquad [6]$$

where $a = \cot\theta$ defines the failure plane location, $b = \tan(\alpha - 90^\circ) = -\cot\alpha$, with α and θ indicated in Figure 4, and γ is the unit weight of the soil. In Figure 4, P_a is the active earth thrust, δ is the friction angle along the soil-wall interface, N' is the effective normal force, and T is the tangential force acting on the failure plane. The pore water pressure force U is the resultant of the pore pressures acting along the failure surface.

The unit weight $\gamma = \gamma_{sat}$ for case 1, whereas $\gamma = \bar{\gamma}$ for case 2, considering that γ_{sat} is the saturated soil unit weight and $\bar{\gamma}$ is the soil mean unit weight inside the soil wedge.

For the determination of the pore water force U, the magnitude of the water pressure at a number of points along the failure surface along with a numerical integration scheme is used. The Gauss-Lobatto numerical quadrature method (Abramowitz and Stegun, 1972), with 12 integration points, was selected for the integration and the water pressure at the integration points was obtained from the BEM seepage solution for the potentials inside the seepage domain. The Gauss-Lobatto method was selected because it includes the end points of the integration interval in the list of integration points, and the water pressure at the end points of that list are zero. Thus, only the water pressure at the ten internal integration points are needed. For case 2, it is necessary to determine the location of the intersection of the phreatic surface and the failure surface, in order to find the location of the integration points.

For the evaluation of U, the unitary $H = 1$ seepage solution is used as basis for both case 1 and case 2. For case 1 the unitary solution is scaled by the actual wall height H. On the other hand, the scale factor for case 2 is taken as z_d / z_0, where z_d is the elevation of the exit point of the phreatic surface in the actual problem.

The equilibrium condition of the forces acting on the soil wedge can be expressed as:

$$N' = W\cos\theta + P_a \sin(\theta - \delta - \alpha + 90^\circ) - U \qquad [7]$$

$$T = N'\tan\phi' = W\sin\theta - P_a\cos(\theta - \delta - \alpha + 90^\circ) \qquad [8]$$

which can be rewritten as:

$$N' = W\frac{a}{\sqrt{1+a^2}} + P_a \frac{1 - a(b+f^*) - bf^*}{\sqrt{1+a^2}\sqrt{1+b^2}\sqrt{1+f^{*2}}} - U \qquad [9]$$

$$T = f\,N' = W\frac{1}{\sqrt{1+a^2}} - P_a \frac{a + b + f^* - abf^*}{\sqrt{1+a^2}\sqrt{1+b^2}\sqrt{1+f^{*2}}} \qquad [10]$$

where $f = \tan\phi'$ is the soil coefficient of internal friction and $f^* = \tan\delta$ is the soil-wall interface coefficient of friction. Solving the above equations for P_a results:

$$P_a = W\frac{1 - af\sqrt{1+b^2}\sqrt{1+f^{*2}}}{f + f^* + b(1-ff^*) + a\left[1 - ff^* - b(f+f^*)\right]} + U\frac{f\sqrt{1+a^2}\sqrt{1+b^2}\sqrt{1+f^{*2}}}{f + f^* + b(1-ff^*) + a\left[1 - ff^* - b(f+f^*)\right]} \qquad [11]$$

The maximum value of P_a as a function of a determines the critical failure surface and the magnitude of the active thrust.

4 NUMERICAL RESULTS

In order to test the calculation procedure presented, a retaining wall having face inclination $\alpha = 100^\circ$ was used as example. For this wall, the active thrust was evaluated for values of the soil internal friction angle ϕ between 25° and 45°, and for $\delta = 2\phi/3$.

The seepage numerical solutions for case 1 and case 2 were obtained, and the pore water pressure force U was evaluated as a function of the inclination θ of the failure surface. The ploted normalized values of U are shown in Figures 5 and 6, for cases 1 and 2, respectively.

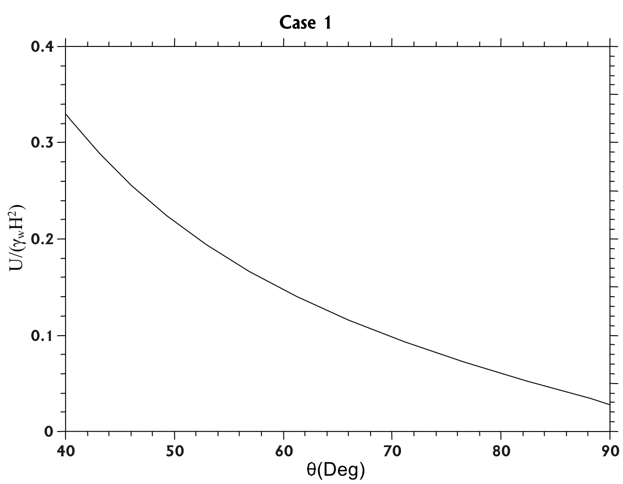


Figure 5: Pore pressure force U as a function of the inclination θ of the failure surface, for case 1 ($\alpha = 100^\circ$)

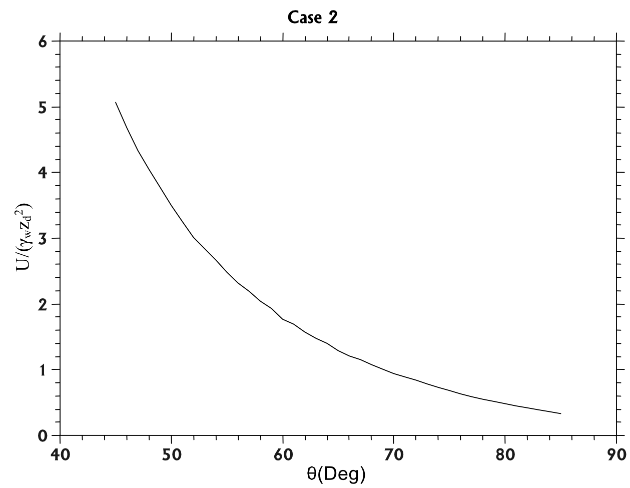


Figure 6: Pore pressure force U as a function of the inclination θ of the failure surface, for case 2 ($\alpha = 100^\circ$)

With the numerical solutions for the seepage, the active earth trust was obtained by means of equation 11. The numerical results for the earth pressure obtained for case 1 and for two elevation ratios $d = z_d / H$ corresponding to case 2 are shown in Figure 7.

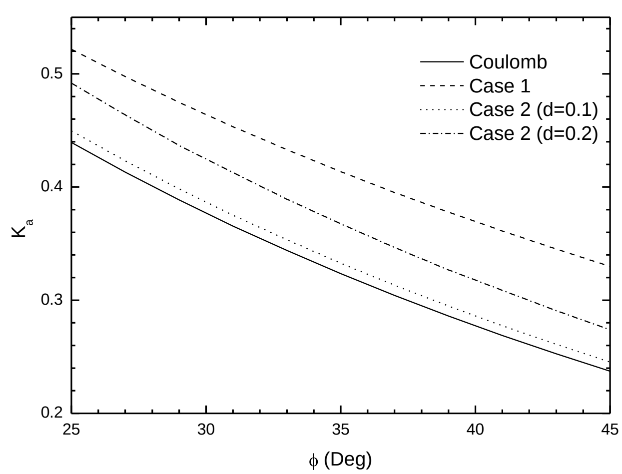


Figure 7: Coefficients of earth pressure for $\alpha = 100^\circ$, $\delta = 2\phi / 3$, $\gamma / \gamma_w = 2$

Figure 7 shows the calculated earth pressure as an equivalent earth pressure coefficient $K_a = P_a / (\frac{1}{2}\gamma H^2)$. Corresponding values of the coefficient of active earth pressure without seepage, calculated by the Coulomb's formula were also plotted in Figure 5 for comparison purposes.

It should be noted that the results for cases 1 and 2 were obtained assuming the ratio $\gamma / \gamma_w = 2$.

The plotted curves in Figure 7 show the influence of the water seepage on the active earth pressure. The seepage increases the earth pressure considerably, with the case 1 showing the largest effect. For case 2, the increase in the active earth pressure is dependent of the elevation of the phreatic surface.

5 CONCLUSION

The evaluation of the active earth pressure by a soil mass including the effect of water seepage can be conveniently and accurately done with the determination of the seepage effect by the Boundary Element Method followed by the trial wedge limit equilibrium procedure. This process can be applied either for the case 1, where the whole soil mass is subjected to the water seepage, or for the case 2, where the seepage occurs only in a portion of the soil mass, which is delimited by a phreatic case. For the last case, the BEM should be applied in a iteractive procedure to find the position of the phreatic surface.

The numerical results obtained with the present method shows that the seepage may significantly increase the active earth pressure against the retaining structure. Therefore, the seepage effect should not be ignored in the structure design.

ACKNOWLEDGEMENTS

This work was partially supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Maccaferri do Brasil Ltda.

REFERENCES

Abramowitz, M. and Stegun, I. A. (Eds.) 1972. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th printing. Dover, New York, NY, USA.

Barros, P. L. A. 2006. A Coulomb-type solution for active earth thrust with seepage. *Géotechnique*, **56**(3): 159-164. doi: 10.1680/geot.2006.56.3.159

Brebbia, C. A. and Dominguez, J. 1992. *Boundary Elements: An Introductory Course*, 2nd Edition. WIT Press/ Computational Mechanics Publications, Southampton, UK.

Coulomb, C. A. 1776. Essai sur une application des regles des maximis et minimis a quelques problemes de statique relatifs a l'arquiteture. *Mem. Acad. Roy. Pres. Divers*, Sav., Paris, **5**, 7.

Harr, M. E. 1962. *Groundwater and Seepage*, McGraw Hill, New York, NY, USA.

Lambe, T. W. and Whitman, R. V. 1969. *Soil Mechanics*, John Wiley & Sons, New York, NY, USA.

Menezes, F. A. and Pulino Filho, A. R. 1984. Uma aplicação do Método dos Elementos de Contorno na determinação de superfície livre em problemas de escoamento não confinado em regime permanente. *Proceedings of the V Congresso de Métodos Computacionais para Engenharia*, Salvador, Brazil.

Santos Junior, P. J. 2010. *Cálculo do Empuxo Ativo com Determinação Numérica da Superfície Freática*. School of Civil Engineering, Achitecture and Urbanism, University of Campinas, Brazil. Master's Dissertation (in Portuguese).
Terzaghi, K. 1943. *Theoretical Soil Mechanics*, John Wiley & Sons, New York, NY, USA.