Shear Wave Velocity Measurements at Municipal Solid Waste Landfills in Michigan

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ABSTRACT

A surface wave based methodology that uses a linear array of 16 geophones and combines active measurements (Multichannel Analysis of Surface Wave technique) and passive measurements (Microtremor Analysis Method) is presented. The methodology is implemented at 13 locations in 4 landfills to measure the shear wave velocity (V_s) of Municipal Solid Waste (MSW) in southeast Michigan. The results in terms of dispersion curves and shear wave velocity profiles are presented. The V_s is generally found to be consistent among landfills and significant differences in V_s are attributed to waste composition variability, site conditions and landfill operation practices. Comparisons are made to evaluate the shear wave velocity variability within each landfill and among different landfills. Shear wave velocities generally increase with depth. Values as low as 75 m/sec are measured near the surface reaching 175-210 m/sec at depths of approximately 25 m. Finally, comparisons are made to MSW V_s profiles reported in the literature. The MSW V_s profiles of the southeast Michigan landfills in this study were generally lower than those reported in the literature for landfills in southern California.

RÉSUMÉ

Une méthodologie basée sur des ondes de surface qui utilise un réseau linéaire de 16 géophones et combine des mesures actives (Technique d'analyse multi-canal des ondes de surface) et des mesures passives (Méthode d'analyse Microtremor) est présentée. La méthodologie est mise en œuvre en 13 endroits dans 4 sites d'enfouissement afin de mesurer la vitesse des ondes de cisaillement (V_s) des déchets solides municipaux (MSW) dans le sud du Michigan. Les résultats sont présentés en termes de courbes de dispersion et de profils de vitesse d'onde de cisaillement .Les valeurs de V_s sont généralement jugées cohérentes dans les sites d'enfouissement et les différences significatives en V_s sont attribuées à la variabilité de la composition des déchets, aux conditions de site et aux pratiques d'exploitation des décharges. Des comparaisons sont faites pour évaluer la variabilité des ondes de vitesse de cisaillement au sein de chaque site d'enfouissement et entre les différentes sites. Les vitesses V_s augmentent généralement avec la profondeur depuis des valeurs aussi basses que 75 m/sec à proximité de la surface jusqu'à 175-210 m/sec à 25 m. Enfin, des comparaisons sont faites avec les profils de V_s de MSW de la bibliographie. Les profils dans les sites d'enfouissement du Michigan étudiés ici sont généralement inférieurs à ceux rapportés dans la bibliographie pour les décharges du sud de la Californie.

1 INTRODUCTION

The small strain shear modulus, G_{max} , is an important parameter in seismic analyses of landfills and is also related to the compressibility of municipal solid waste (MSW). It can be calculated using elasticity theory through the following equation:

$$G_{\rm max} = \rho V_s^2 \tag{1}$$

where ρ is the mass density of the material (equal to the total unit weight of the material divided by the gravitational acceleration) and V_s is the shear wave velocity of the material. In the field, V_s is commonly measured using seismic geophysical methods, such as downhole, crosshole, suspension logging and surface wave methods. Surface wave methods are especially appealing in measuring V_s in landfills, because they are non-intrusive (i.e. they do not require drilling), efficient, and reliable (Zekkos and Flanagan 2011). The most common surface wave technique used in landfills is Spectral Analysis of

Surface Waves (SASW) method (Stokoe et al. 1994). Shear wave velocity of MSW using SASW has been measured at various landfills including south California (Kavazanjian et al., 1996; Matasovic and Kavazanjian, 1998), north California (Lin et al. 2004), Georgia (Rix et al. 1998), Spain (Pereira et al. 2002), and elsewhere. The Multichannel Analysis of Surface Waves (MASW) technique (Park et al. 1999a) has not previously been used in MSW landfills. Also, "passive" techniques (Okada 2003) are increasingly used in engineering practice, but have not been used in landfills.

Shear wave velocity of MSW at four modern landfills in Michigan was measured using a combination of the active MASW technique and the passive Microtremor Analysis Method (MAM) and the results are presented.

Similar to other surface wave methods, both the MASW and the MAM techniques consist of 3 stages: field measurements (or data acquisition), dispersion curve analysis, and the inversion process. The procedures used for the implementation of the combined MASW/MAM methodology are presented.

2 FIELD MEASUREMENTS

2.1 MASW (active) Measurements

Data acquisition in active MASW entails recording the ground roll generated by a vibrator or a sledge hammer. A schematic of the test configuration is shown in Figure 1. A linear array of sixteen 4.5-Hz geophones was used. Based on evaluation of initial measurements, a geophone spacing of 3 m was selected for most locations. Thus, the spread length was equal to 45 m. This geophone spacing or interval (dx) was selected to prevent aliasing and maximize the depth of investigation for the purposes of characterizing the MSW material. A large spread length (i.e. > 100 m) may increase the risk of higher-mode domination and may reduce signal to noise ratio (S/N) for the fundamental mode (Park et al. 2002). This latter aspect is particularly important for MSW that has relatively high material damping (Zekkos et al. 2008) and may have softer and stiffer zones throughout the waste thickness.

A 10-lb sledge hammer was used as the active source at a source offset (or near offset) of 4.5 m. Depending on the background noise level, stacking was performed to improve the signal to noise ratio. In general, 5-8 stacks were performed to generate one active MASW record. An example of a MASW dataset with five stacked records from location #1 at Carleton Farms landfill is presented in Figure 3. In this figure, zero distance indicates the position of the geophone which is farthest from the source.



Figure 1. General test setup for MASW



Figure 2. View of MASW test in a landfill in Michigan



Figure 3. Example of an active record from Carleton Farms landfill location 1.

2.2 MAM (passive) Measurements

MAM utilizes surface waves that are generated by ambient activities, such as cultural noise (e.g. highway traffic), wind movement, ocean waves, and construction activities. For passive measurements, typically a circular, triangular, or L-shaped geometric configuration (2-D arrays) is recommended to ensure that the collected data is not affected by directionality of the background noise.

Re-configuring the geophones from the linear array used in MASW, requires significant effort in the field, may cover a wide area that may disturb landfill operations, needs careful surveying, and reduces the efficiency of the technique. In this investigation, MAM data were collected with the same linear geophone configuration used for the MASW tests (Figure 1). Use of a linear array for passive measurements has been attempted before (Louie 2001, and Park et al. 2008). The authors have investigated the impact of background noise directionality on the reliability of the data and these findings will be presented in more detail elsewhere. The authors found that depending on the relative orientation of the linear array and the source of noise, the information yielded from the MAM measurements may prove to be reliable or unreliable. Whether the passive data is reliable or not, becomes clear in the dispersion curve analysis stage.

For the MAM measurements, surface waves that are generated by cultural activities (e.g. highway traffic and construction activities) and other sources were recorded for 32 seconds. In landfills, it is typically easy to identify the predominant vibration sources. An example of a 32second passive record from this study is shown in Figure 4. At least 20 recordings are collected from each location.



3 DISPERSION CURVE ANALYSES

The field measurements record is transformed to a dispersion curve. The dispersion curve shows the variation of phase velocity, V_{ph} , with frequency beneath the geophone spread. V_{ph} and Rayleigh wave velocity are similar and are commonly used interchangeably (Nazarian 1984). This analysis allows the identification of unwanted waves, such as body waves, higher-mode Rayleigh waves, and other noises (Park et al. 1999a). Generally, the dispersion curve is extracted from the fundamental mode of the Rayleigh waves, unless lower V_s layers underlying higher V_s layers are identified (Tokimatsu et al. 1992).

MASW and MAM records are transformed to a dispersion curve using different signal-processing methodologies. In MASW, the transformation can be performed using f-k transform, f-p transform, Park et al. (1999b) transform, or cylindrical beamformer (Zywicki 1999). In this study the dispersion curve analyses were implemented using the Park et al. (1999b) transform. Figure 5 shows the resulting dispersion curve (highlighted in white color) generated from the active data shown in Figure 3.



Figure 5. Dispersion curve from MASW data at Carleton Farms landfill location 1

In MAM, the twenty 32-second recordings are transformed to a single dispersion curve using the Spatial Autocorrelation (SPAC) method (Aki 1957). An example of the resulting dispersion curve (highlighted in white color) generated from the MAM data (Figure 4) is presented in Figure 6. The MASW signal is typically rich in high frequency (short wavelength) content, whereas the MAM

signal is richer in low frequency content (long wavelength), providing information at higher depths. MAM data may also include high frequency content, depending on the generating source and distance. In general, passive data below the resonant frequency (4.5 Hz) of the geophone can be collected. The signal at frequencies below the resonant frequency is damped according to the geophone's calibration curve, but if it is coherent, then the phase velocity at this frequency can be estimated (Park, Hayashi, personal communication).



Figure 6. Dispersion curve from MAM data at Carleton Farms landfill location 1

The independently developed dispersion curves from the MASW and the MAM data are then compared. In some cases, the passive dispersion curve agrees well with the active dispersion curve and provides additional information on frequencies that were not available from the MASW data. Generally, for this study, active dispersion curves contain high frequency data (10-30 Hz), whereas the passive curve contains lower frequency data (<15 Hz). An example of such case is shown in Figure 7. which illustrates the dispersion curves from the active (Figure 5) and passive (Figure 6) data. In such cases, a smoothed combined dispersion curve is generated from the active and passive data and used in the inversion process. In other cases, the MAM data do not agree well with the MASW data. This discrepancy may be attributed to the method of analysis of the passive data (SPAC method) and the use of a linear array in tandem with strong directionality of background noise. The SPAC method assumes that the signal is stable and omnidirectional (Aki 1957, Okada 2003). A linear aeophone does not accommodate arrav the omnidirectionality assumption when a passive noise originates primarily from one direction. When active and passive dispersion curves are not consistent for the overlapping frequencies, the inversion process is performed using the active dispersion curve only. The combination of dispersion curves from active and passive records is often valuable. It broadens the frequency range of the dispersion curve. In addition, it helps differentiate modes of Rayleigh waves in the dispersion curve (Park et al. 2005).

4 INVERSION PROCESS

The resulting measured dispersion curve (i.e. the measured dispersion curve from the MASW or combined

MASW/MAM data) is used in the last stage of the analyses to derive the V_s profile through an inversion process. An assumed V_s profile is back-calculated to obtain a theoretical dispersion curve. The theoretical curve is compared against the measured one, and changes in the assumed profile are made iteratively until the two curves closely match. A non-linear least squares method is implemented to evaluate the fitness between the theoretical dispersion curve and its measured counterpart (Xia et al. 1999).

It is important to note that as part of the inversion process, the V_s at shallow layers affect the inversed V_s at deeper layers. For this investigation, as shown below, the highest frequencies recorded were in the order of 25-30 Hz with phase velocities of 100-160 m/sec. This means that the shortest wavelengths for which data was recorded are in the order of 2.4-5 m resulting in uncertainty in the V_s for the top 0.8-1.7 m approximately, which was considered acceptable since the objective of this study was to characterize the change in V_s with increasing depth. The V_s profiles shown in subsequent figures include only the V_s of MSW material and not of the foundation soils.



Figure 7. Combination of active and passive dispersion curves (Carleton Farms landfill, location 1)

5 LANDFILL DESCRIPTIONS AND RESULTS

5.1 Arbor Hills Landfill

Arbor Hills landfill is located in Northville and has been receiving MSW from southeast Michigan since 1991. The unit weight, estimated by the owner, is 1.48 ton/m³ (2267 lbs/yd³) based on an average estimate for all disposed waste including MSW as well as construction and demolition debris. The maximum thickness of waste is 61 m (200 ft). Figure 8 presents the dispersion curves generated from data collected at Arbor Hills Landfill. Dispersion curves at locations 2 and 3 were derived by combining their corresponding MASW and MAM dispersion curves. Dispersion curves at locations 1 and 4 were developed using the MASW data only.

The majority of the dispersion curves indicate that V_{ph} decreases with increasing frequency, implying a "normal" site with V_s increasing with depth. In some cases V_{ph} may

increase with increasing frequency, indicating the presence of a high velocity layer over a low velocity layer. Such is the case for location 4. In those cases, a consideration of higher modes of Rayleigh waves is recommended in the inversion stage (Tokimatsu et al. 1992). Thus, the $V_{\rm s}$ profile at location 4 was calculated taking into account the higher modes of Rayleigh waves. Figure 8 shows the V_s profiles at Arbor Hills. The uncertainty in the V_s profiles for locations 2, 3 and 4 is higher than the uncertainty in the V_s profile for location 1 because the reliable dispersion data are fewer. For example, in the worst case, for location 2, reliable data were collected only for frequencies between 5 Hz and 12 Hz, which is equivalent to wavelengths between 9.2 and 24 m only. In such cases, although the inversion process can be completed, the reliability of this inversed V_s profile is significantly lower than in the case of location 1 where reliable data is collected for wavelengths varying from 4 m to 24 m. Dispersion curves and V_s profiles are shown in Figures 8 and 9. The V_s profiles from location 1 and 2 are consistent. The V_s at location 3 is also consistent for depths up to 10 m, but appears to increase with depth faster at greater depths than in locations 1 and 2. Location 4 is on top of an unpaved landfill road at a closed section of the landfill with older waste. Although, the V_s appears to be similar to locations 1 and 2 for depths greater than 7 m, at shallower depths the V_s is much higher, probably due to the cover soils and the fill material used for the landfill access road.



Figure 8. Dispersion curves at Arbor Hills landfill



Figure 9. V_s profiles at Arbor Hills landfill 5.2 Oakland Heights Landfill

The Oakland Heights landfill is located in Auburn Hills and has been receiving MSW from McCone and Oakland County since the 1980s. The MSW unit weight, estimated by the landfill owner, is $1.2-1.5 \text{ tn/m}^3$ (2000-2600 lbs/yd³). Approximately 12% by volume soil is used for daily cover operations and the waste is compacted with Caterpillar 836 compactors. The thickness of waste is 30 m (100 ft) in location 1 and 49 m (160 ft) in locations 2 and 3. The array in location 1 is situated along a bench of the landfill that is underlain by at least 2 m of soil as observed by a trial test pit followed by MSW from the 1980s. This observation is consistent with the observation in location 1 at the Arbor Hills landfill. Locations 2 and 3 are at the crest of the landfill on waste placed since 1994. Dispersion curves are presented in Figure 10. Dispersion curves at locations 1 and 2 were generated using the MASW data only, whereas the dispersion curve at location 3 was derived by combining MASW with MAM. The inversion process of the dispersion curve at location 1 considered higher modes of Rayleigh waves and MAM data. Figure 11 shows V_s profiles at the Oakland Heights landfill. The V_s profiles are similar in locations 2 and 3. A high $V_{\rm s}$ layer is observed in location 1 overlying the MSW which has approximately the same V_s at a depth of 7 m in locations 2 and 3.



Figure 10. Dispersion curves at Oakland Heights landfill



5.3 Sauk Trail Hills Landfill

Sauk Trail Hills landfill is located in Canton, Michigan. It has been receiving MSW from southeast Michigan since 1974. The estimated MSW unit weight is 1.4 ton/m³ (2300 lbs/yd³). Approximately 7% by volume soil is used for daily soil cover operations. Large compactors (Caterpillar 836) are used for the compaction of the waste. The thickness of waste is 30 m (100 ft), 70 m (230 ft) and 37 m (120 ft) in locations 1, 2, and 3, respectively. Auto shredder residue (auto fluff) is used as daily cover for the top 3 lifts in location 1. Auto fluff consists of non-metallic shredded pieces of vehicles, typically soft and stiff plastics, cushions, foam and other parts of the interior of vehicles that are typically light in weight (Figure 12). Local soil was used as daily cover in locations 2 and 3.

Figure 13 shows dispersion curves at Sauk Trail Hills landfill. Dispersion curves at locations 1 and 3 were generated combining the MASW and MAM dispersion curves. The dispersion curve at location 2 was only derived from the MASW data. Derived V_s profiles at Sauk Trail Hills landfill are presented in Figure 14. Significant variations in V_s are observed between the three profiles at the Sauk Trail Hills landfill. The V_s of the waste where auto fluff is used (location 1) is significantly lower than the $V_{\rm s}$ in locations where local soils are used. An abrupt increase in V_s is observed below depths of 4.5 m. This increase is consistent with the thickness of the waste that was covered with autofluff as opposed to daily soil cover. The V_s for the top 20 m in location 3 is higher than the V_s in locations 1 and 2, probably because of the co-disposal of contaminated soils. According to the landfill operator, contaminated soil may represent as high as 40% of the total volume of waste disposal in the summer months.



Figure 12. View of auto shredder residue (auto fluff) with geophone for scale





Figure 14. V_s profiles at at Sauk Trail Hills landfill

5.4 Carleton Farms Landfill

Carleton Farms landfill has been receiving MSW from southeast Michigan and southwest Ontario (Canada) since 1993. The MSW unit weight, estimated by the owner, is 1.4 ton/m³ (2300 lbs/yd³). Soil cover is 7% by volume on the exterior slopes with auto shredder residue being used throughout the landfill with the exception of the exterior permanent slopes. Large compactors (Caterpillar 836) are used for the compaction of the waste. Location 1 is on a bench of the landfill. The thickness of waste in locations 1, 2 and 3 is 30 m (100 ft), 73 m (215 ft) and 40 m (130 ft). Locations 1 and 3 receive borrow soil as daily cover. Location 2 is at the crest of the landfill where auto shredder residue (auto fluff) is used as daily cover. Locations 1 and 3 are at a MSW and sludge combineddisposal area. Sludge was placed in trenches excavated in the MSW and then was covered with MSW. Dispersion curves at Carleton Farms landfill are presented in Figure 15. Dispersion curves at locations 1 and 3 were derived using their MASW and MAM dispersion curves. The dispersion curve of Carleton Farms 2 was generated using MASW data only.

Figure 16 shows V_s profiles at Carleton Farms landfill. Inversion at location 3 was conducted taking into account the higher-mode of Rayleigh waves. Similarly to the observations made at Sauk Trail Hills landfill, the V_s of the waste where auto fluff is used as alternative daily cover is lower than the V_s in locations where local soils are used. Overall the V_s profiles at the three locations are consistent.



Figure 16. V_s profiles at Carleton Farms landfill

6 V_S PROFILES FROM LITERATURE

The V_s profiles from the landfills in Michigan are compared against the data in the literature. The most extensive study of V_s measurements in landfills is that performed by Kavazanjian et al. (1996) who reported V_s profile measurements in six landfills in south California using the SASW technique. Figure 19 presents the range of V_s profiles from Kavazanjian et al. (1996) with Vs profiles from this study. Measured $V_{\rm s}$ profiles are lower than those recommended by Kavazanjian et al. (1996) in the upper 22 m, although two soundings that extend to greater depths are more consistent. The differences may be attributed to a number of factors: Operation practices are different. For example, as observed in this study, the use of auto fluff material results in lower Vs profiles. The amount of daily soil cover used in these Michigan landfills may also be lower than those studied by Kavazanjian et al. (1996). Previous studies (Zekkos et al. 2008) indicate that waste-rich MSW has significantly lower V_s than soilrich MSW. In cases of disposal of soils (e.g. contaminated soils) higher velocities are measured. Also, the waste streams (and as a consequence, the waste composition) at the Michigan landfills are different from that of the south California landfills. Southeast Michigan has a continental climate with much higher seasonal temperature fluctuations (warm summers and cold winters) and greater precipitation (in the order of 750-1000 mm), compared to south California that has a Mediterranean climate with much lower precipitation (in the order of 250-380 mm). These differences in climate may affect the degradation of MSW and its composition.



Figure 17. $V_{\rm s}$ profiles from this study and literature

7 CONCLUSIONS

Shear wave velocity measurements have been performed in four landfills in southeast Michigan. Data were generated using a surface wave based methodology that combines information from active (MASW) and passive (MAM) techniques using a linear array. The shear wave velocity profiles were generally lower than those measured in south California by Kavazanjian et al. (1996). Shear wave velocities are generally consistent throughout the sites and increase with depth with values as low as 75 m/sec at the surface to 175-210 m/sec at depths of 25 m. Major differences in V_s profiles could be identified depending on the site conditions (e.g. sounding on a landfill access road vs. the crest of the landfill), waste composition (e.g. typical MSW vs. contaminated soils) or landfill operations conditions (e.g. use of auto shredder residue vs. conventional soil cover).

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