Stochastic analysis of tailing dams stability using numerical modelling

2011 Pan-Am CGS
Geotechnical Conference

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ABSTRACT

Deterministic approaches such as the limit equilibrium method (LEM) and finite element analysis have been traditionally used to evaluate the stability of tailing dams. However, the inherent uncertainty associated with the material properties necessitates the use of the probabilistic method to account for the influence of this uncertainty on the reliability of the deterministic approaches. As a result, a number of stochastic analysis techniques have been introduced in the last decade as a complementary tool to the LEM for assessing the stability performance of tailing dams. However, the LEM based studies are limited in their scope and cannot accurately assess the stability of tailing dams that is governed by the deformation-pore water pressure coupling behavior. In this paper, the use of stochastic and strength reduction technique for the assessment of the hydromechanical performance of tailing dams is introduced. The paper emphasizes the integration of the Monte Carlo Method with the elasto-plastic coupled finite difference analysis for this assessment. A set of recommendations relating to the use of the Monte Carlo method in combination with the coupled finite difference analysis for evaluating the stability performance of tailing dams are provided.

RÉSUMÉ

Les approches déterministes telles que la méthode d'équilibre limite et l'analyse d'éléments finis sont des méthodes traditionnellement utilisées pour l'évaluation de la stabilité des digues de résidus. Cependant, l'incertitude reliée aux propriétés du matériau nécessite le recours à la méthode probabiliste afin de considérer l'influence de cette incertitude sur la fiabilité des approches déterministes. Ainsi, un nombre de techniques d'analyses stochastiques a été introduit durant cette dernière décennie comme outil complémentaire à la méthode d'équilibre limite pour évaluer la stabilité et les performances des digues de rejets miniers. Toutefois, la méthode d'équilibre limite est basée sur des études limitées par leur étendue et ne peuvent évaluer la stabilité des digues de rejet miniers soumises aux déformations et une pression interstitielle couplée. Dans cet article, l'utilisation de techniques de réduction stochastique et résistance est introduite pour évaluer les performances hydromécaniques des digues. L'article met l'emphase sur la méthode d'intégration Monté Carlo pour l'évaluation de stabilité en considérant une analyse de différences finis élasto-plastique couplée. Des recommandations sont proposées sur l'usage de la méthode Monté Carlo en combinaison avec l'analyse de différences finis couplées pour l'évaluation de la stabilité des performances des digues.

1 INTRODUCTION

The disposal and management of mine wastes, mainly tailings, has been continuously evolving over the past years with almost every impoundment having its unique needs, capacity, service and functionality. An increase in the social awareness regarding the risks and liabilities associated with the permanent existence of tailings impoundment facilities coupled with the intense regulatory attention and public scrutiny have delineated a conceptual framework within which all stake holders have to work collectively to ensure the safety of these impoundments, from cradle to grave. Over the past few decades, the principles of geotechnical engineering were being applied to tailings impoundments, starting with the design practices for water-retention dams. But the past experiences and failures have taught us that tailings impoundments are complex in nature requiring an integrated approach and as such necessitating the development of novel analysis techniques that shall address the hidden uncertainties governing the system's dynamics.

At the present, embankments are rising to unprecedented heights; thus magnifying the damage resulting from a break in the embankment of a tailings impoundment that will typically unleash a tidal wave of slimes and sediments heavily contaminated with toxic compounds. Never the less around 45% of tailings dam failure cases occurred in dams under 15m in height as illustrated in Figure 1 (Rico et al 2008). As such the strategic concerns should be flattened across small, medium and large sized dams since the dam failure cases are more or less equal. The causes of failure vary, but can be categorized into 11 classes as presented in Figure 2 (Rico et al 2008). It illustrates that the majority of the failures were a result of slope stability, overtopping and unusual rain. Thus, there is a clear need for further research on tailings dams not only to better understand the causes of past failures, but also to provide tools capable of operating such facilities reliably and safely.

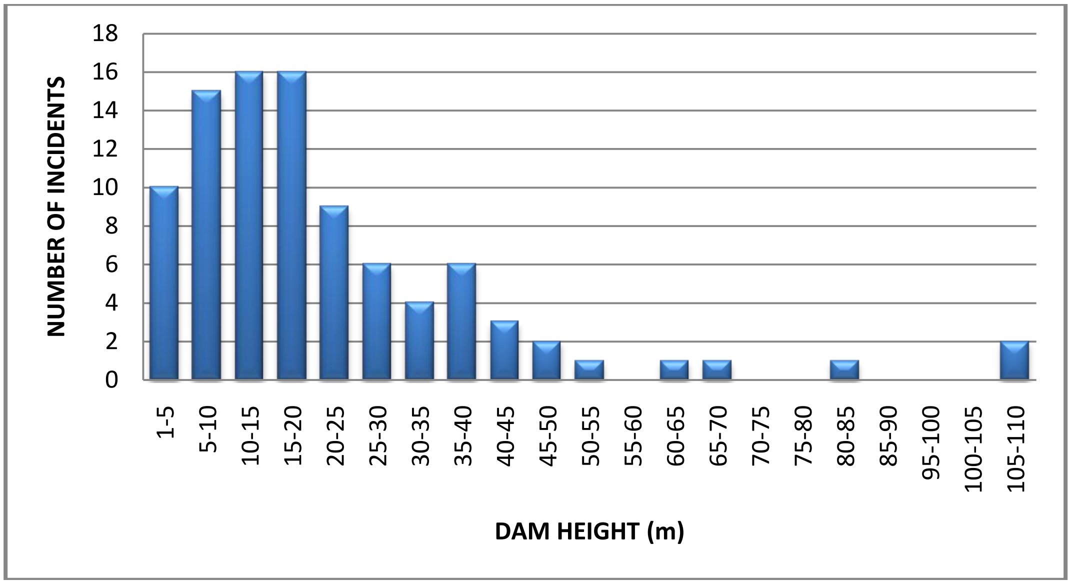

Figure 1: Number of incidents versus dam height from Rico et al database (2008)

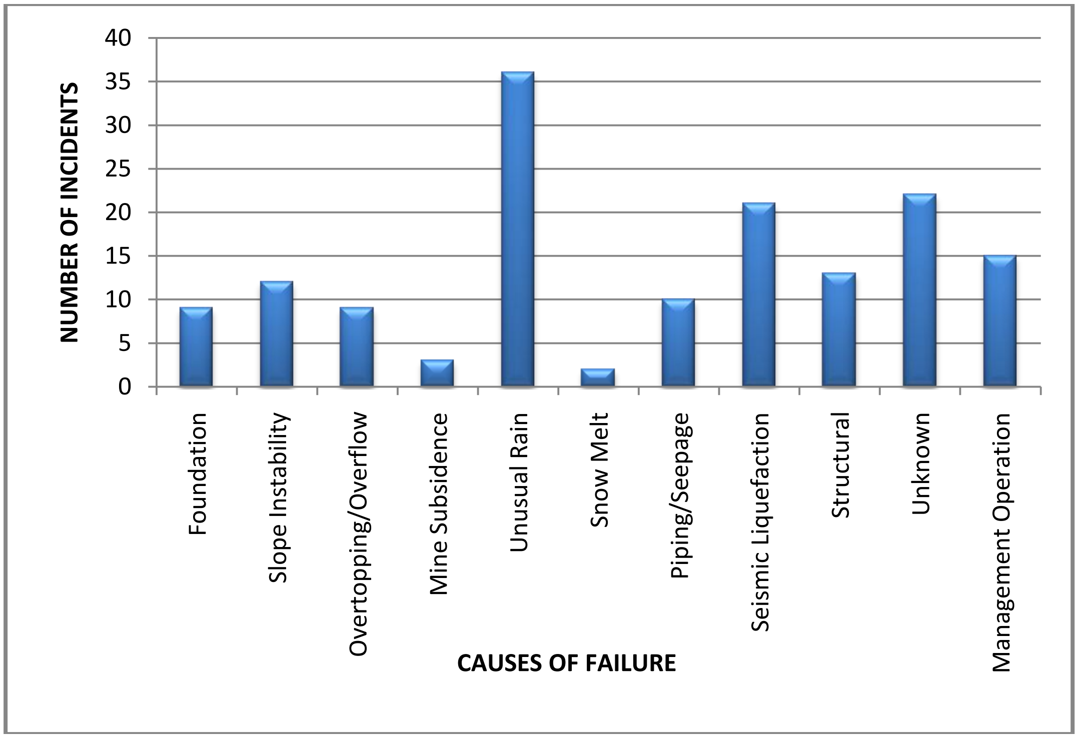

Figure 2: Number of incidents versus causes of failure from Rico et al database (2008)

2 SLOPE STABILITY ANALYSES

2.1 Deterministic approaches

Different classical geotechnical slope stability analyses such as the limit equilibrium method (LEM) have been applied for calculating a factor of safety (FOS) for tailings impoundments. Also, many stability analyses have included the use of finite element models (FEMs). Several researchers have worked on defining the material properties of different zones within the impoundment to help tune up these existing models. Thus, predicting the stability and its evolution in the tailings impoundments during its construction and operation can be limited using the LEM. However, the full interaction between the pore pressure evolution and the on-going deformation induced by the construction process could be accounted for with greater accuracy if a coupled deformation-based analysis is made. The latest development in this area included developing a finite element model capable of performing a fully coupled hydromechanical, transient analysis of upstream tailings disposal facilities (Saad and Mitri 2011). A number of conclusions were presented in this paper; and one states that the “maximum plastic shear strain zones do not appear along surfaces of well defined shapes, but rather they spread over a volume of irregular shape. Such findings confirm the inappropriateness of the use of LEM for the prediction of the potential failure surface” (Saad and Mitri, 2011). This observation was confirmed analytically and from field observations reported in the literature (Lade 1999).

Another deterministic approach developed over the past decade is best known as the strength reduction technique (SRT) (Dawson et al. 1999; Griffiths and Lane 1999). In this approach the slope is brought to a state of limit equilibrium by progressively reducing the shear strength of the material. As such, the material shear properties will be reduced by the same factor until the slope reaches the state that is the onset of failure. At this point, the end factor is deemed the as the dam’s FOS.

2.2 Probabilistic approaches

Engineers have routinely utilized deterministic approaches for design analyses. This approach which leads to calculating the FOS does not deal with the uncertainties in the input soil parameters. Moreover, the ease of interpreting the results in terms of the FOS obtained from LEM has made this deterministic approach both popular and effective in many respects. Bowles et al. (1996) highlights the application of risk assessment in dam engineering, as the FOS is no longer a sufficient measure of risk. Whitman (2000) and Duncan (2000) argue that it is difficult to evaluate how much safer a structure becomes as the factor of safety increases. Theoretically, a structure with a FOS greater than 1.0 is deemed stable, but in practice the design factor of safety is typically taken significantly greater than unity due to uncertainties related to material variability, measurement and model transformation uncertainty (Phoon and Kulhawy, 1999). As such, mathematical frameworks that take into account the uncertainties in design parameters are developed using the probabilistic approach. Such an approach establishes a direct linkage between uncertainty in the design parameters and the probability of failure or reliability (Babu et al. 2007).

Elements of soil spatial variability have to be identified to proceed with a stochastic analysis to assess the effect of the type of variability (Elkateb, Chalatumyk et al. 2003); mainly, the statistical characteristics such as mean, coefficient of variation (COV), and the probability distribution of the soil data. Thus, accounting for the variability in soil properties can be done by employing stochastic analysis techniques along with developing algorithms to estimate the soil design parameters on a probabilistic basis; and consequently enabling us to quantify the associated risk. These stochastic techniques have been applied to multiple geotechnical problems, including: liquefaction assessment, slope stability analysis, seepage through an earth fill dam, and foundation settlement. As such, three different stochastic techniques are present in the literature, and they are (Elkateb, Chalatumyk et al. 2003): 1) application of reliability principles to limit equilibrium analyses (including the First Order Second Moment Method); 2) stochastic finite element analysis; and 3) application of stochastic input soil parameters into deterministic numerical analysis (such as the Point Estimate Methods and Monte Carlo simulation).

Thus, the purpose of undertaking these stochastic techniques is to define probability of failure or what the US Army Corps of Engineers refers to as the probability of unsatisfactory performance. The Corps of Engineers uses the term “probability of unsatisfactory performance” in recognition of the distinction between failure and less significant performance problems (USACE 1998). Ultimately, the probability density functions produced from the analyses will be used to quantify risk, which is defined as the product of probability and consequence of an event; and typically, the probability being that of failure and the consequence referring to the cost of failure. In a LEM dam analysis, a probability of failure will correspond to the fraction of the total runs that generated a factor of safety less than one.

3 OBJECTIVES AND ANALYSIS APPROACH

In this paper, the aim is to analyze the stability of a typical tailings impoundment by formulating a model that includes the following features: first, the deformation-diffusion process of coupled mechanical and hydraulic modeling; second, randomness in the material property to account for uncertainty; third, calculating the factor of safety using the strength reduction technique after the construction and operation of the tailings impoundment. The geometry of the modeled tailings impoundment used in this study is characteristic of typical tailing impoundment sites.

First, we perform a classical slope stability analysis using the LEM to calculate the FOS. Then we carry a sensitivity analysis on the different material properties constituting the impoundment to identify the parameter that most influences the FOS. Note, that the LEM analysis does not take into account the effect of the staged construction of the impoundment; however this stochastic study will factor it in.

Second, we perform a deterministic hydro-mechanical coupled analysis of the staged construction of the impoundment using finite difference analysis. The main feature of this model is that the impoundment is constructed in stages and at the end of construction the FOS is calculated using the SRT. Thus, the FOS is calculated after the pore pressure and stress regimes have developed and deformations in the impoundment have taken place. Then the FOS calculated from this analysis is compared with that obtained from the LEM one.

Third, we perform a Monte-Carlo (MC) stochastic analysis on the finite difference model consisting of 100 runs by varying the material property for a defined zone according to a probability distribution function. And after each run the FOS is calculated using the SRT. Then the FOS probability density function is constructed.

Fourth, we perform another Monte-Carlo stochastic analysis on the finite difference model consisting of 100 runs by varying the material property spatially within a defined zone. The material property assigned at the local level will be chosen randomly from a normal (Gaussian) distribution of a specific standard deviation and mean. Similar to the previous step, after each run the FOS is calculated using the SRT; and then the FOS probability density function is constructed. It is important to note that while the spatial variation of the material properties is accounted for in the current analysis, there is no spatial correlation between the property values within the defined grid zone.

At the end of the analyses we compare the FOS probability density functions obtained using the different approaches and identify the advantages and disadvantages of each for the purpose of calculating the probability of failure.

4 DETERMINISTIC ANALYSES

4.1 Impoundment features

The geometry of the model is presented in Figure 3. The foundation is made of a bedrock layer topped with a silty-clay layer. The slope of the embankment is 2.5H:1V. The top of the dam is 8m wide and the bottom is 98m wide. The dam is 16m in height and is designed to retain 14m of tailings material. The dam is of water-retention type as it has a low permeability core keyed into the bedrock to dramatically minimize seepage and migration of toxic contaminants and maximize water circulation on site which is common practice in tailings management facilities.

The material properties used in the analysis are presented in Table 1.

Table 1: Material properties of tailings impoundment

Material	Properties				
	γ(kN/m^3)	c(kPa)	ϕ(°)	k (m/s)	n
Core	21.5	12	28	1E-07	0.35
Borrow	18.5	0	35	1E-01	0.5
Tailings	16	0	28	1E-06	0.4
Silty Clay	16.5	50	0	1E-07	0.2
Bedrock	27	6000	42	1E-08	0.02

γ=unit weight; c=cohesion; ϕ=angle of friction; k=permeability; n=porosity

4.2 FOS calculation using LEM

To calculate the FOS using the LEM for the proposed tailings impoundment, the model was set up in GeoStudio SLOPE/W. The constitutive model governing the material is the Mohr-Coulomb and the Morgenstern-Price analysis type was chosen for calculating the FOS.

The resulting FOS generated from the model is 1.292. The slip surface corresponding to this FOS is presented in Figure 4.

It was determined after undertaking an extensive material properties sensitivity analysis that the dam's FOS was most sensitive to the core's angle of friction. The sensitivity analysis chart is illustrated in Figure 5.

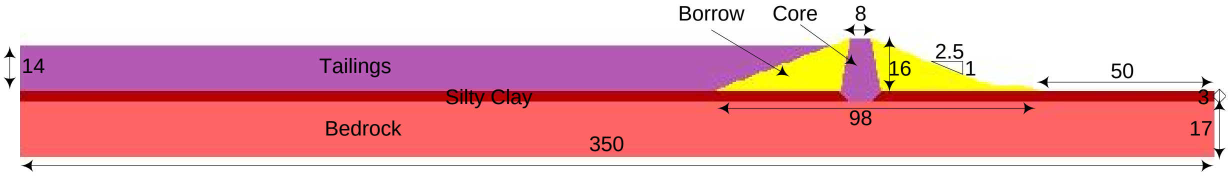

Figure 3: Model geometry (dimensions in meters)

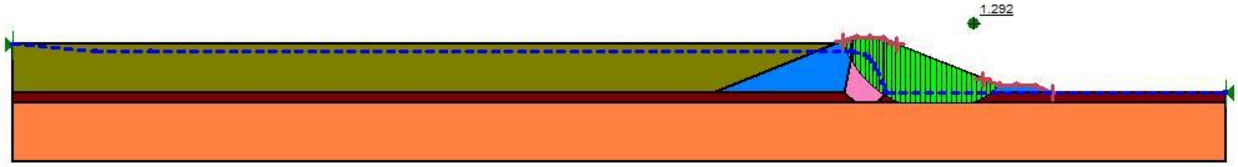

Figure 4: Slope FOS obtained from LEM; dashed line representing piezometric line

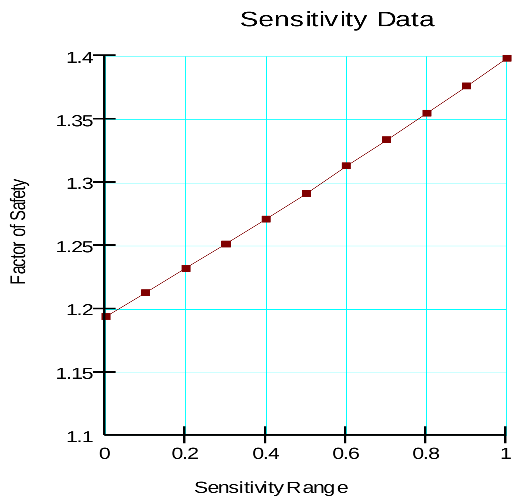

Figure 5: Sensitivity analysis for FOS vs. Dam’s core angle of friction (Range on x-axis: 0 for Φ= 21° and 1 for Φ=35°)

4.3 FOS calculation using SRT; coupled analysis

In this section, a hydromechanical-coupled analysis approach is presented to model the staged construction of the impoundment; and after the dam has deformed and the pore pressure regimes are redefined after each stage, the FOS of the tailings dam is calculated using the SRT.

The cross-section illustrated in Figure 3 was setup in the Fast Lagrangian Analysis Continua (FLAC) - Single Phase option, finite difference software. The purpose of setting up the model and running the analysis using FLAC was to take advantage of the deformation-diffusion process of coupled mechanical and hydraulic modeling. Using FLAC's capabilities in hydro-mechanical coupling we were able to construct the tailings impoundment in five stages. The first stage included building the dam. The second, third and fourth stages included a lift of 4 meters of tailings each and the fifth stage included a lift of two meters of tailings material. The model was run for steady state. At the end of the coupled analysis and the complete deformation of the impoundment the FOS of the dam was calculated using the SRT. The main advantage in this approach was calculating the FOS of the dam after it has completely deformed at the end of construction of the tailings impoundment.

The material input parameters presented in Table 1 were used in the FLAC model. In addition the constitutive model used was the Mohr-Coulomb to take advantage of the SRT capabilities in FLAC. Figure 6 illustrates the pore pressure distribution contour lines at the end of the staged construction in the vicinity of the retaining dam.

The FOS calculated using the SRT was 1.13, which is approximately 12.5% lower than the FOS calculated using the LEM. The literature on SRT does specify that the FOS calculated from the LEM is typically an upper bound to the FOS obtained from the SRT (Davis and Selvadurai 2002). In addition to the FOS calculation, the maximum shear strain rate contours that delineate the location of the failure surface along with the velocity vectors indicating the failure mode, which in this case is rotational, are presented in Figure 7.

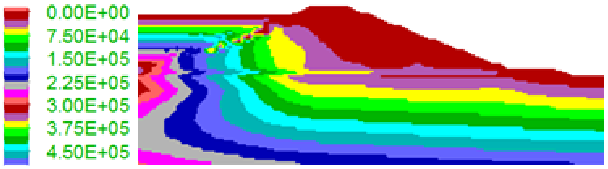

Figure 6: Pore pressure contour lines (units in Pa)

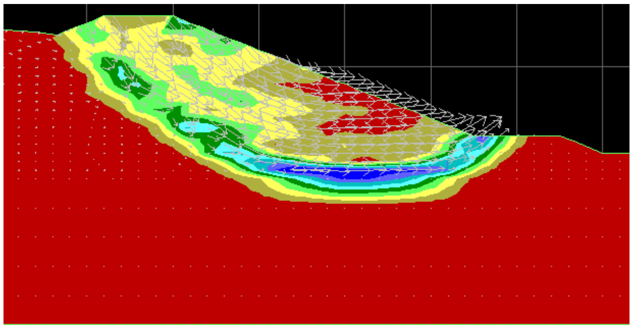

Figure 7: Zone of maximum shear strain rate in dam and arrows indicating velocity vectors

5 STOCHASTIC ANALYSES

5.1 Monte-Carlo Stochastic Analysis

The sensitivity analysis conducted on the LEM model identified the dam’s core angle of friction as the material input parameter that mostly influenced the dam’s FOS. As such, we defined the core’s angle of friction as a stochastic variable following a Gaussian distribution with a mean of 28° and a standard deviation of 7°. The choice of 7° for the standard deviation corresponds to a COV of 25% which is within the range of COVs for the core’s material properties (Baecher and Christian 2003).

As mentioned earlier 100 simulations were conducted in this analysis in FLAC. The sample size included a minimum angle of friction of 12.1° (~2.2 standard deviations below the mean) and a maximum of 41.3° (~1.9 standard deviations above the mean). Then for each run, the FOS generated using the SRT after the completion of the staged construction of the tailings impoundment was tabulated. From the M-C simulations a probability distribution for the FOS can be generated. Figure 8 presents the probability density function (PDF) of the FOS as a lognormal distribution. The choice of the lognormal distribution stems from the fact that the FOS cannot take a negative value.

As a result, if we define the probability of failure corresponding to the area under the PDF for FOS less than 1; then the probability of failure $p_f(FOS<1)$ is calculated to be 0.0002 or 0.02%.

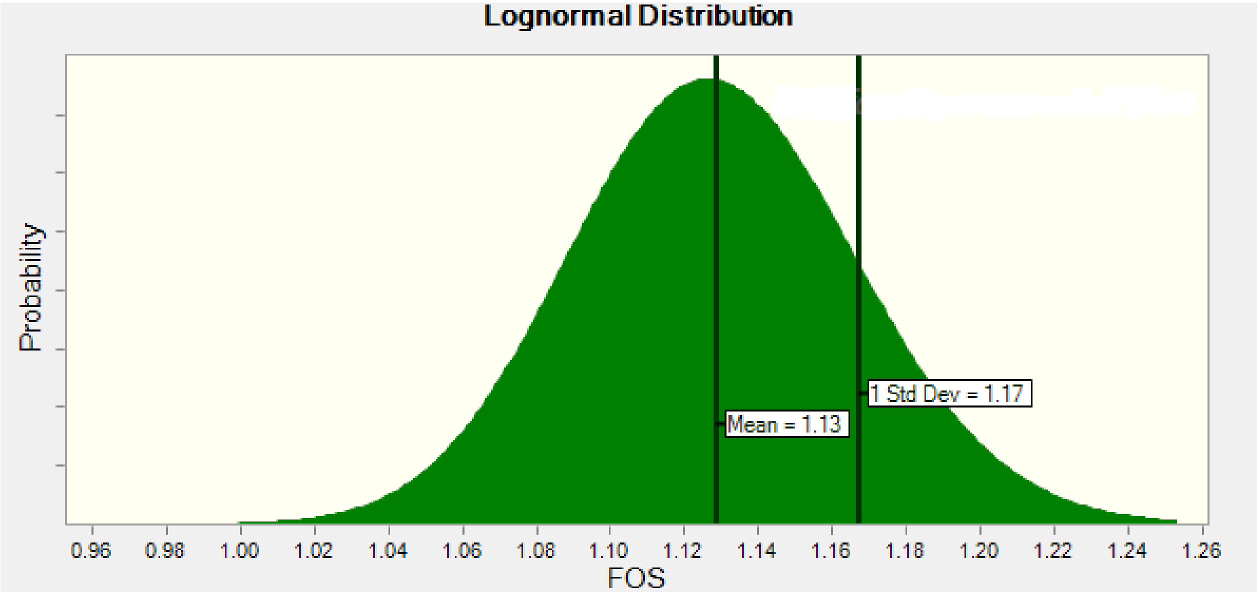

Figure 8: Lognormal distribution PDF for FOS after 100 M-C simulations

5.2 FLAC Random Monte-Carlo Stochastic Analysis

In this section, the uncertainty in the dam's core is taken at the local level by introducing the spatial variability of the material property. As in the previous case, for every run of the M-C simulations in FLAC the hydro-mechanical coupled analysis of the staged construction of the impoundment is completed before calculating the FOS using the SRT.

Similarly, the dam's core angle of friction was considered as the stochastic variable following a Gaussian distribution with a mean of 28° and a standard deviation of 7°. However in this case, every zone in the finite difference grid of the dam's core has a unique angle of friction different from the zones surrounding it. In every simulation in FLAC a new seed was input to generate a new set of random input parameters for the zones in the dam's core. Figure 9 illustrates the spatial variability in the dam's core with the angle of friction varying between 15.38° and 43.94° for a certain specific seed. For every seed input in the model, the range for the dam's core angle of friction will spread wider or narrower. For illustrative purposes the left side of the dam's core shows the variation in the material property at the local level with the right side showing the overall spatial variability.

From the random M-C simulations a probability distribution for the FOS can be generated. Figure 10 presents the probability density function (PDF) of the FOS as a lognormal distribution. Comparing Figures 8 and 11, it is evident that the spread is narrower for the lognormal distribution PDF for the FOS generated from the random M-C analysis.

As a result, if we define the probability of failure corresponding to the area under the PDF for FOS less than 1; then the probability of failure $p_f(FOS<1)$ is virtually zero. This implies the following: 1. introducing a spatial correlation might influence the generated PDF; 2. the uncertainty in the material input parameters must be applied to more than one variable, e.g. including cohesion as a stochastic variable; 3. increasing the COV in the dam's core to account for more heterogeneity in the material.

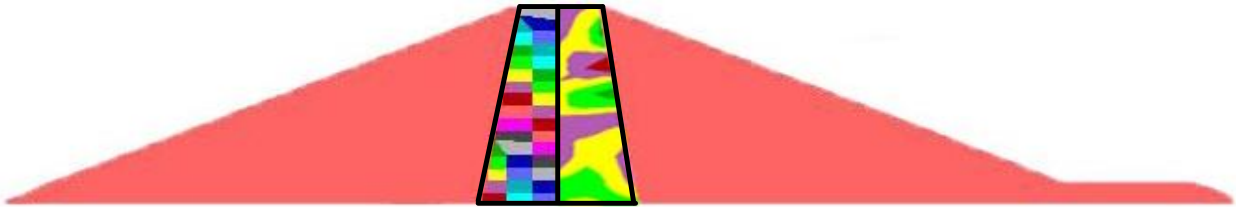

Figure 9: Material uncertainty in the core's angle of friction represented at the local level

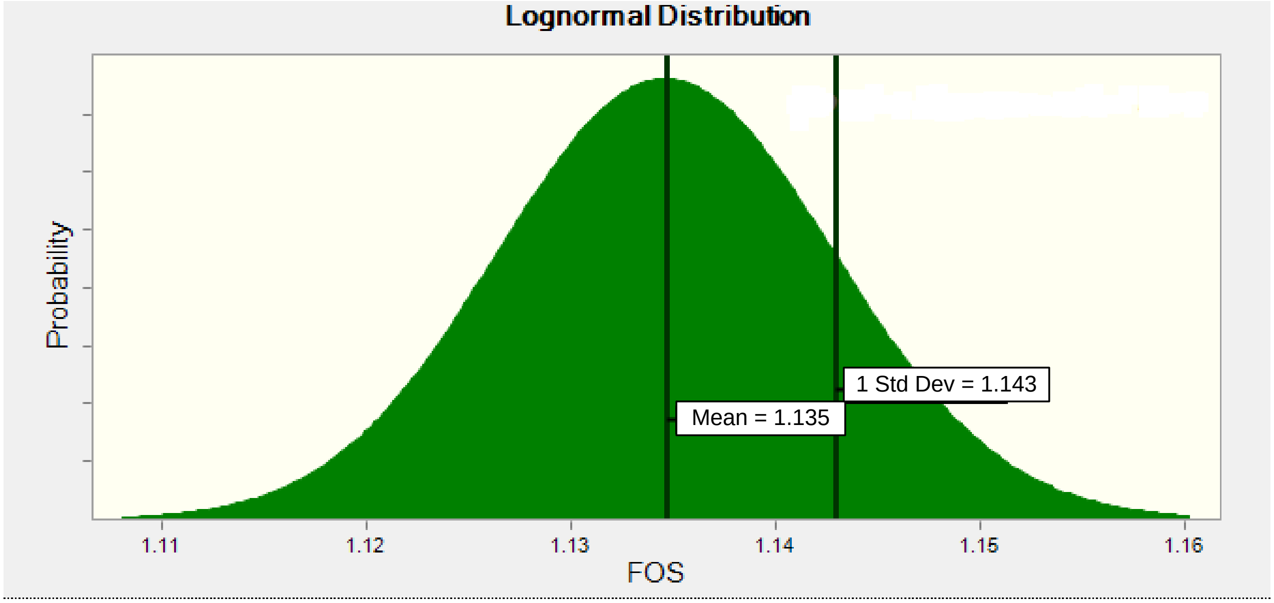

Figure 10: Lognormal distribution PDF for FOS after 100 Random M-C simulations

6 DISCUSSION

In this paper an attempt was made to study the stochastic analysis of tailings dam stability using numerical modelling. First we start with a characteristic tailings impoundment site and illustrate the common practice in the industry which uses the LEM to compute the FOS. This method has a number of limitations; mainly that it does not consider strain and displacement compatibility and as a result the "local variations in the safety factors cannot be considered and the computed stress distributions are often unrealistic" (Krahn 2003). Established studies such as those presented by Saad and Mitri (2011) show that the hydromechanical coupled analysis of tailings dam stability is an appropriate method for identifying "the maximum plastic shear strain zones that typically do not appear along surfaces of well defined shapes, but rather spread over a region of irregular shape".

Thus the motivation for this study stems from the need to use advanced and more accurate tools for modeling the stability of tailing dams while translating the output into parameters commonly used in the industry, namely the FOS. As such, FLAC was chosen for its capabilities in performing a hydromechanical coupled analysis of a tailings impoundment and then proceed in calculating the FOS using the SRT after the completion of the staged construction. To account for the uncertainty in the model parameters, two stochastic techniques were implemented and compared: the M-C and the Random M-C stochastic analyses. Although, the number of simulations for each being at 100 is considered low in the realms of the M-C world, it is interesting to note the random M-C results converge closer to the mean; whereas the results of the M-C simulation are spread wider and thus generating a probability of failure that is typical of such impoundments. For the purpose of this paper, the number of simulations was limited to 100 as stochastic analyses are heavily expensive on the time

Further research is currently pursued to better understand the applicability of the random M-C method and its capabilities in calculating a representative probability of failure for the ultimate purpose of quantifying risk and mitigating against further tailings dam failures in the future.

ACKNOWLEDGEMENTS

This work is financially supported by the Faculty of Engineering at McGill University and a research grant from Agnico-Eagle Mines Ltd. The authors are grateful for their financial support.

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