Influence of Particle Size and Particle Orientation on the Shear Strength Parameters of Unbound Slate Slag

Seth Knihtila – *Student, Norwich University, Northfield, VT, USA*,
Adam F. Sevi, PhD – *Professor, Department of Civil Engineering – Norwich University, Northfield, VT, USA,*
Brad Ruderman P.E. Adjunct Professor. Department of Civil Engineering – Norwich University, Northfield, VT, USA,
John B. Stevens P.E.- *Professor Emeritus . Department of Civil Engineering – Norwich University, Northfield, VT, USA,*

2011 Pan-Am CGS Geotechnical Conference

ABSTRACT

The objective of this study is to determine if particle orientation and particle size have an influence on the angle of internal friction for unbound slate slag with an emphasis on low normal stresses. Slate slag samples were retrieved from the Paine Mountain slate tract, located in Northfield VT, USA, and sieved. Using parallel gradations and particle scalping techniques, four different gradation samples were produced for testing. In order to compare data between gradations, a relative density of 50% was used for all tested samples. In order to assess angle of repose of the slag under no confinement a "pour test" was performed. Direct shear testing was also performed on all gradations to determine the angle of internal friction under normal stresses of 6.9, 14, 21, 69, 138, and 207 kPa (1, 3, 5, 10, 20, and 30-psi) normal loads. An additional direct shear test was performed using samples with particles carefully oriented perpendicular to the shear plane. The four random oriented granular gradation samples angle of internal friction ranged between 28-38 degrees while the vertically oriented sample found an apparent angle of internal friction of 75-degrees. The size of particles in the pour test was directly proportionate to the angle of repose found using this test. The smaller the grain sizes present in the sample, the steeper the angle of repose. The direct shear testing showed that as the particle size within the sample increased, the angle of internal friction also increased. The critical void ratio for all compiled slate samples was found to be between 14-21 kPa normal stresses. Most pronounced was the finding of the apparent angle of internal friction differing considerably with careful particle orientation.

RÉSUMÉ

L'objectif de cette étude est de déterminer si les orientation de particules et de la taille des particules ont une influence sur l'angle de frottement interne pour laitier ardoise mettant l'accent sur la basses des accouchements. Ardoise laitier échantillons récupérée par les voies de la montagne de Paine ardoise, situé à Northfield VT, USA, ont été acquis et tamisés. À l'aide des gradations parallèles, quatre échantillons ont été produites pour les tests. Afin de comparer les données entre les gradations, une densité relative de 50 % a été testée pour tous les échantillons. Afin d'évaluer à angle de repos des scories sous aucune confinement un test pour a été effectuée. Diriger le cisaillement tests ont été effectués sur tous les gradations pour déterminer l'angle de frottement interne sous accouchements de 6.9, 13.79, 20.7, 68.9, 137.9, and 206.8 kPa (1, 3, 5, 10, 20, and 30-psi) charges normales. L'essai est également réalisé sur des échantillons avec particules soigneusement orientés perpendiculairement au plan de cisaillement. La gradation granulaire quatre échantillons angle de frottement interne que se situait entre 28-38 degrés, alors que l'échantillon orienté verticalement a conclu avec un angle de 75 degrés de frottement interne. La taille des particules, du test pour, est directement proportionnelle à l'angle de repos à aucune confinement. Plus la taille des grains présente plus l'angle de repos. L'essai de cisaillement direct a montré que plus les particules présentes dans le cisaillement échantillon, plus l'angle de frottement interne. Le rapport de vide critique pour tous les échantillons ardoises compilés s'est avéré entre 3 et 5 psi. Aussi, l'angle de frottement interne diffère considérablement avec orientation de particules.

1 INTRODUCTION

Located in Northfield Vermont, USA, Paine Mountain is currently a location of an abundance of slate slag. The volume of the slag present is estimated, from a topographic survey, at 30,000 cubic yards, and is the result of mining performed in the late 1800's. Norwich University owns several of the inactive slate quarries. With an active University, and a soon to be developed trail system located on this mountain, the slate slag could be used to construct and rehabilitate the trail system. Due to these uses, the slate slag testing was performed at low normal stresses. Since the trail system will only endure light weight use, hikers, bikers, and all terrain vehicles, it is important to find out how the slag acts under low normal stresses and placed with low compaction effort.

2 TESTING PROCEDURES

2.1 Sieve Analysis

The slate slag was retrieved and transferred to the soils laboratory located at the David Crawford School of Engineering at Norwich University. The sample was air dried and underwent a sieve analysis ranging from the 76-mm to the #200 (0.075mm) sieve. Utilizing a grain size distribution chart a 76–mm prototype sample was compiled in accordance to the percent finer of each sieve in the original sieve analysis. In order to test the angle of internal friction of the samples, a parallel gradation-modeling scheme was used to make three scaled down samples, 19-mm, 9.5-mm and #4 (4.75-mm) (Lowe, 1964). The three model gradation samples consisted of a larger portion of smaller particle sizes. Additionally, scalping had to be performed on the 76–mm prototype , and 19–mm model samples for direct shear testing (Zeller and Wullimann, 1957). The scalping removed particles larger than 9.5-mm sieve. According to ASTM D3080 – 04 Standard Test Method for Direct Shear Testing of Soils Under Consolidated Drained Conditions no particle should be greater than one-tenth the nominal width of the shear box. With the shear box being 102-mm in width, all samples larger than the 9.5-mm sieve had to be removed for the test. As seen below in Figure #1, the original gradation curves (blue) and scalped gradation curves (black extension), the amount of scalping done to the 76-mm and 19-mm samples is portrayed.

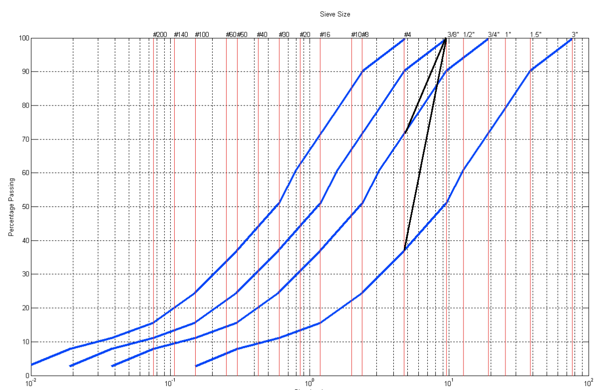

Figure #1 Original/Scalped Parallel Gradation

With the scalping performed, a look into the soil characteristics was performed. The coefficient of uniformity (C_u) was found for the original and scalped samples. As seen in Table #1 below, the only significant change after the scalping was done was to the 76-mm sample.

Table #1 Coefficient of Uniformity Comparison

Gradation Size (mm)	Original C_U	Scalped C_U	% Scalped (mass)
79	26	13	48
19	25	25	10
9.5	25	25	-
4.75	27	27	-

The shear strength parameters of a granular soil are directly correlated to the maximum particle size, the coefficient of uniformity, the density, the applied normal stress, and the gravel and fines content of the sample . It can be said that the shear strength parameters are a result of the frictional forces of the particles, as they slide and interlock during shearing (Yagiz, S. 2001).

2.2 Relative Density Testing

To determine the effect of particle size on the angle of internal friction, a set of data between samples needed to be compared. A relative density, (D_R%) test was completed on each sample to acquire the minimum and maximum density of the respective samples using ASTM C128 - 07a Standard Test Method for Density, Relative Density (Specific Gravity), and Absorption of Fine Aggregate. A D_R% of 50% was chosen for all samples to provide comparable results between the different gradation samples. This D_R 50% allowed testing to be completed at the same relative density for all samples, hence a set of comparable results between samples was produced (Burmister, 1948).

2.3 Angle of Repose Testing

The angle of repose testing was performed in general accordance to ASTM C1444, The Standard Method for Measuring the Angle of Repose of Free-Flowing Mold Powders. While this testing specification has been discontinued, it has proven useful in assessing granular material shear properties (Lee, 1993 and Vallejo, 2001). While attempting to perform this test it was found that the slag plugged even large outlet funnels. The plugging occurred due to the angularity, and high length to width ratio of the particles. The larger particles interlocked in the funnels exit, producing segregation of particle size in the conical pile. Therefore, a sample pouring test was developed (in the lab) to determine the angle of repose of the samples at no normal stress (σ_n). This test was performed with the sample freely flowing out of a 50-mm PVC tube at a specified height, thus producing a conical pile up to the exit of the pipe, 8-cm above the horizontal surface. Using the height and width of the pile the angle of repose was determined according to ASTM C1444.

2.4 Direct Shear Testing

After completing the angle of repose testing, direct shear testing was performed on each of the four granular samples. The direct shear testing was used to determine the angle of internal friction of each sample under normal stresses of 6.9, 13.8, 20.7, 68.9, 137.9, and 206.8 kPa (1, 3, 5, 10, 20, and 30-psi). With the D_R = 50% as determined above each sample was tested at each normal stress (σ_n) in an ELE International Direct Shear Machine.

To evaluate the effect of particle orientation on the angle of internal friction, a specimen was constructed using a #100 sieve bedding layer and vertically oriented 9.5-mm particles throughout the shear plane. The sample was tested under the same normal stress (σ_n) as the randomly oriented granular samples. As seen in Figure #2 below, the particles were oriented perpendicular to the shear plane, modeling a stacking situation.

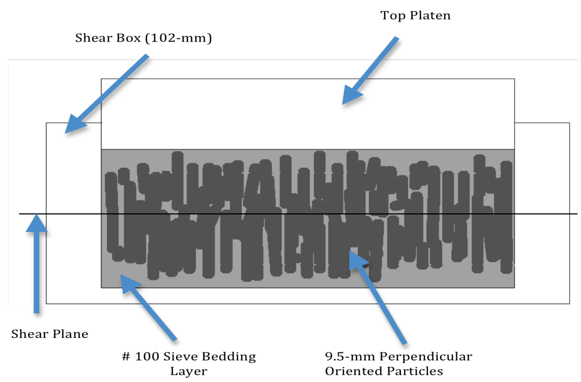

Figure #2 Vertical Oriented Shear Diagram.

3 TESTING RESULTS

3.1 Relative Density Results

In relative density testing, it was found that after scalping the maximum densities of the 9.5-mm, 19-mm and 76-mm samples were similar, varying between 2090.4 and 2095.2 kg/m^3. The #4 sample had the lowest minimum and maximum density. The 50% relative densities (D_R 50%) had corresponding densities ranging from 1493 to 1646 kg/m^3, with the #4 sample resulting in the lowest relative density of 1492.9 kg/m^3.

Table #2: Relative Density Results

Sieve	ρ_{min} (kg/m^3)	ρ_{max} (kg/m^3)	D_R50%
#4	1214.2	1941.4	1492.9
9.5-mm	1358.3	2090.4	1646.6
19-mm	1347.1	2088.7	1635.4
76-mm	1457.6	2095.2	1638.6

3.2 Angle of Repose Testing Results

Using the "pour test", ASTM C1444, the 76 mm gradation had the lowest angle of repose at 43 degrees while the #4 gradation had the highest angle, 47 degrees. These results are summarized in Table 4.

3.3 Direct Shear Testing Results

Using MatLab three plots were produced for analysis for each sample gradation tested, a vertical vs. horizontal strain plot, a shear stress vs. horizontal strain plot, and a Mohr Diagram. The Mohr diagram depicts the maximum shear stress versus the normal stress of each direct shear test. A best-fit and a second order polynomial line was placed on the Mohr Diagrams for analysis. The best-fit line was used to determine the angle of internal friction for the samples at the confinements tested, while the second order polynomial line was used to depict the variation in angle of internal friction as the normal stress (σ_n) increased.

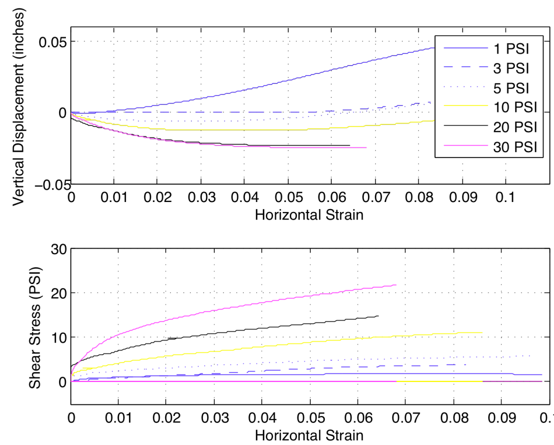

Figure #3: 9.5-mm Gradation Horizontal Strain vs. Vertical Displacement and Shear Stress

In relation to the Strain vs. Vertical deflection graph seen in Figure #3, while the sample was at low normal stress (σ_n) the sample dilated (positive vertical displacement) during shear, while at higher normal stress (σ_n) contraction (negative vertical displacement) was observed. This test helped determined that the critical void state, at D_R=50%, was observed for all samples between the 13.8 – 20 kPa normal stress tests. As seen in Figure #3, the 13.8 kPa normal stress trial follows the zero vertical deflection line until very high horizontal strain. The critical void state is the state at which no contraction or dilation occurs. At the specific normal stress (σ_n) this critical state is the sample condition where the particles on the shear plane are "sliding" by each other with no measurable vertical displacement. As seen in Table #3, this critical void state has a correlation to the shear strength. The void ratio of the #4 sample at this state is higher than that of the other three larger samples. The 19-mm sample has the lowest critical void ratio (indicating highest density) and respectively has the largest angle of internal friction.

Table #3 Critical Void Ratio

Gradation Size (mm)	$e_{Critical}$	$\rho_{Critical}$ (kg/m^3)
4.75	0.214	1493
9.5	0.165	1647
19	0.127	1636
76	0.17	1683

Significant data was obtained in direct shear testing of the parallel gradation and scalped gradation samples. With relation to the granular samples, the angle of internal friction varied from 28 degrees for the #4 (4.75-mm) sample to 38 degrees for the 19-mm sample. A similarity between the angle of internal friction of the 9.5-mm 19-mm and 76-mm is pronounced. The angle of internal friction for all samples represents the entire range of normal stresses tested.

With the perpendicular oriented particles, the angle of internal friction was found to be much larger than that of the bulk gradation samples with random orientation of particles. The angle of internal friction for the stacked sample was 75 degrees, as compared to the random particle orientation gradation samples between 28 and 38 degrees as seen in Figure #6.

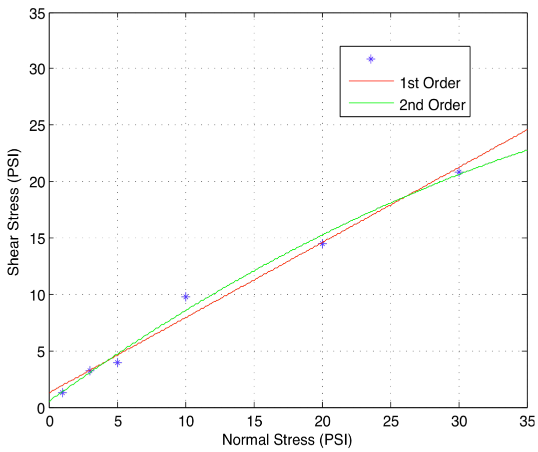

Figure #4 : #4 (4.75-mm) Mohr Diagram

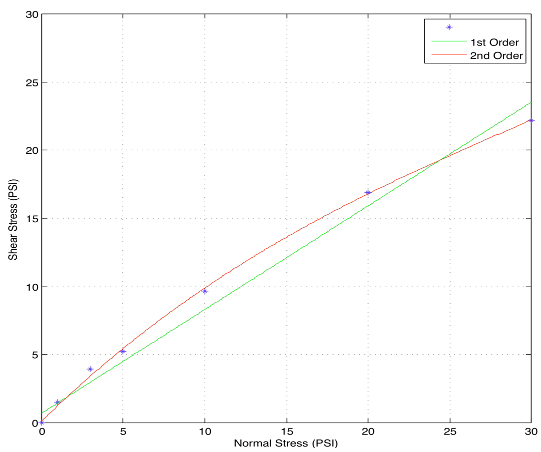

Figure #5: 19-mm Gradation Mohr Diagram

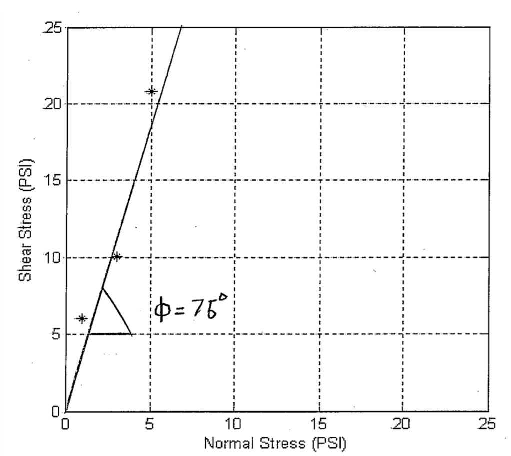

Figure #6: Vertically Oriented Particles Mohr Diagram

4 DISCUSSION

The angle of repose test or "pour test" revealed a consistent trend of smaller sized gradations having a higher angle of internal friction (ϕ)). The smaller angle of repose for the larger particles appeared to result from the larger particles striking the top of the material cone and thus compacting the pile when falling from a specified height while building the pile. This compaction tends to result in a lower angle of repose for the larger particles, as seen in Table #4 (Frette, 1996). The angle of repose, as found from the "pour test", for the given samples is significantly higher than the angle of internal friction as found using the direct shear test. This can be defined as the maximum angle of stable piles (θ_{ms}), or the angle at which a pile becomes unstable. Since the unstable piles, for the determination of the angle of repose, were at their loosest state the angle of repose shall be higher than under a normal stress as seen in direct shear testing (Lee, 1993).

Table #4: Testing Data Summary

Sieve Gradation	Direct Shear ϕ (deg)	Angle of Repose Testing ϕ (deg)	ρ_{min} (kg/m^3)	ρ_{max} (kg/m^3)	D_R 50% -
#4	28	47	1214.2	1941.4	1492.9
9.5-mm	35	47	1358.3	2090.4	1646.6
19-mm	38	45	1347.1	2088.7	1635.4
76-mm	36	43	1457.6	2095.2	1638.6

The relative density testing found that the smaller gradations exhibited slightly lower minimum density, as seen in Table #4 above. In relation to the maximum densities, the results show great similarities between the 9.5-mm, 19-mm, and the 76–mm gradation. The main reason for this is likely the scalping that was done previously. The scalping removed any particles larger than the 9.5-mm sieve causing the three gradations to have similar grain size distribution as tested and properly sized for the direct shear testing. These similar maximum densities resulted in similar D_R50%'s for the three larger gradations. The #4 gradation exhibited the lowest minimum and maximum densities, and corresponding D_R50% density (1493 kg/m^3) of all four gradations. This gradation consisted of particles finer than the #4 sieve ranging down below the #200 sieve, and containing 16% fines, as compared to 12%, 7% and 0% fines for the 9.5-mm, 19-mm, and 76-mm gradations respectively. The #4 gradation, while testing for maximum density, was so fine it produced dust while compacting. Relative density is important for preparing samples for direct shear testing since not all of the samples had the same minimum and maximum density. This resulted in the use of the D_R=50% for comparisons of shear data between gradations.

The direct shear testing produced a variety of tables and graphs, which convey important data. As seen in Table #3, the angle of internal friction varies significantly between the #4 gradation and the three other (larger) gradations. The trend in internal friction data could be due to the presence of increased fines in the smaller gradation. Also observed was that larger particles were typically more angular than smaller particles. This would result in greater interlocking between particles when direct shear testing was performed. The three larger gradations are in fact quite similar in gradation after scalping was performed. The angle of internal friction for these larger samples is significantly higher than the finer #4 sample, 35-38 degrees compared to 28 degrees for the #4 gradation sample.

Finally the vertically oriented sample tested in the direct shear machine provided proof that particle orientation has a large role in the strength of slate slag samples at low normal stresses. Slate slag is known for its flat-plate-like particle shape. With the particles oriented perpendicular to the shear plane in the shear box, the apparent angle of internal friction was found to be 75 degrees, as seen in Figure #6. This finding indicates that this material stacks extremely well and exhibits excellent strength at low normal stresses. The high friction angle is likely a result of the frictional and interlocking resistance of the 9.5-mm stacked particles, while #100 sieve bedding material is used for constructing the sample only (Vallejo, Luis, E.).

5 CONCLUSION

Upon completing the testing and analysis of the slate slag samples acquired, the following observations were made. The critical void ratio of the gradation samples produced lies between 13.8 and 20 kPa at a D_R=50%. Likewise, in relation to the angle of repose and direct shear testing, the angle of internal friction (ϕ) ranges from 28-38 degrees for all the tested gradation samples. Particle size plays a role in the angle of internal friction, since the #4 sample has an angle of internal friction of 28, while the other three similar scalped samples have an angle of internal friction between 35 and 38 degrees. If a shear box of larger size was used, a larger variation of angle of internal friction may be found between the four samples (Hight and Leroueil 2003).

If the particles are stacked perpendicular to the shear plane, within the shear box the angle of internal friction is much steeper, 75 degrees. Therefore, we conclude that particle orientation plays a significant role in the shear strength of slate slag. With the angle of internal friction at low normal stress (σ_n) ranging from 28-38 degrees the granular samples are well suited for a surface cover for light duty roads and trails. Additionally, slate materials could be extremely useful if careful stacking of particles can be performed, as in a stone wall formation.

REFERENCES

ASTM C1444-00. The Standard Method for Measuring the Angle of Repose of Free-Flowing Mold Powders (C1444-00) adopted 2000 and withdrawn 2005. 2001 Annual Book of ASTM Standards, vol. 15.01. American Society of Testing and Materials (ASTM), Philadelphia, Pa, pp. 694-695, 2001.

According to ASTM D3080 – 04 Standard Test Method for Direct Shear Testing of Soils Under Consolidated Drained Conditions 2001 Annual Book of ASTM Standards, vol. 15.01. American Society of Testing and Materials (ASTM), Philadelphia, Pa, 2001.

Burmister, Donald, M. The Importance and Practical Use of Relative Density in Soil Mechanics. Proceedings American Society for Testing Materials. Vol. 48, pp 1249-1268, 1948.

Cartensen, J. T. & Chan, P. L. Repose Angles and Flow Properties of Non-Cohesive Powders. Journal Pharm. Sci., 65, 1965.

Frette, V., Christensen, K., Malthe-Sorenssen, A., Feder, J., Torstein, J. & Meakin, P.. Avalanche Dynamics in a Pile of Rice. Nature vol. 379, January 4, 1996, pp 49-52.

Hight, D. W. and Leroueil, S., 2003, “Characterisation of Soils for Engineering Purposes,” Proceedings of the Characterisation and Engineering Properties of Natural Soils, Vol. 1, pp. 255– 360.

Lee, Jysoo & Herrmann, Hans. Angle of Repose and Angle of Marginal Stability: Molecular dynamics of Granular Particles. Journal of Physics A: Mathematical and General, Vol. 26, No. 2, pp 373-383, January 1993.

Lowe, John. Shear Strength of Coarse Embankment Dam Materials. Proceedings, 8th Congress on Large dams, pp. 745-761, 1964.

Vallejo, Luis, E. Interpretation of the Limits in Shear Strength in Binary Granular Mixtures. Canadian Geotechnical Journal, 38, pp. 1097-1104, 2001.

Yagiz, S. (2001) Brief note on the influence of shape and percentage of gravel on the shear strength of sand and gravel mixture. Bulletin of Engineering Geology and the Environment, 60(4), 321-323.

Zeller, J. and Wulliman, R. The Shear Strength of the Shell Materials for the Goschenenalp Dam, Switzerland. Proceedings, 4th Conference on Soil Mechanics and Foundation Engineering, Vol II, pp. 399-404. 1957.