Numerical investigations on vibration screening by in-filled geofoam trenches

Ashref Alzawi, M. Hesham El Naggar, Ph.D. P. Eng.
Department of Civil and Environmental Engineering – The University of Western Ontario, London, Ontario, Canada

ABSTRACT
In-filled concrete and bentonite trenches have been used in practice for many years as wave barriers to control the transmitted surface steady-state ground vibration induced by machine foundations. On the other hand, experimental investigations conducted by the authors showed that in-filled geofoam trenches can be used as an effective wave barrier to reduce the transmitted surface waves. In this paper, numerical models were developed to further investigate the effectiveness of geofoam trenches as wave barriers. The developed numerical models have been verified by comparing the numerical results with those obtained from the field experiments and a favourable agreement was observed. A comprehensive parametric study has been performed to examine the influence of various geometrical and material parameters on the protective effectiveness of geofoam trenches as wave barriers. The results are analyzed, interpreted and some guidelines regarding the design of geofoam wave barriers are outlined.

RÉSUMÉ
En béton rempli et tranchées de bentonite ont été utilisés dans la pratique depuis de nombreuses années comme des obstacles à la vague de la surface de contrôle transmis vibrations du sol l'état d'équilibre induite par des fondations de la machine. D'autre part, des études expérimentales menées par les auteurs ont montré que les tranchées geofoam en-rempli peut être utilisé comme une barrière d'onde efficace pour réduire la transmission des ondes de surface. Dans cet article, les modèles numériques ont été développées pour étudier plus avant l'efficacité des tranchées geofoam que les obstacles d'onde. Les modèles développés numériques ont été vérifiées par la comparaison des résultats numériques avec ceux obtenus à partir des expériences de terrain et un accord favorable a été observée. Une étude paramétrique exhaustive a été réalisée pour examiner l'influence de divers paramètres géométriques et des matériaux sur l'efficacité de protection de tranchées geofoam que les obstacles d'onde. Les résultats sont analysés, interprétés et des lignes directrices concernant la conception des écrans d 'ondes geofoam sont décrites.

1. INTRODUCTION

The use of geofoam material in vibration wave screening has been the focus of a few studies. Davies (1994) carried out a series of 20-g centrifuge tests to explore the screening effect of the expanded polystyrene (EPS) wall, concrete wall and their composites on adjacent buried structures. The centrifuge test results indicated that barriers containing low acoustic materials were highly effective in the attenuation of stress wave propagation. Wang (2008) has conducted numerical investigations on the performance of EPS geofoam barriers to protect buried structures against the effect of blast-induced ground shock. An open trench, an inundated water trench, three in-filled geofoam walls with different densities, and a concrete wall have been included in the numerical simulation. The numerical model simulated the prototype dimensions of a centrifuge test carried out by Davies (1994). Based on the numerical model findings, the geofoam barriers were found to significantly reduce the blast-induced stress waves. Moreover, Wang (2008) noted that the geofoam barrier is considered to provide flexibility in design that can be easily and efficiently implemented in the field. However, it should be noted that vibration sources in the above-mentioned two studies were blast-induced ground shock.

Murillo et al. (2009) performed centrifuge tests to simulate the traffic vibration and investigate the effectiveness of EPS barriers in scattering this type of ground borne vibrations. The centrifuge tests involved a parametric study to examine the EPS barriers performance in terms of the barrier dimensionless geometry and its distance from the source of disturbance. The results showed that the barrier performance depends mainly on its depth and location from the vibratory source. Also, it was concluded that the barrier width had a minor influence in the case of deeper barriers. On the other hand, a remarkable influence of the barrier width was observed for the case of shallow barriers.

The above mentioned studies demonstrated that geofoam wave barriers can be used as an effective tool to screen ground-borne vibrations, and that geofoam polymers provide an attractive construction material for these barriers. Alzawi and El Naggar (2009) conducted a comprehensive parametric study to investigate, numerically, the behavior and effectiveness of the geofoam material under periodic harmonic loadings as active and passive wave barriers in the form of box-wall, single-continuous wall, double-continuous and double-staggered wall systems. The finite element method (FEM) has been adopted as a numerical tool to simulate the problem of wave propagation in soil medium. The results showed that all the proposed configurations performed well in scattering the surface waves; however, the single-continuous wall system was considered the economic and practical alternative for wave scattering.

Alzawi and El Naggar (2011) investigated, in a field study, the constructability of a single-continuous wall

system and its effectiveness in vibration isolation for harmonic excitations. A series of full scale vibration tests have been conducted to examine the effects of both open and in-filled geofoam trench barriers geometry and location from the source of disturbance on the system protective effectiveness. Moreover, an experimental parametric study is conducted to investigate the effect of varying the ratio between the barrier depth and its location. An innovative approach to construct open and geofoam trenches was introduced as well. The experimental results confirmed that both open and geofoam barriers can effectively reduce the transmitted vibrations. Thus, some guidelines and recommendations, based on the experimental findings, were proposed for dimensioning the in-filled geofoam trench to achieve a protective effectiveness up to 68% or higher.

Furthermore, the performance of the two types of wave barriers in scattering ground vibrations has been assessed numerically by developing a 2D FE model (Alzawi and El Naggar, 2010). It was concluded that the developed 2D FE model could be used to extrapolate the experimental results and conduct a parametric study on the geofoam barrier performance with different configurations and in different soil profiles.

Therefore, the objective of this study is to conduct a comprehensive parametric study on the in-filled geofoam trench barrier performance under vertical harmonic loadings. The parametric study is conducted by varying the key parameters such as barrier geometric dimensions, location, and soil properties. Both 2D and 3D numerical models were developed in the time domain by utilizing the finite element package, ABAQUS (2007). The adequacy of the 2D FE model in comparison with 3D FE model is discussed as well.

2.   METHODOLOGY OF STUDY

Calibrated 2D and 3D FE models have been established in order to conduct an extensive parametric study. The calibration process of the FE models was performed using two well-documented reference studies. Given the fact that the 2D FE model has much lower computational cost, such model would be more efficient in conducting an extensive parametric study to better understand the behaviour of in-filled geofoam trench barrier with different dimensions, locations and different soil conditions. Therefore, the accuracy of the 2D FE plane-strain model was verified by comparing the obtained results with those obtained from 3D FE model. A staged mesh refinement has been carried out to obtain an optimized meshing configuration.

The adopted key parameters in this parametric study are: the barrier depth; distance between the barrier and source of disturbance; and dynamic soil properties including shear wave velocity, density, Poisson's ratio, and material damping. All geometric parameters are normalized by the Rayleigh wavelength, λ_R. The numerical model results are analyzed and interpreted to provide recommendations for design purposes. A typical schematic of the vibration isolation system and geometric parameters are shown in Figure 1.
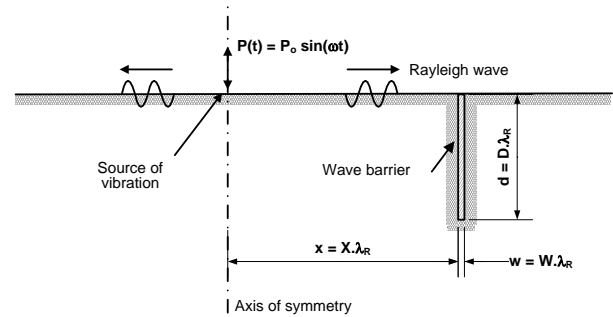

Figure 1. Typical schematic presentation of the vibration isolation system and geometric parameters.

3.   FINITE ELEMENT MODELS

The 2D and 3D FE models were developed by utilizing the finite element package, ABAQUS (2007). The explicit dynamic analysis procedure has been adopted in performing the numerical modelling using direct integration solution. The soil and wave barriers in the 2D model were modeled using 4-noded bilinear, reduced integration, plane-strain rectangular elements with relevant properties. On the other hand, the 3D FE model was mainly used to verify the accuracy and reliability of the 2D FE model in simulating the problem of wave propagation. Thus, the soil and wave barriers were modeled using 8-noded first-order, reduced integration, hexahedron elements with relevant properties. Moreover, the soil was modeled as a homogeneous, isotropic, elastic, half-space.

It is worth mentioning that higher-order elements perform much better in wave propagation problems. However, linear elements can be a successful choice to reach an adequate accuracy in simulating wave propagation problems if the element size number is kept smaller than one-eighth of the shortest possible Rayleigh wavelength λ_R, (Kramer, 1996). Hence, in the present models, the minimum number of elements per wavelength was at least 28 and 14 elements for 2D and FE models respectively. Further, to ensure complete energy dissipation, non-reflecting semi-infinite elements have been imposed at the artificial boundaries to simulate the far field conditions (Suhol, 1997). First-order 8-noded solid continuum, one-way semi-infinite elements were assigned to represent the non-reflecting boundaries in the 3D FE model while first-order plane-strain 4-noded solid continuum, one-way semi-infinite elements were used to represent the non-reflecting boundaries in the case of the 2D FE model.

Based on the symmetrical nature of the problem, symmetry boundary conditions were applied by restraining the displacement in the perpendicular direction to the symmetry surfaces. Hence, the axis of symmetry was placed across the point of load application. The analysis has been extended until the conditions of steady state response conditions were reached.

The surface waves have been generated by applying vertical harmonic dynamic loading represented by a sine function. For modeling purposes, the footing carrying the dynamic load was eliminated as it did not practically affect

the vibration results (Kattis 1999). Thus, the load was applied at varying distances from the barriers and pointed directly on the ground surface.

3.1   Finite Element Models Verification

The developed 2D FE model was verified by comparing the FE model results with the results of a passive isolation problem that was investigated by Ahmad and Al-Hussaini (1991) and Beskos et al. (1986). The results were presented in the form of amplitude reduction ratio, A_r. The amplitude reduction ratio is defined as the normalized post-trench installation maximum vertical response amplitude, $(U_v)_{After}$, to the maximum vertical response amplitude before trench installation, $(U_v)_{Before}$, as shown in Equation 1. The maximum vertical response amplitudes were obtained at specified nodes along the monitoring path from their response time histories. Woods (1968) considered the averaged vertical response amplitude reduction ratio to be smaller or equal to 0.25 for an effective isolation system.

$$A_r = \frac{(U_v)_{After}}{(U_v)_{Before}} \qquad [1]$$

The passive isolation problem is defined as an open trench of depth $d=1.0\lambda_R$ and width $w=0.1\lambda_R$ located at a distance $x = 5.0\lambda_R$ from the source of vibration, which was a periodic harmonic load of magnitude of 1.0 kN frequency of 31 Hz, in an elastic half-space soil. The dynamic properties of the soil medium were in accordance to Yang and Hung (1997): shear wave velocity V_s=101 m/sec, Poisson's ratio ν=0.25, Rayleigh wave velocity V_R=93 m/sec, Rayleigh wave length λ_R =3.0 m, unit weight, γ =18 KN/m^3 and Rayleigh damping ξ=5%. Figure 2 shows a good agreement between the obtained 2D FE model results and those reported by Ahmad and Al-Hussaini (1991) and Beskos et al. (1986).

As aforementioned, the validity of using 2D instead of 3D FE model in conducting the parametric study was assessed by solving a problem of vibration isolation using in-filled geofoam trench barrier. Therefore, a geofoam trench of depth $d=1.0\lambda_R$, and width w=0.25m located at a distance $x=1.0\lambda_R$ from the source of vibration in an elastic half-space soil was considered. The material properties of the soil medium were in accordance to Kattis *et al.* (1999): shear wave velocity V_s=272 m/sec, Poisson's ratio ν=0.25, Rayleigh wave velocity V_R=250 m/sec, Rayleigh wave length λ_R =5.0 m, unit weight γ=17.5 kN/m^3 and Rayleigh damping ξ=5%. The source of vibration is modeled as a vertical harmonic load of magnitude of 1.0 kN and frequency of 50Hz. The geofoam material used in this study is a two-component Polyurethane lightweight material supplied by URETEK Canada. The geofoam material has a density of 61kg/m^3 when it is installed in the trench under no pressure, i.e., free to expand. The dynamic properties of geofoam were evaluated using Bender Elements test and Resonant Column test and were found to be: shear wave velocity of 312 m/sec, and

Poisson's ratio close to zero. Figure 3 illustrates that the results obtained from 2D and 3D FE models are in excellent agreement. Thus, it is concluded that the 2D FE model can be used, with confidence, in conducting the parametric study.
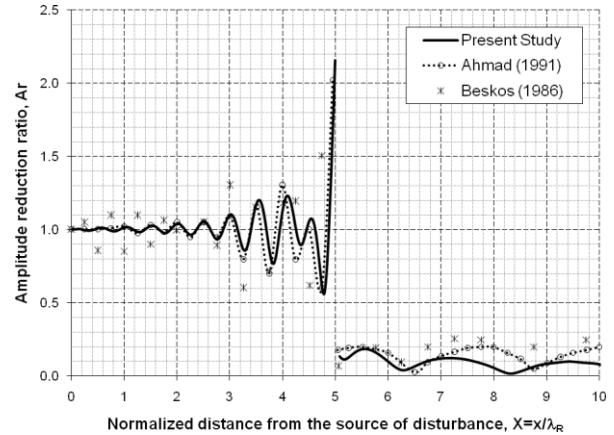

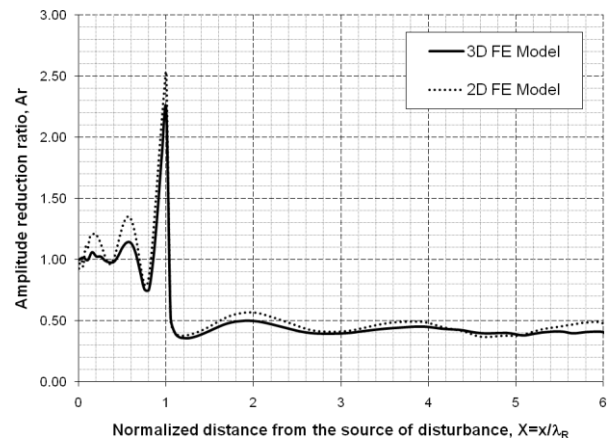Figure 2. 2D FE model verification, open trench (W=0.1, D=1, X=5)

Figure 3. 2D verses 3D FE model, geofoam trench (w=0.25m, D=1, X=1)

4.   PARAMETRIC STUDY AND RESULTS

An extensive parametric study has been carried out to study the influence of various key parameters such as the trench geometric dimensions, location, and the soil properties on the screening effectiveness of an in-filled geofoam trench barrier.

The results of the parametric study will be presented in the form of averaged amplitude reduction ratio, $\overline{A_r}$. As described in the previous section, the amplitude reduction ratio, A_r. along the monitoring path is evaluated first using Equation 1. Then the averaged amplitude reduction ratio, $\overline{A_r}$, over a distance of interest (x=5λ_R) measured behind

the wave barrier can be calculated by using the following equation:

$$\overline{A_r} = \frac{1}{x}\int A_r\ dx \qquad\qquad [2]$$

The system effectiveness is then calculated as:

$$Eff_A = \left(1 - \overline{A_r}\right) \times 100 \qquad\qquad [3]$$

4.1 Influence of barrier normalized dimensions and location from the source of disturbance

The in-filled geofoam trench wall depth and its proximity to the vibration source have been varied independently. The normalized depth D is varied from 0.4 to 2.0 and the normalized distance X between the source of disturbance and the barrier is varied from 0.3 to 4.0. The influence of normalized width W will be ignored in this parametric study as reported by Alzawi and El Naggar (2011). It was recommended that the practical width to construct such type of geofoam trench barrier system is 0.25 m, which was found to provide an excellent performance in scattering the induced ground vibration. Figures 4 and 5 present the coupled influence of D and X on the averaged amplitude reduction ratio $\overline{A_r}$ for the in-filled geofoam trench installed in an elastic half-space soil which has the following dynamic properties: shear wave velocity V_s=250 m/sec, Poisson's ratio of ν=0.3, unit weight γ=19.5 kN/m^3, and Rayleigh damping ξ=5%.
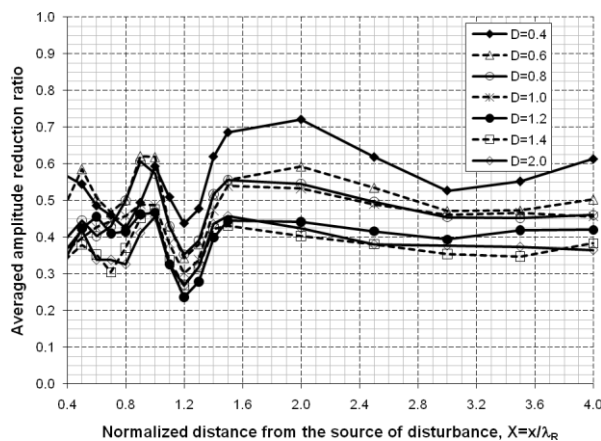

Figure 4. Influence of normalized distance from the source of disturbance, X

It can be seen that changing the normalized distance between the barrier and the source of disturbance, X, from 0.4 to 1.5 has a significant influence on the barrier performance in a complex manner with the effect of the normalized depth D. Figure 4 shows that X appears to

govern the barrier's protective effectiveness for X ranging from 0.4 to 1.5. For X>1.5, the effectiveness remains almost constant regardless of the normalized depth. For the considered configuration, a significant screening effectiveness can be achieved when the barrier is placed at X=0.4 and 1.2 (averaged amplitude reduction ratio $\overline{A_r}$ is minimum). For deeper depths (D ≥1.2), placing the barrier at any distance X can result in acceptable screening effectiveness. Another important observation is that it is apparent from Figure 5 that increasing the normalized depth D beyond 1.2 does not provide any remarkable improvement for in-filled geofoam. Hence, it may be conservatively assumed D = 1.2 as an optimum depth for geofoam trenches. It is also concluded that as the geofoam barrier's proximity to the source of disturbance increases, a deeper trench is required to achieve a significant improvement in the system's effectiveness.
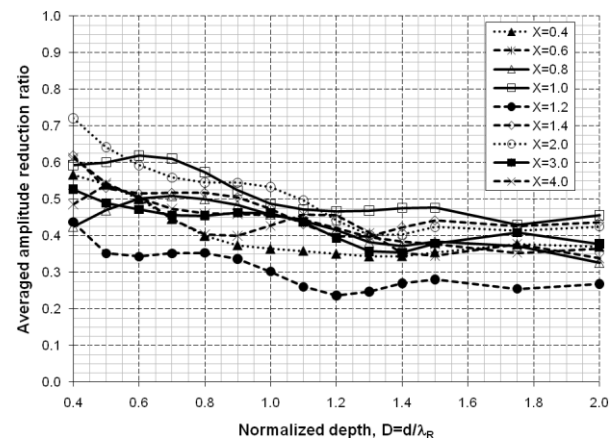

Figure 5. Influence of normalized depth, D

4.2 Influence of soil shear wave velocity

The soil shear wave velocity, V_s, and density, $\rho = \gamma/g$, are the most important soil dynamic properties in wave propagation problems, which govern the amount of reflection, refraction and mode conversion when a wave is incident at the interface between the geofoam barrier and soil medium (impedance difference). Therefore, the effect of V_s on the vibration screening effectiveness of an in-filled geofoam trench barrier is demonstrated in this section while the influence of soil density will be discussed in the subsequent section.

The soil shear wave velocity is varied from 200 m/sec to 400 m/sec. The results of a geofoam trench installed in an elastic half-space soil and located at X=0.4 and 1.2 from the source of disturbance are presented in Figures 6 and 7, respectively. Figures 6 and 7 show the general trend of the influence of variation of V_s on the averaged amplitude reduction ratio for various trench normalized depths. It is clear that vibration screening using geofoam trenches is more effective in soils with higher V_s (i.e. stiffer soils).
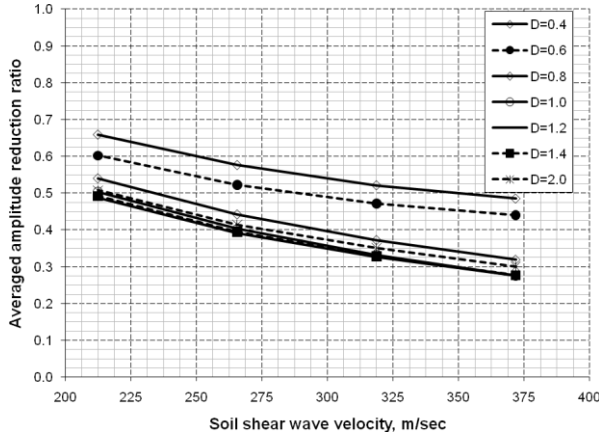
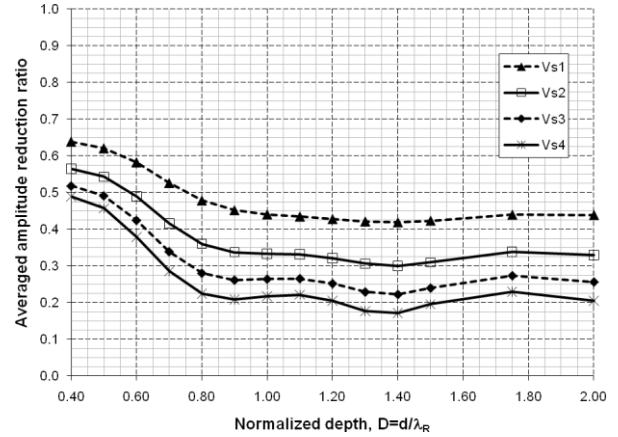Figure 6. Influence of of soil shear wave velocity (X=0.4, ν=0.25, γ=19.3kN/m^3, ξ=5%)

Figure 8. Influence of of soil shear wave velocity (X=0.4, ν=0.35, γ=19.3kN/m^3, ξ=5%)
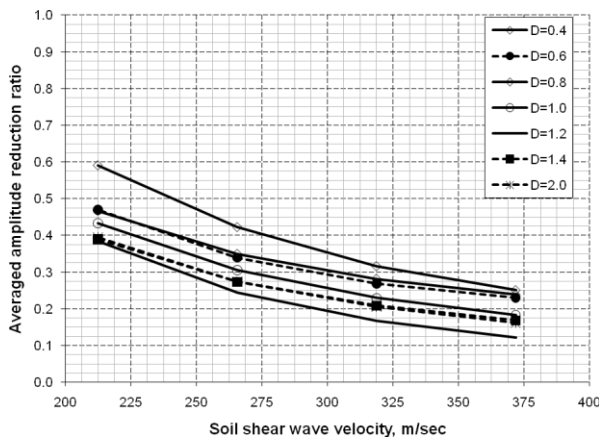

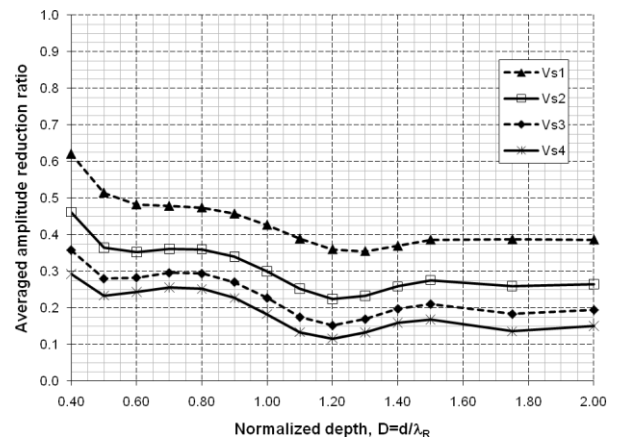Figure 7. Influence of of soil shear wave velocity (X=1.2, ν=0.25, γ=19.3kN/m^3, ξ=5%)

Figure 9. Influence of of soil shear wave velocity (X=1.2, ν=0.35, γ=19.3kN/m^3, ξ=5%)

By varying the normalized depth from 0.4 to 2.0 and the soil shear wave velocity from V_{s1} to V_{s4} (where $V_{s1}<V_{s2}<V_{s3}<V_{s4}$), Figures 8 and 9 clearly indicate that as the shear wave velocity increases, the averaged amplitude reduction ratio decreases. For example, the efficiency of a geofoam trench installed in a soil with V_s=380 m/sec will be greater than the protective efficiency of the same system installed in a soil with V_s= 210 m/sec by about 45%. Hence, the soil shear wave velocity should be considered as the most important criteria in designing in-filled geofoam barriers. It can be concluded that such type of isolation system performs more effectively in soils with relatively high shear wave velocity values (stiff soils) rather than in soils with low shear wave velocity (weak soils).
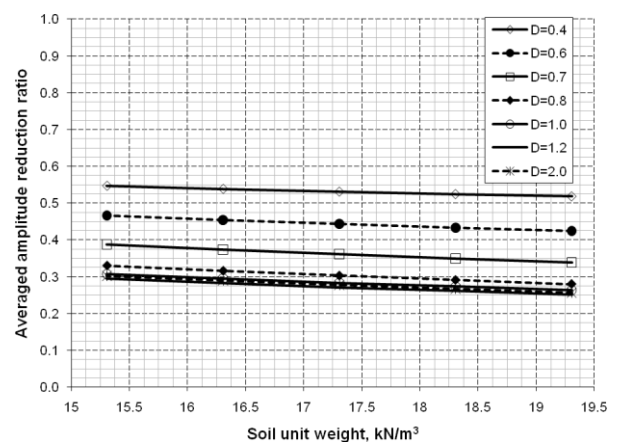

Figure 10. Influence of soil density (X=0.4, V_s=318m/sec, ν=0.35, ξ=5%)

4.3 Effect of changing the soil density

To examine the effect of soil density on the vibration screening effectiveness of geofoam barriers, the soil unit weight is varied from 15.5 kN/m^3 to 19.5 kN/m^3 while the shear wave velocity is kept constant. A sample from the obtained results are presented in Figures 10 and 11 for barriers located at X=0.4 and 1.2, respectively. As it can be seen, the effect of soil density on screening effectiveness has the same trend as that of V_s. However, the effect of soil density on the screening effectiveness is less significant. For example, the vibration screening effectiveness is higher by about 9% for a soil with unit weight of 19.5 kN/m^3 compared to that observed for soil with γ = 15.5 kN/m^3.

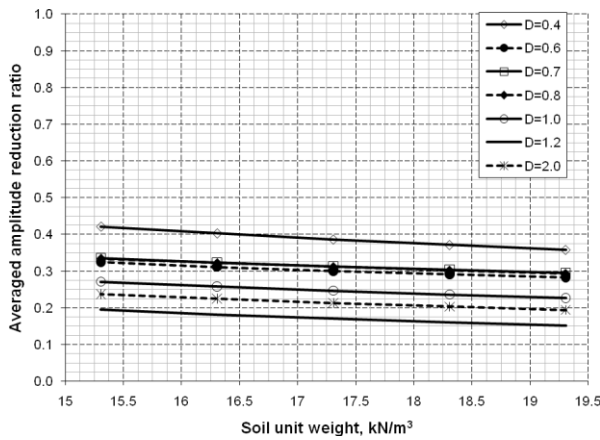Figure 11. Influence of soil density (X=1.2, V_s=318m/sec, v=0.35, ξ=5%)

4.4 Influence of Poisson's ratio

The soil Poisson's ratio, v, is varied from 0.25 to 0.4. Figures 12 and 13 demonstrate the influence of v on the performance of in-filled geofoam trench. Figure 12 shows that the performance of a geofoam trench barrier with normalized depths ranging from D=0.6 to 1.2 and located at X=0.4 from the source installed in soil with v = 0.4 is higher than the effectiveness of the same barrier installed in a soil with v = 0.25 by about 15% or less. On the other hand, for barriers with proximity to the source X =1.2, Figure 13 shows that the effect of v on screening effectiveness is unclear and insignificant. It can be concluded that the effect of the soil Poisson's ratio on the protective effectiveness of in-filled geofoam trench barriers is not important.
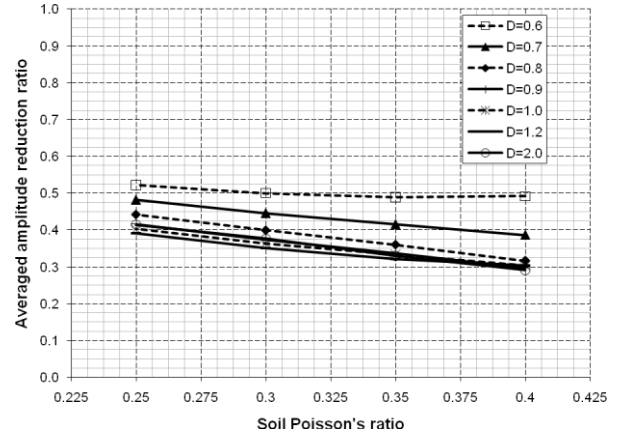

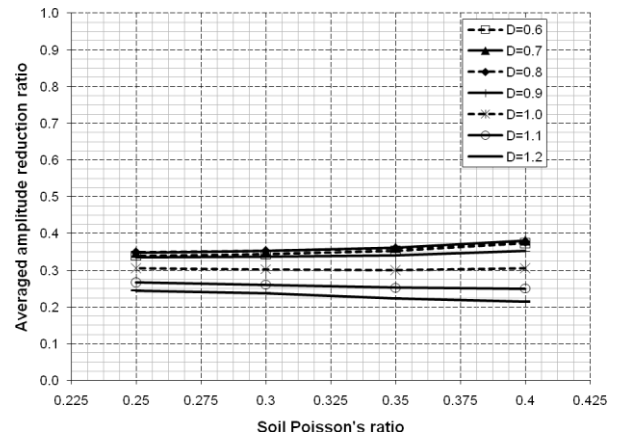Figure 12. Influence of soil Poisson's ratio (X=0.4, V_s=265m/sec, γ=19.3kN/m^3, ξ=5%)

Figure 13. Influence of soil Poisson's ratio (X=1.2, V_s=265m/sec, γ=19.3kN/m^3, ξ=5%)

4.5 Influence of material damping

The damping represents the system ability to dissipate energy, which has to be accounted for in FE models used to investigate dynamic phenomena. In this parametric study, the soil material damping has been implemented in the FE models in the form of Rayleigh damping. Rayleigh damping is defined by specifying two Rayleigh damping factors which are mass and stiffness proportional damping. The soil damping is varied from 1% to 10%.

A sample from the obtained results is presented in Figure 14, which demonstrates the influence of changing the soil's material damping on the performance of in-filled geofoam trench. According to the definition of amplitude reduction ratio and as expected, small differences can be observed. Thus, it can be concluded that changing the soil material damping has a minor influence on the system screening performance.
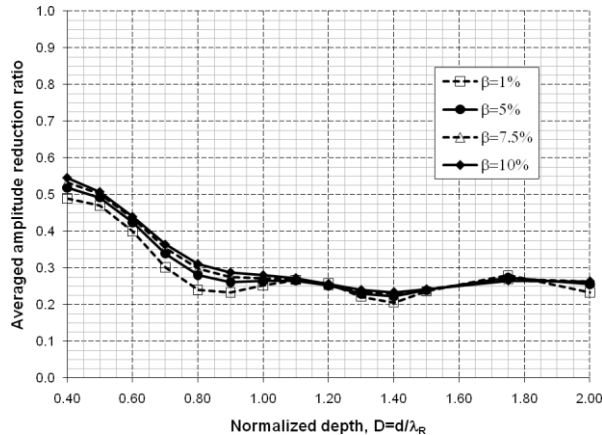
Figure 14. Influence of material damping (X=0.4, V_s=318m/sec, γ=19.3kN/m^3, ν=0.35)

5. CONCLUSIONS

This paper summarizes the results of a numerical investigation of the protective effectiveness of in-filled geofoam trenches as wave barriers to scatter the steady state vibration induced by machine foundations. The methodology used involved conducting an intensive parametric study employing 2D FE numerical models. The barrier depth and location were varied independently as well as the soil dynamic properties. The obtained results were for geofoam barriers installed in an elastic half-space soil. The wave barriers protective effectiveness was evaluated based on the achieved reduction in soil particle response. Based on the results obtained and their analysis, the following conclusions can be made:

1) The key parameters are found to be the barrier depth, barrier's proximity to the source of disturbance, and the shear wave velocity of soil medium. The soil density, Poisson's ratio, and material damping also have some influence but less significant.

2) As the barrier's proximity to the source of disturbance increases, a deeper trench is required to achieve a significant improvement in its effectiveness.

3) For practical design, the normalized depth D should be greater than 1.2 for maximum performance. However, D can be as low as 0.8 for X = 0.4. Also, for practical construction purpose, the width of geofoam barrier can be kept at 0.25m.

4) In-filled geofoam trench barrier performs more effectively in stiff soils (i.e. with relatively high V_s values) rather than in soft soils (i.e. with low V_s values). Accordingly, the soil shear wave velocity should be considered as the most important criteria in designing in-filled geofoam trench barriers.

REFERENCES

Ahmad, S. and Al-Hussaini, T.M. 1991. Simplified Design for Vibration Screening by Open and In-Filled Trenches. *Journal of Geotechnical Engineering*, 117: 67-88.

Ahmad, S., Al-Hussaini, T.M. and Fishman, K.L. 1996. Investigation on Active Isolation of Machine Foundations by Open Trenches. *Journal of Geotechnical Engineering*, 122: 454-461.

Al-Hussaini, T.M. and Ahmad, S. 1991. Design of Wave Barriers for Reduction of Horizontal Ground Vibration. *Journal of Geotechnical Engineering*, 117: 616-636.

Al-Hussaini, T.M. and Ahmad, S. 1996. Active Isolation of Machine Foundations by In-Filled Trench Barriers. *Journal of Geotechnical Engineering*, 122: 288-294.

Al-Hussaini, T.M., Ahmad, S., and Baker, J.M. 2000. Numerical and Experimental Studies on Vibration Screening by Open and In-Filled Trench Barriers, in : S. Chouw(Ed.), *Wave 2000*, Balkema, Rotterdam, 241-250.

Alzawi, A. and El Naggar, M.H. 2009. Vibration Scattering Using GeoFoam Material as Vibration Wave Barriers, *62nd Canadian Geotechnical Conference*, Halifax, NB, Canada, 997-1004.

Alzawi, A. and El Naggar, M.H. 2010. Experimental Investigations on Vibration Isolation Using Open and GeoFoam Wave Barriers: Comparative Study, *63rd Canadian Geotechnical Conference*, Calgary, AB, Canada, 360-368.

Alzawi, A. and El Naggar, M.H. 2011. Full scale experimental study on vibration scattering using open and in-filled (geofoam) wave barriers, *Soil Dynamics and Earthquake Engineering*, 31: 306–317.

Beskos, D.E., Dasgupta, G. and Vardoulakis, I.G. 1986. Vibration Isolation Using Open or Filled Trenches, Part1: 2-D Homogeneous Soil, *Computational Mechanics* 1: 43-63.

Dassault Systèmes. 2007. ABAQUS 6.7 User's Manual, Providence, RI, USA.

Davies, M.C.R. 1994. Dynamic Soil Structure Interaction Resulting from Blast Loading. Leung, Lee and Tan(Eds.), *Centrifuge 94*, Balkema, Rotterdam, 319-324.

El Naggar, M.H. and Chehab, A.G. 2005. Vibration Barriers for Shock-Producing Equipment, *Canadian Geotechnical Journal*, 42: 297–306.

Haupt, W.A. 1977. Isolation of Vibrations by Concrete Core Walls, In Proceedings of the Ninth International Conference on Soil Mechanics and Foundation Engineering, vol. 2. Tokyo, Japan, 251-256.

Haupt, W.A. 1981, Model Tests on Screening of Surface Waves, Proceedings of the 10th Conference on Soil Mechanics and Foundation Engineering, Vol.3, Stockholm, 215-222.

Kattis, S.E., Polyzos, D. and Beskos, D.E. 1999. Vibration Isolation by a Row of Piles Using a 3-D Frequency Domain BEM. *International Journal of Numerical Methods in Engineering*, 46: 713-728.

Kattis, S.E., Polyzos, D. and Besko, D.E. 1999. Modeling of Pile Wave Barriers by Effective Trenches and Their Screening Effectiveness, *Soil Dynamics and Earthquake Engineering*, 18: 1–10.

Kramer, SL. 1996. *Geotechnical Earthquake Engineering*, Prentice-Hall Inc, Upper Saddle River, NJ, USA.

Murillo, C., Thorel, L. and Caicedo, B. 2009. Ground vibration isolation with geofoam barriers: Centrifuge modeling, *Geotextiles and Geomembranes*, 27:423–34.

Suhol, B. 1997. Infinite Boundary Elements for the Dynamic Analysis of Machine Foundations, *International Journal for Numerical Methods in Engineering*, 40: 3901-3917.

Wang, JG, Sun, W. and Anand, S. 2008. Numerical investigation on active isolation of ground shock by soft porous layers, *Journal of Sound and Vibration,* 321: 492–509.

Woods, R.D. 1968. Screening of Surface Waves in Soils, *Journal of Soil Mechanics and Foundation Engineering Div,* ASCE, 94(4): 951–79.

Yang, Y.B., Kuo, S.R. and Hung, H.H. 1996. Frequency-Independent Infinite Elements Analyzing Semi-Infinite Problems, *International Journal for Numerical Methods in Engineering*, 39: 3553-3569.

Yang, Y.B. and Hung, H.H. 1997. A Parametric Study of Wave Barriers for Reduction of Train Induced Vibrations, *International Journal for Numerical Methods in Engineering*, 40: 3729-3747.