Modeling of tailings flow using Smoothed Particle Hydrodynamics for risk assessment of tailings dam breaches

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ABSTRACT
The potential risk associated with tailings dam failures can be considerable due to the release of toxic chemicals into the environment. The extent of outflow and region of potential damage caused by tailings dam failures can be difficult to assess due to the inherent uncertainty in the estimation of tailings properties and the interaction of tailings flow with the topology of the terrain. The tailings flow is modeled using smoothed particle hydrodynamics (SPH) to capture the evolving nature of the flow. Due to the large number of parameters involved, a back-analysis of literature-reported dam failures was used for the testing and validation of the applicability of SPH method, with the results showing a considerable agreement with the literature-reported ones. Thus, the use of SPH simulations could enable analysis of the extent of tailings outflow for the design of future tailings facilities and to assess the potential damage arising from failure of existing ones.

PRESENTACIONES TÉCNICAS
El riesgo potencial asociado a la falla de represas de relave puede ser considerable debido a la descarga de químicos tóxicos al ambiente. La extensión de esta descarga y región de daño potencial causado por estas fallas pueden ser difíciles de evaluar debido a la incertidumbre inherente en la estimación de las propiedades de los relaves y a la interacción de este fluido con la topografía del terreno. El flujo de relaves es modelado utilizando hidrodinámica de partículas suavizadas (HPS) para captar el avance natural del flujo. Debido a la gran cantidad de parámetros involucrados, un análisis inverso de varias fallas en represas reportadas fueron utilizadas para comprobar y validar la aplicabilidad del método HPS. Los resultados mostraron una considerable concordancia con aquellos reportados en la literatura. Por lo tanto, el uso de simulaciones HPS puede permitir el pronóstico de la extensión de la descarga en el diseño de futuras instalaciones de desechos de relave y para evaluar el daño potencial que puede provocar la falla de represas existentes.

1 INTRODUCTION

Tailings dam failures are often sources of considerable negative environmental impact affecting nature, the built environment and the human population. Traditional geotechnical assessment of tailings facilities concentrates on the stability of tailings dams and other infrastructure (Vick, 1983), while relatively few studies have investigated the flow of tailings released following a dam failure. In particular, how far the tailings will reach and what would be the extent of damage. Knowing these could aid in assessing the risk associated with disposing tailings and plan for remedial measures in case a dam break occurs. Perhaps what hinders research in modeling tailings flow is the complex fluid-like behavior of tailings coupled with its interaction with the terrain it is flowing on. The traditional numerical methods of analysis such as the finite element of finite difference formulations need to address the free surface and moving boundaries with frequent re-meshing during the course of a simulation to comply with the changing geometry of the flowing tailings mass. However, methods employing a mesh-free formulation easily resolve these issues in simulating a flowing mass such as tailings. Smoothed Particle Hydrodynamics (SPH) in particular, is a method used in developing fluid flow models having been applied from astrophysics, underwater explosion and general fluid mechanics problems (Liu and Liu, 2010). The aim of this paper is to use SPH to model the flow of tailings in conjunction with developing a quantitative assessment of parameters such as the topology of terrain and surface roughness in recreating a model of a well-documented tailings dam breach.

2 SALIENT FEATURES OF THE SMOOTHED PARTICLE HYDRODYNAMICS FORMULATION

The essence of the SPH formulation is presented next, however for a detailed treatment of SPH the reader is referred to Liu and Liu (2003) and more recently, Liu and Liu (2010). Generally, in the SPH method particles having properties such as mass, velocity, density and position are used to discretize a continuum. Since particles move with the general velocity of the material a Langrangian-type set of equations can be formulated. Thus the governing partial differential equations can be transformed into integral equations via a kernel, or smoothing function, approximation for expressing the first and second derivatives. Therefore, the main development behind an SPH method is the interpolation of a function such as $f(x)$

based on Equation 1, where it is evaluated with a help of a kernel function *W(x, h)*

$$f(x) = \int_D f(x')W(x-x',h)dx' \quad [1]$$

which, in terms of discrete particles (*N*) in the neighborhood of a particle *i* can be written as

$$f(x) = \sum_b^N f(x_b)W(x-x_b)V_b = f_h(x) \quad [2]$$

where *h* is the smoothing length – the significance of the area around a particle in which the dynamics of the first particle is considerably affected by all neighbouring particles.

The smoothing function *W(x,h)* has to satisfy the conditions of consistency and positivity (Crespo et al. 2008). In addition Swegle et al. (1995) postulate that the smoothing function has to possess the following properties

a. has the property of a Delta function,

$$\lim_{h\to 0} W(x,h) = \delta(x) \quad [3]$$

b. satisfy the normalization (or unity) condition,

$$\int W(x,h)dx = 1 \quad [4]$$

c. being zero everywhere except in the smoothing domain (compactness)

$$W(x,h) = 0 \text{ for } |x| \geq 2h \quad [5]$$

Thus, a general form for a kernel function is

$$W(x,h) = \frac{\alpha}{h^{d_m}} f(\xi) \; \xi = \frac{\|x\|}{h} \quad [6]$$

where d_m is the number of dimensions, *h* is the smoothing length, *x* is the distance between two particles and *α* is a factor that ensures the satisfaction of the consistency condition.

The location of the interpolating points is tracked within the fluid as a simulation progresses. The SPH adheres to the conservation law equations, material constitutive models and equations of state. A brief overview is given of each:

Conservation of momentum: A general form for the momentum conservation equation is

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\nabla P + \boldsymbol{g} + \boldsymbol{\Theta} \quad [7]$$

where *g* is the acceleration due to gravity, *P* is pressure and *Θ* represents diffusive terms. Among different diffusive equations used to develop a momentum equation, the ‘Artificial Viscosity’ approach has been used by Monaghan (1994) and Crespo et al. (2008) which results in the following equation for particle *a*, with respect to a particle *b*,

$$\frac{dv_a}{dt} = -\sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab}\right) \nabla_a W_{ab} + g \quad [8]$$

where $\nabla_a W_{ab}$ is the gradient of the smoothing kernel and

$$\Pi_{ab} = \begin{cases} \frac{-\alpha \overline{c_{ab}} \mu_{ab} + \beta \mu_{ab}^2}{\overline{\rho_{ab}}} & v_{ab}x_{ab} < 0 \\ 0 & v_{ab}x_{ab} > 0 \end{cases} \quad [9]$$

where

$$\mu_{ab} = \frac{h v_{ab} x_{ab}}{x_{ab}^2 + \eta^2} \quad [10]$$

where *x* and *v* present the position and velocity of a particle. x_{ab} and v_{ab} are the relative displacements and velocities, $\eta = 0.01h^2$, and *c* is the speed of sound. The constants α and β are characteristic of the problem, Rodriguez-Paz and Bonet (2003) and Monaghan (1994) give examples for these constants.

Density and continuity: According to law of conservation of mass:

$$\frac{d}{dt}\int \rho dV = 0 \quad [11]$$

The objective of the SPH method is to compute a rate of change for density, then to find a smoothed density for each particle and sum over the particles in the smoothing domain of a sample point, therefore density is expressed as

$$\rho = \sum_b m_b W_{ab} \quad [12]$$

While the fluid is considered incompressible, it easier to solve the problem if a slight compressibility is assumed. For water-like fluids the density on a free surface falls to zero. Thus, to be able to avoid this abrupt density decrease near the interfaces and achieve correct pressure and simplify the calculations, Crespo et al. (2008) suggested using a modified equation, which assigns the same initial density to all particles. This state only changes when the particles are in motion (Monaghan, 1994)

$$\frac{d\rho_a}{dt} = \sum_b m_b v_{ab} \nabla_a W_{ab} \quad [13]$$

Particle motion: The rate of change in the position of a particle can be expressed by

$$\frac{dx_a}{dt} = v_a \quad [14]$$

This general overview of SPH is not indented to be a comprehensive one. Nevertheless, it introduces the basic concepts of SPH that will be referred to in the modeling of tailings flow.

3 MODELING OF TAILINGS DAM FAILURES USING SPH - MERRIESPRUIT TAILINGS DAM FAILURE

The Merriespruit tailings facility, enclosed with a 38-meter high dam, was located 250 kilometers from Johannesburg, South Africa. The total capacity of the facility was about 270 million cubic meters (Van Niekerk and Viljoen 2005). Due to a heavy rainfall in February 1994, the dam breached along a 150-meter wide section. It took 600,000 cubic meters of tailings (2.5 million tons (Van Niekerk and Viljoen 2005)) 5 minutes to travel a distance of 1960 meters through the valley and into an adjacent urban area (Davies 2001), which resulted in 17 deaths and serious damages to properties (Wagener 1997). The general layout of the facility in relation to the breach and the area affected by tailings is shown on Figure 1. Table 1 summarizes the tailings dam geometry and the extent of tailings flow as reported by Van Niekerk and Viljoen (2005) for the Merriespruit tailings facility.

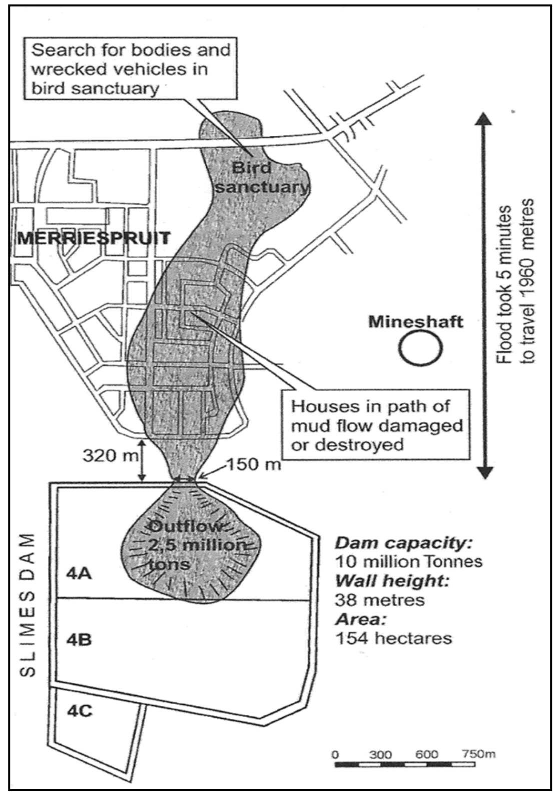

Figure 1. The flow path and distribution of tailings in the Merriespruit tailings dam breach (Van Niekerk and Viljoen 2005)

Table 1. Merriespruit tailings dam data and characteristics of tailings flow

Dam height (m)	Breach length (m)	Runout distance (m)	Runout time (min)	Nature of tailings
38	150	1960	5	Low Viscosity

Generally, the flow of tailings is affected by two key parameters; the characteristics of the terrain and the physical properties of tailings. Characteristics of the terrain influence tailings run out distance and rate of advance. The terrain affects the tailings' advance rate due to the gradient of terrain; the direction and routing of flow due to large-scale topographic features. On a much smaller scale, local irregularities can affect the direction of flow or retard the flow. The incorporation of large-scale features in the model was achieved using digital elevation models (DEM) of the terrain affected the by tailings dam failure. The DEM model was selected with the highest available degree of resolution, which is often not better than 30 meters. Therefore, the large-scale features of DEM maps needed to the supplemented with relatively accurate small-scale parameters such as the roughness of the terrain.

The properties of tailings used in the modeling affect the simulation; the travel distance of tailings, apart from the effect of topography, depends on parameters such as viscosity and density of tailings. The more viscous the tailings are, the greater driving forces they need to travel longer distances. As reported by Van Niekerk and Viljoen (2005), the Merriespruit tailings exhibited a more fluid behavior and were represented by relatively low values of viscosity and density. Therefore, both density and viscosity were parameters in the simulation that were varied to obtain the literature reported flow characteristics of the tailings.

The geometry of the tailings facility was formed from a triangulated DEM map along with basic primitives representing the dam itself and the zone of breach. To create the breach, a retaining object was embedded in the dam where the breach developed and was removed at the beginning of simulation when the area behind the dam was filled, in essence, representing a dam-breach situation. The SPH simulation was carried out using RealFlow (Botella 2006), which is an SPH-based modeling environment for fluid simulation. The goal of the simulation was to match the run out length of tailings and thus necessitated the back calculation of most important parameters for obtaining the literature-reported tailings behavior. In particular, parameters such as friction and surface roughness, which are difficult to quantify on a larger scale, required considerable adjustment. Nevertheless, based on the results of the mesh-free particle-based formulation, the simulation accurately captured the physics of tailings flow.

The literature suggested that the tailings in the pond were considerably diluted by the heavy precipitation prior to dam breach (Van Niekerk and Viljoen 2005). Therefore, the tailings had characteristics similar to water, thus the viscosity was selected to be more representative of a watery material. The key model parameters are summarized in Table 2. The friction and roughness, representing the terrain, were reduced to simulate a condition where tailings can flow unimpeded attaining a large run out length. Note that some of the parameters in RealFlow do not have physical units, so considerable calibration was necessary to map from the physical domain to the range of values used by RealFlow.

Table 2. Final model parameters for Merriespruit tailings dam failure

Density (kg/m^3)	Viscosity	Friction and roughness
1500	3	0.1

After the removal of an object representing the breached section of the dam, the low-viscosity tailings flowed out of the impoundment and traveled a distance of 1960m in approximately 12 minutes. The plots of distance and velocity versus time of the advancing front of tailings are shown in Figures 2 and 3 respectively.

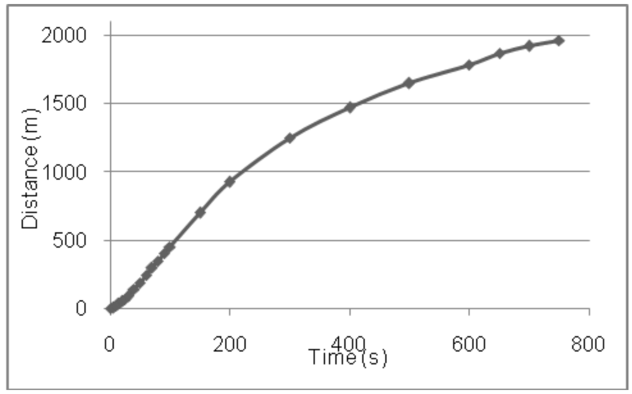

Figure 2. Run out distance versus time for Merriespruit tailings flow simulation

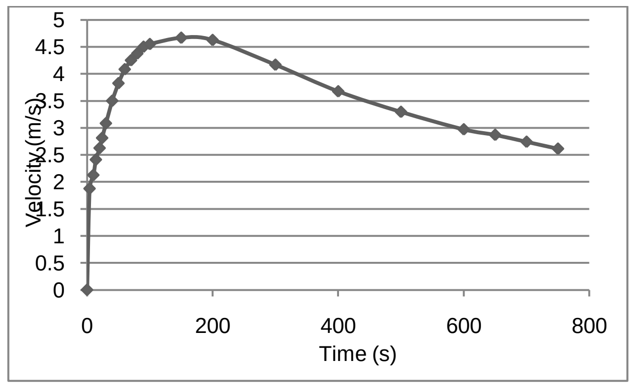

Figure 3. Velocity versus time for Merriespruit tailings flow simulation

As shown on Figures 1 and 2, within the first hundred seconds the tailings flow advanced rapidly and the velocity increased continuously. During the first hundred and fifty seconds, the velocity, due to the head of the tailings behind the dam under the influence of gravity, continued to increase. However, after reaching a maximum value of 4.7 m/s it started to decrease. The effects of friction, roughness and small-scale irregularities of the terrain became significant along with the general flatness of the terrain (from the DEM map) resulting in decrease in the momentum of flow as observable from the decline in velocity.

According to reports and observed from satellite imagery, the tailings had entered a pond upon reaching their terminus on land. Thus establishing what the final condition to terminate the simulation was a challenge. It was assumed that once the tailings had entered the water they got diluted and dispersed or settled.

Key frames of the tailings advancement are shown on Figure 4. The upper image shows the front of tailings flow at 80 meters from the breach. The middle image is taken at 1650 meters from the breach while the bottom one is at the terminus reaching the pond at 1960 meters from the tailings facility. Due to the relative flatness of the terrain, where the housing development was located, the tailings flow has widely dispersed as shown on the middle and bottom images on Figure 4.

Although, the tailings flow exhibited a behaviour of thinning and fanning out on a flat terrain an important shortcoming was identified; the location and arrangement of houses forming streets in the subdivision could act as channels to guide and divert the flow and reduce velocity of tailings. These obstacles to flow were not modeled in the simulation, however, the inclusion of such objects representing houses is feasible and it is proposed for a future study.

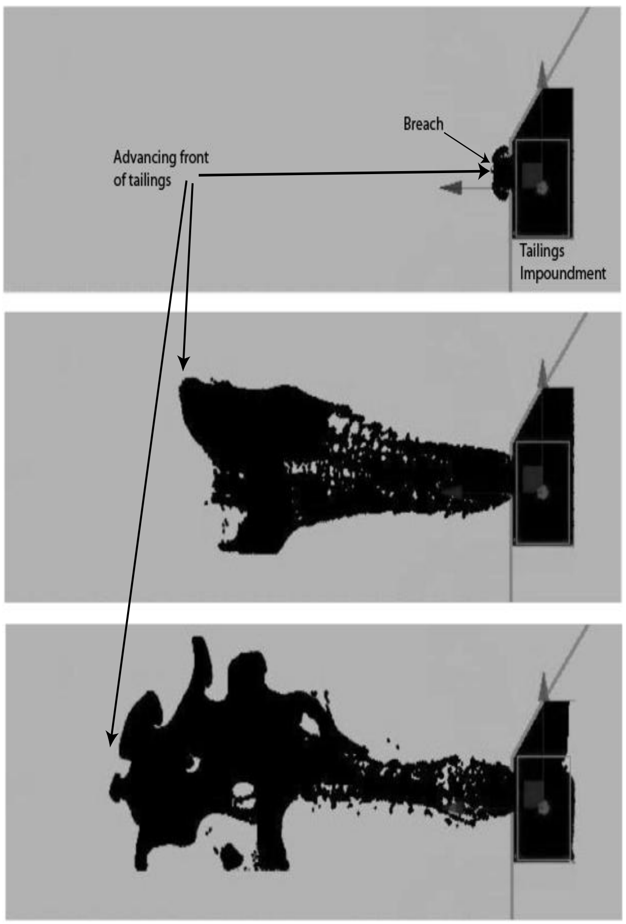

Figure 4. Key frames of Merriespruit tailings dam failure simulation showing advancement of tailings at 80m, 1650m and 1960m from the dam

In conclusion, if the results of simulation are superimposed on a satellite imagery of the terrain, a substantial agreement is evident, as seen from Figure 5. The distribution of flow, in comparison to what the literature reported encompasses (cf. with Figure 1) the residential area, where most of the damage occurred, and terminates at the pond of a bird sanctuary.

Figure 5. The result of Merriespruit tailings dam failure simulation superimposed on satellite imagery (Google Earth 2010)

4 CONCLUSIONS

The threat and damage caused by tailings dam failures can be a costly and often fatal result of many mining operations around the globe. The interaction of fluid-like tailings with the terrain generally dictates the direction and velocity of flow. Characteristics of the terrain, such as large-scale topologic features along with small-scale roughness can retard and direct the flow of tailings. To simulate the tailings flow, RealFlow, a SPH-based tool, was used after a set of calibration tests. The SPH model was applied to a literature-reported tailings dam failure with low viscosity tailings (Merriespruit). Although the results generated by the simulation exhibits strong correspondence with the actual flow pattern and distribution, it was identified that perhaps the most important and least understood parameter in the present simulation is associated with the roughness of terrain. Therefore, the simulation was repeated as part of a back-analysis to arrive at values for roughness that yielded close match with the reported values. Similar analyses, coupled with simple experimental flume tests of tailings could be used to calibrate a model, which could be employed to assess the risk and damage should a tailings facility fail and map out the affected area as a preparation for remedial measures.

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