Reinforcement systems subjected to complex loadings

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ABSTRACT
A new computer based technique has been recently developed in order to investigate the performance of reinforcement systems subjected to multiple and complex combinations of axial, shear and rotational displacements. The analysis technique allows for large displacements and the nonlinear behaviour of the component materials and the interactions at the interfaces between them. The computer program is described and examples of its use are presented.

ABSTRACTO
Este articulo presenta un nuevo programa computacional recientemente desarrollado. El programa investiga el comportamiento de sistemas de reforzamiento de roca sujetos a combinaciones multiples y complejas de desplazamiento axial, de corte y rotacional. La tecnica analitica permite deplazamientos significantes con comportamiento no lineal de los materials que forman el sistema de reforzamiento, a la vez permitiendo inetraccion en las interfases de los componentes. El program de computadora es descrito y ejemplos de su uso son presentados en el articulo.

1 INTRODUCTION

Reinforcement systems are used widely for stabilisation of natural slopes and man-made excavations formed during civil construction and mining projects. These reinforcement systems are used in materials ranging from soil to hard rock. They are commercially available in a wide range of lengths and force capacities. There are various designations such as soil and rock nails, rock bolts, cable bolts and ground anchors. However, many of these reinforcement systems have common factors such as being installed in boreholes and encapsulated with grout based on either cementitious or resinous materials. Other systems rely on mechanical anchors or simply use friction to transfer load.

In practice, reinforcement systems are evaluated on the basis of their response to axial loading in tension. This type of test provides an indication of the load transfer between the element (i.e. bar or multiple wire strand) and the grout, and the grout and material surrounding the borehole. Many investigators have successfully used various analytical and computational methods to simulate and explain the behaviour observed in axial tension tests.

In practice, reinforcement systems are subjected to shear as well as axial loadings. A lesser number of attempts have been made to analyse the performance of reinforcement subjected to shear and axial loading. Also, a reinforcement system, particularly for applications involving long lengths, may be subjected to loadings at multiple points. Again, this has been successfully simulated for axial loading but not when accompanied by shear loading.

A new computer based technique has been developed to simulate the response of reinforcement systems to complex loadings involving combinations of axial, shear and rotational displacements.

2 REINFORCEMENT SYSTEMS

The key to understanding reinforcement systems is to recognise that they involve a limited number of components and the interactions between them. Windsor and Thompson (1993) used the concepts embodied in Figure 1 and Table 1 to demonstrate that all reinforcement systems could be classified into one of three categories, namely:

- Continuously Mechanically Coupled (CMC)
- Continuously Frictionally Coupled (CFC)
- Discretely Mechanically or Frictionally Coupled (DMFC)

Accordingly, a generic model should embrace these concepts.

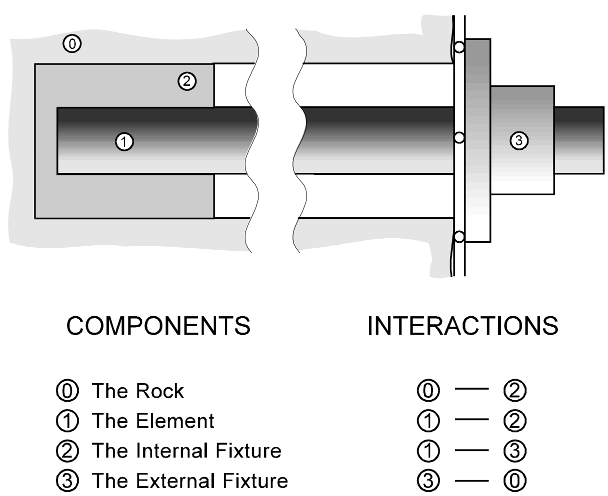


Figure 1. Generic reinforcement system.

Table 1. Load transfer mechanisms.

Type	Description
CMC Continuously Mechanically Coupled	Full column cement/resin grouted solid or hollow bars Cement grouted strand – cable bolts and ground anchors
CFC Continuously Frictionally Coupled	Friction rock stabilisers
DMFC Discretely Mechanically or Frictionally Coupled	Mechanical anchors Short resin anchored Decoupled cable bolts and ground anchors

3 OVERVIEW OF REINFORCEMENT SYSTEM MODELS

3.1 Previous Models

Previously published reinforcement system models only embrace some of the concepts given in the previous section and do not allow for all the loading conditions to which reinforcement systems may be subjected. In general, the previous models can be classified as being axial tension only or a combination of shear and axial tension. Some examples of analytical methods for prediction of axial behaviour have been presented by Farmer ((1975), Diederichs et al. (1993), Hyett et al. (1996), Li and Stillborg (1999), and Yang et al. (2002)

Examples of analytical methods for prediction of shear and tension behaviour have been published by Dulacska (1972), Bjurstrom (1974), Haas (1976), Brady and Lorig (1988) and Holmberg and Stille (1992). The shear and tension models generally assume the formation of a plastic hinge in the reinforcement element near the interface caused by a shearing displacement of the discontinuity intersecting the reinforcement system. Various assumptions are then made regarding the behaviour of the grout and material surrounding the borehole. Possibly the most sophisticated model has been presented by Ferrero (1995) who recognised and took account of the influence of large displacements and the effects of axial forces on the bending moments. Complex stress analyses have been performed (e.g. Windsor 1985 for single plane shearing) or for comparison with laboratory tests involving double shear with reinforcement at an angle or normal to the plane (e.g. Grasselli 2005 and Jalalifar and Aziz 2010, respectively).

3.2 Model Deficiencies

Figure 2, first presented by Windsor and Thompson (1993) shows simply how reinforcement systems respond at discrete discontinuities to movement of single sliding block. The mechanisms at a closed sliding discontinuity are distinctly different from those at a dilating discontinuity and may involve compressive loading of the reinforcement element, an aspect not considered in existing models.

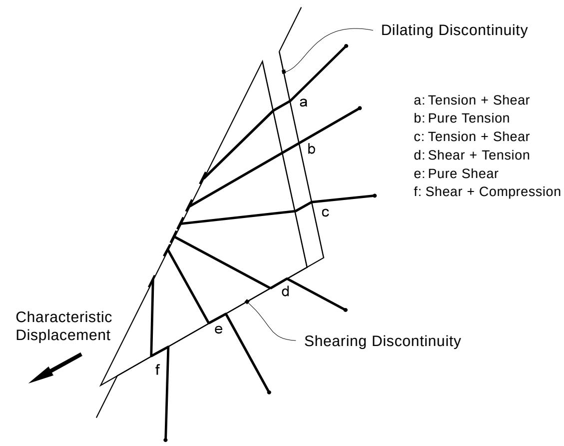


Figure 2. Various possible reinforcement system responses to block movement.

Another important aspect is to be able to consider loading of a reinforcement system in a “continuous” displacement field such as might occur in soft material or close to surface or underground excavations in massive or closely jointed rock masses. This problem has been addressed for axial behaviour by Hyett et al. (1996) and Li and Stillborg (1999) but not for shear.

4 GENERIC REINFORCEMENT SYSTEM MODEL

A generic reinforcement system model is most useful if it is capable of being able to simulate the various element types, internal fixtures and external fixtures and the response to displacements of the materials surrounding the borehole.

4.1 Model Description

The segmental model shown in Figure 3 was developed following careful consideration of the requirements needed to simulate the response of all reinforcement systems in various configurations and subjected to complex combinations of displacements.

Each segment of the reinforcement system model consists of three major components to represent:

- The reinforcement element.
- The material surrounding the borehole.
- The internal fixture used to transfer load between the reinforcement element and the borehole wall.

The soil/rock segment movements involve translations and rotations about the centre of mass. The element distortion involves translations and rotations of Node i and Node i + 1. Relative movements between the element and the material surrounding the borehole cause distortion of the Internal Fixture segment and load transfer between the Element and the Borehole.

In addition, the model allows for different load transfer mechanisms at the interfaces between the element and internal fixture and the internal fixture and the borehole wall. All the aforementioned components will be involved in a model of a reinforcement system in which the annulus between the element and the borehole is completely filled with cement or resin grout.

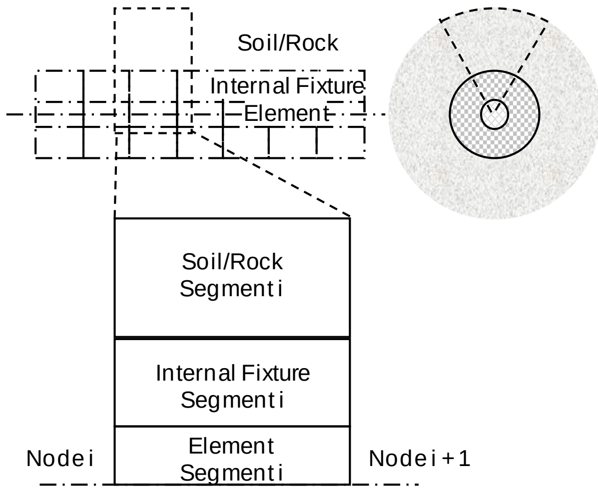


Figure 3. Generic reinforcement system model discretisation into segments.

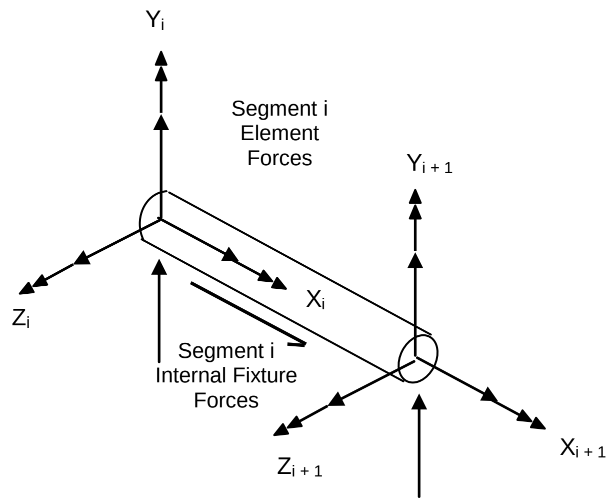


Figure 4. Forces for the reinforcement element segment.

The following options must also be available in order to simulate various configurations of reinforcement systems:

- Annulus empty (e.g. point anchored bolt).
- Annulus filled but decoupled from the element (e.g. sleeved and lubricated ground anchor).
- Zero thickness annulus (e.g. friction rock stabiliser).

The model also needs to allow for special load transfer mechanisms:

- Expansion shell anchor.
- Modified profile strand (e.g. bulbed strand).
- External fixture (e.g. barrel and wedge strand anchor).

Each node involves translations and rotations and forces and moments. In the following sections translations and forces are designated by single headed arrows while rotations and moments are designated with double-headed arrows. The forces are shown acting in the positive axis directions; no attempt has been made to assume the directions in which the forces would act to achieve equilibrium.

4.2 Element Model

The element model is shown in Figure 4 with its internal forces and the external forces imposed by the internal fixture. The element is modelled as a three-dimensional prismatic member with allowances made for the influences of changes in geometry and the influence of axial forces on bending as detailed by Meek (1991).

4.3 Internal Fixture Model

The internal fixture is modelled as inelastic, nonlinear springs that transmit radial forces between the element nodes and the borehole wall. Separate nonlinear relationships are used to model the longitudinal forces between the element and the internal fixture and the internal fixture and borehole wall.

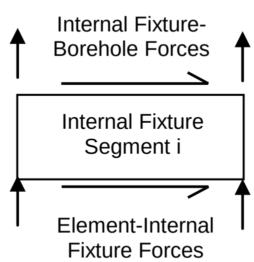


Figure 5. Forces transmitted by internal fixture.

4.4 Borehole Model

The borehole is modelled as a rigid segment that acts to resist forces transmitted from the element to the internal fixture or is loaded to cause transmission of forces through the internal fixture to the element. The forces are assumed to act through the centre of the borehole segment. No longitudinal forces are transmitted between the borehole segments to allow for displacements to be specified to simulate extensional or compressional effects in the material surrounding the borehole.

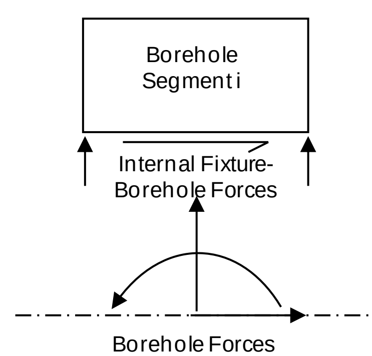


Figure 6. Borehole segment model.

4.5 Equilibrium Equations

Equilibrium equations are developed using the direct stiffness method to relate nodal displacements to nodal forces through a stiffness matrix. Further, the equations are expressed in a form suitable for solution of inelastic, nonlinear problems with large displacements that result in a change of geometry. In general, the equilibrium equations are expressed in the following partitioned matrix form:

$$\begin{bmatrix} F_a \\ F_u \end{bmatrix} = \begin{bmatrix} P_a \\ P_u \end{bmatrix} + \begin{bmatrix} K_{au} & K_{aa} \\ K_{uu} & K_{ua} \end{bmatrix} \begin{bmatrix} d_u \\ d_a \end{bmatrix} \quad [1]$$

where:

- F_a = vector of applied forces
- F_u = vector of unknown forces
- P_a = vector of existing known forces
- P_u = vector of existing unknown forces
- d_a = vector of applied displacement increments
- d_u = vector of unknown displacement increments
- K_{ij} = 'stiffness' matrix relating forces and moments to displacements and rotations at each node

Forces and displacements may be directed in any of the three coordinate directions. Applied displacement may be used to simulate either loading or restraint of the borehole or element. That is, zero applied displacement at a node implies rigid restraint.

Equation 1 may be assumed to consist of two separate matrix equations:

$$\{F_a\} = \{P_a\} + [K_{au}]\{d_u\} + [K_{aa}]\{d_a\} \quad [2]$$

$$\{F_u\} = \{P_u\} + [K_{uu}]\{d_u\} + [K_{ua}]\{d_a\} \quad [3]$$

These simultaneous matrix equations are solved in two parts. Firstly, Equation 2 is used to solve for the unknown displacement increments.

$$\{d_u\} = [K_{au}]^{-1}\left(\{F_a\} - \{P_a\} - [K_{aa}]\{d_a\}\right) \quad [4]$$

where $[K_{au}]^{-1}$ is the inverse of $[K_{au}]$

Note that it is more efficient to solve Equation 4 using a banded solver. Finally, the unknown forces at the nodes may be calculated by substituting $\{d_u\}$ into Equation 3.

This process appears to be simple and straight forward. However, nonlinearities in the material properties and changes in the geometry of the reinforcement element require that a new set of equations be developed at each iteration and loading step. The solution procedure is repeated until equilibrium of forces and compatibility of displacements are satisfied or failure is predicted for the element (rupture) or sliding occurs on one of the internal fixture interfaces with the element or borehole (pull out). In the latter case, special procedures are used to take account of partial interactions and changes in association between element and internal fixture segments.

5 SOFTWARE

Computer software has been developed in Visual Basic and uses ComponentOne Chart and OpenGl for graphical displays and visualisation, respectively. The main menu form is shown in Figure 7. The input will be described for an example of a 15.2 mm diameter steel strand, fully encapsulated with cement grout in a 60mm diameter borehole drilled in three concrete blocks and subjected to double plane shear in a laboratory.

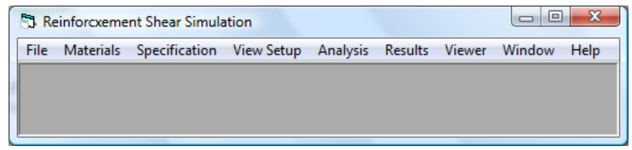


Figure 7. Software main menu.

Prior to simulating a reinforcement system response to loading, it is necessary to define a number of nonlinear relationships for the various material responses for the components. There are ten possible components for which the responses may need to be defined. These are:

1. Element axial
2. Element bending
3. Element internal fixture shear
4. Element Internal fixture bearing
5. Element– discrete anchor
6. Discrete anchor-internal fixture bearing
7. Discrete anchor-borehole shear
8. Discrete anchor-borehole shear
9. Element-external fixture
10. External fixture-plate-surface bearing

A variable number of linear segments are used to approximate measured or assumed responses. For example, the nonlinear response curve based on a test certificate for 15.2mm diameter, 7-wire, steel strand is shown in Figure 8.

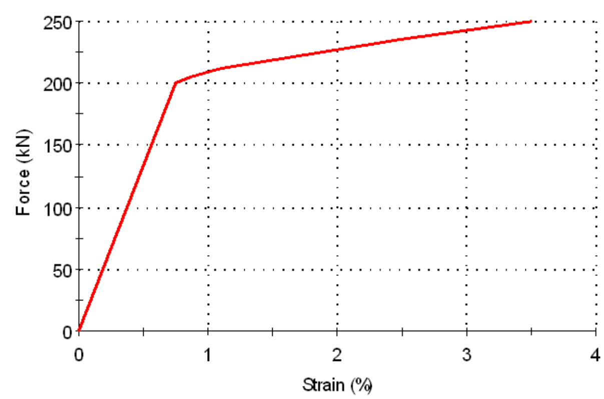


Figure 8. Nonlinear force strain response for 15.2mm diameter, 7-wire, steel strand.

Figure 9 and Figure 10 are based on data obtained from unpublished laboratory tests designed specifically to measure the required material responses for the interactions between steel strand and cement grout.. The radial force-displacement response as expected shows a stiffening response as the element approaches the borehole wall. The shear stress-displacement shows an initially very stiff response until shearing of the grout at the strand circumference. Following failure, the resistance to shearing is from friction only. Many investigators have observed this phenomenon for both concrete and rock reinforcement.

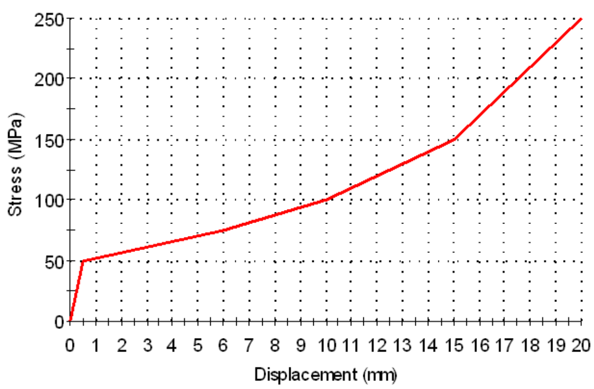


Figure 9. Nonlinear radial force-displacement response for cement grout.

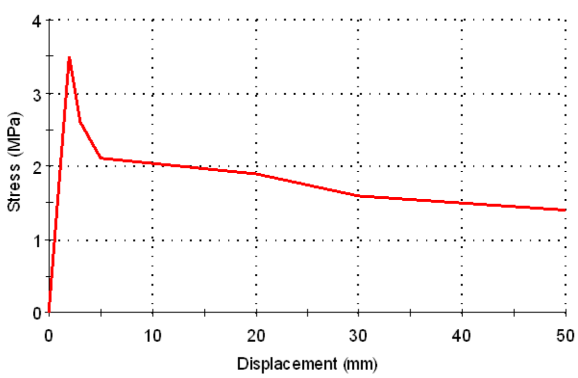


Figure 10. Nonlinear shear stress-displacement response for the strand/cement grout interface.

5.1 Reinforcement System Configuration

The reinforcement system configuration is defined using the interface shown in Figure 11. This part of the interface is used to define:

- Element type (other details obtained from database)
- Hole diameter
- Hole Length

The hole may be a borehole in soil/rock or the internal diameter of a hole in a concrete block or a steel pipe used in a laboratory test. The length of hole is divided in a user defined number of segments from which the segment length is calculated. Alternatively, the segment length may be defined and the number of segments is derived.

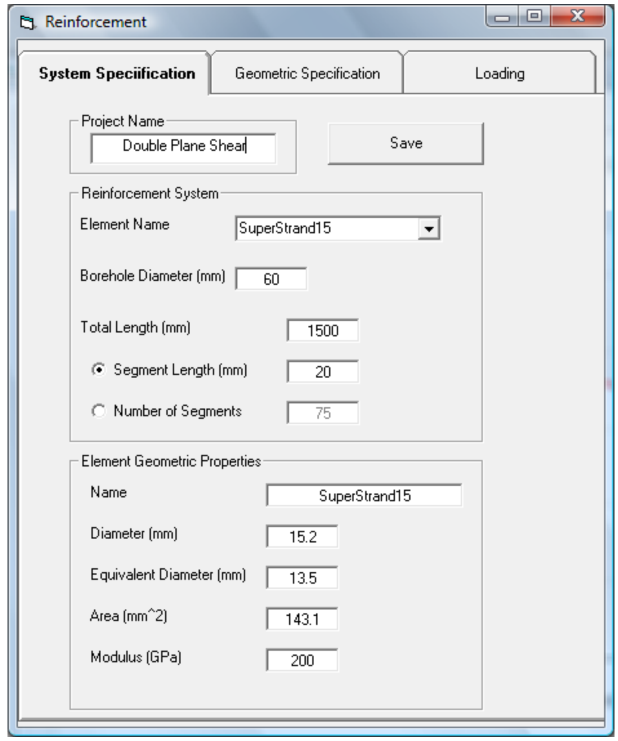


Figure 11. Reinforcement system configuration interface.

The form shown in Figure 12 is used to define the interactions between the reinforcement element, internal fixture and the hole in which they are installed. As shown, the element surface finish may be plain, bulbed or corrugated. The options for the internal fixture are none, friction, cement or resin filled, and mechanical anchor. The options for load transfer between the element and internal fixture and the internal fixture and borehole are nil, friction or fixed. Different responses are defined for the element-internal fixture and internal fixture-borehole interfaces.

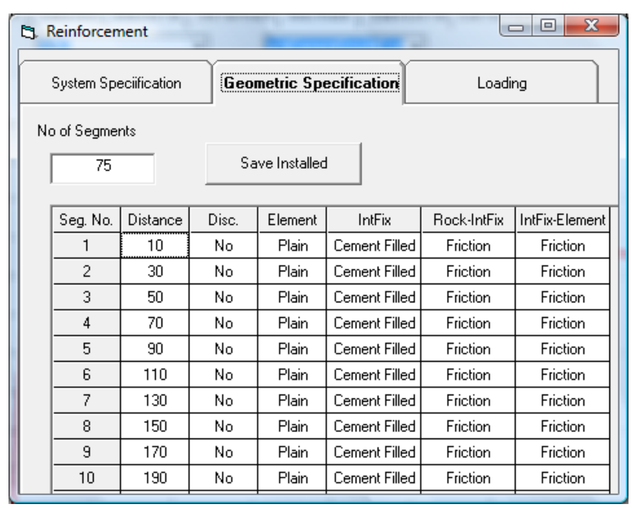


Figure 12. Interface used to specify the installed reinforcement system configuration.

5.2 Reinforcement System Loading

The reinforcement system loading is specified using the interface shown in Figure 13. Loading consists of specified numbers of segments for which components are fixed as shown in Figure 14 or displaced in a specified direction as shown in Figure 15. Alternatively, loading may be a force applied in a specific direction. In order to be able to follow responses in which a peak force is accompanied by a lower force at higher displacement, it is most usual to use the defined displacement option.

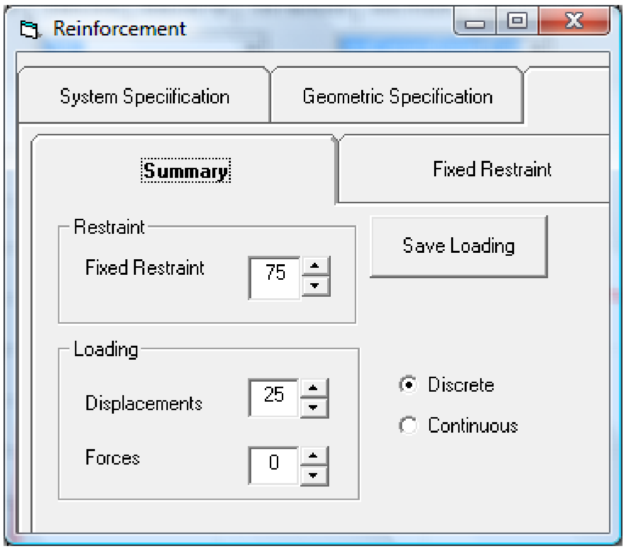
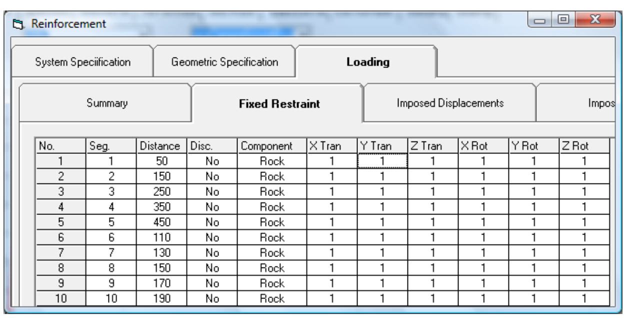


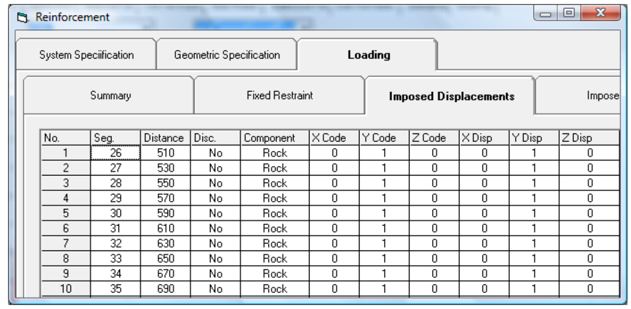
Figure 13. Reinforcement loading interface.



No.	Seg.	Distance	Disc.	Component	X Tran	Y Tran	Z Tran	X Rot	Y Rot	Z Rot
1	1	50	No	Rock	1	1	1	1	1	1
2	2	150	No	Rock	1	1	1	1	1	1
3	3	250	No	Rock	1	1	1	1	1	1
4	4	350	No	Rock	1	1	1	1	1	1
5	5	450	No	Rock	1	1	1	1	1	1
6	6	110	No	Rock	1	1	1	1	1	1
7	7	130	No	Rock	1	1	1	1	1	1
8	8	150	No	Rock	1	1	1	1	1	1
9	9	170	No	Rock	1	1	1	1	1	1
10	10	190	No	Rock	1	1	1	1	1	1

Figure 14. Interface used to define the components and their fixity against translation and rotation.

Figure 16 shows the reinforcement system configuration and the locations of any discontinuities. In this figure, the borehole to the left and right are fixed and the central section will be displaced to simulate double plane shear in a laboratory test. This figure is used to check visually that the expected configuration has been specified correctly prior to analysis.



No.	Seg.	Distance	Disc.	Component	X Code	Y Code	Z Code	X Disp	Y Disp	Z Disp
1	26	510	No	Rock	0	1	0	0	1	0
2	27	530	No	Rock	0	1	0	0	1	0
3	28	550	No	Rock	0	1	0	0	1	0
4	29	570	No	Rock	0	1	0	0	1	0
5	30	590	No	Rock	0	1	0	0	1	0
6	31	610	No	Rock	0	1	0	0	1	0
7	32	630	No	Rock	0	1	0	0	1	0
8	33	650	No	Rock	0	1	0	0	1	0
9	34	670	No	Rock	0	1	0	0	1	0
10	35	690	No	Rock	0	1	0	0	1	0

Figure 15. Interface used to define system components to be displaced.

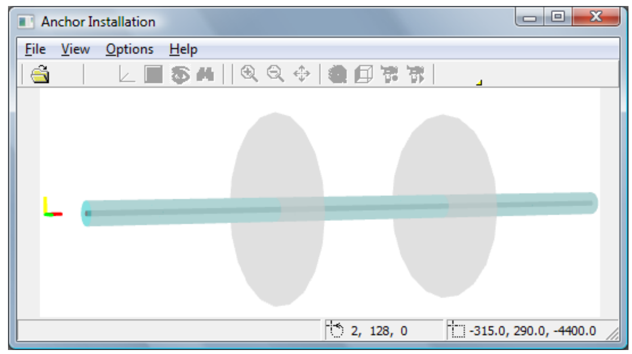


Figure 16. Visualisation of the reinforcement system configuration.

6 EXAMPLE OF ANALYSIS

The interface used for specifying and controlling calculations is shown in Figure 17.

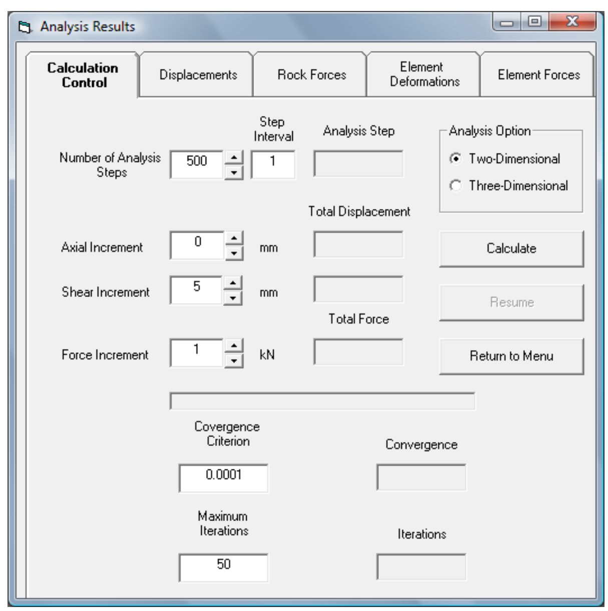


Figure 17. Interface form used to specify and control analysis and examination of detailed results.

Since the analysis method involves nonlinear material responses, the calculations may be interrupted at a specified interval and all the detailed results up to that stage of analysis examined. The discontinuity shear and axial displacements are specified as increments. The ratio of shear and axial displacements are used to simulate sliding parallel to a discontinuity or sliding accompanied by dilation or compression. In this simple example, sliding is vertical. It is known that this will induce a combination of axial tension, shear and bending. The horizontal displacements of the reinforcement element relative to the borehole are given in Figure 18. This figure indicates that significant axial displacements result from shearing and is not taken into account in some of the simple models.

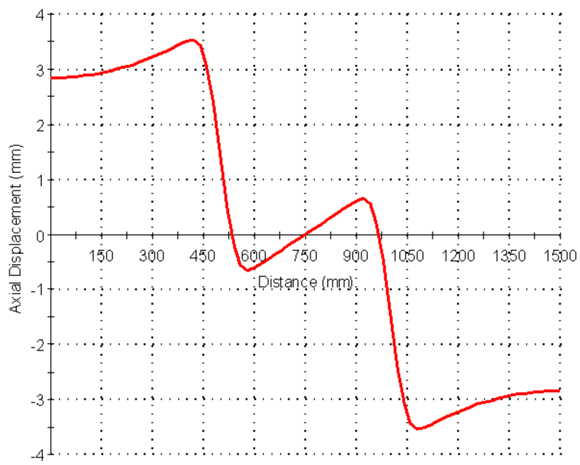


Figure 18. Distribution of axial displacement relative to the borehole.

The deformed shape of the reinforcement element is given in Figure 19. As expected, the element is deformed laterally in both he fixed and displaced lengths of embedment.

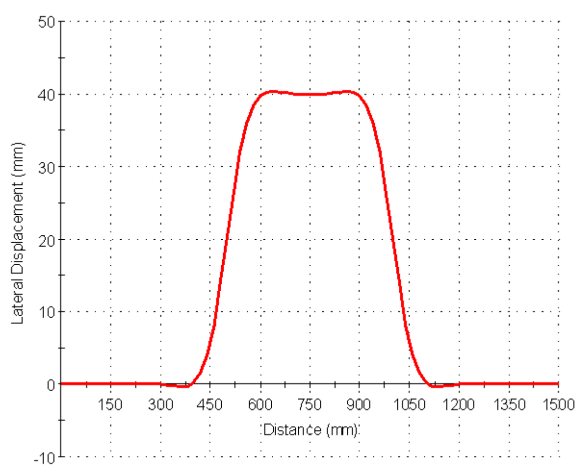


Figure 19. Deformed shape of the reinforcement element caused by axial tension, shear and bending.

The rotation of the element is given in Figure 20. The rotations are confined to within about 100mm near the planes of shearing, This is in accordance with the simple models reported previously in Section 3.1.

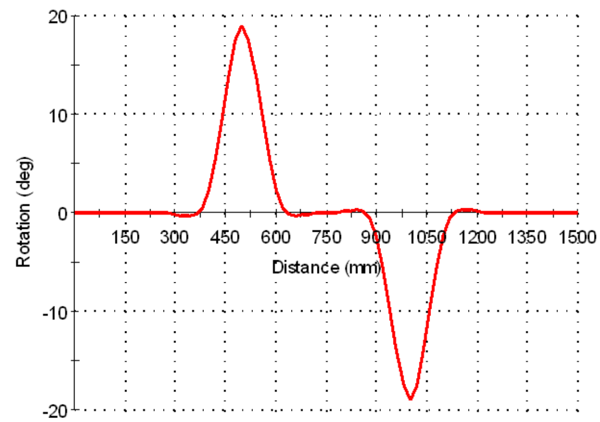


Figure 20. Element rotation distribution with length.

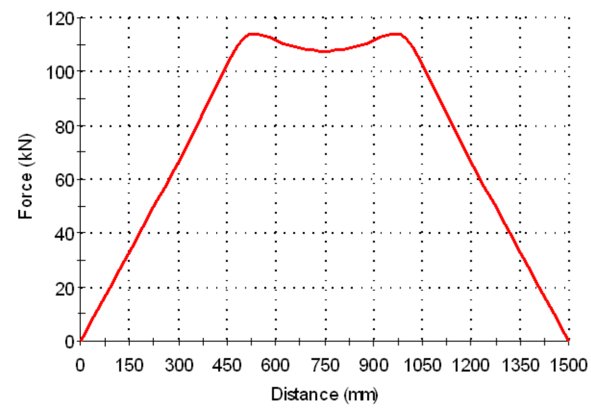


Figure 21. Element axial tensile force distribution.

Finally, the total shear force-displacement response is shown in Figure 22. The shape of this response is similar in nature to the response measured in double plane shear tests on strand embedded in cement grout.

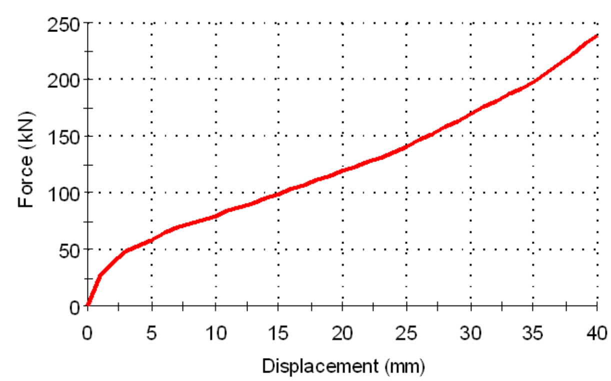


Figure 22. Total shear force-displacement response.

7 CONCLUDING REMARKS

Software based on well established engineering principles has been developed to simulate reinforcement systems subjected to complex loadings. The software is suitable for the analysis of all types of reinforcement systems in many and varied configurations. For example:

- Mechanically anchored bolts.
- Fully cement or resin grouted rock bolts.
- Friction rock stabilisers.
- Ground anchors.

In all cases, the load transfer can be decoupled for a defined length or supplemented with intermittent anchors or variations in the element cross-section (e.g. birdcaged or bulbed strand).

Loading applications include:

- single ended axial pull tests in both the laboratory and field.
- double embedment axial pull tests in the laboratory, single and double plane shear tests in the laboratory.
- dilation and shear loading at multiple discrete planes intersecting the axis of a reinforcement system.
- continuous displacement field - dilation/shear.

The generally inelastic, nonlinear behaviour of all materials and the interfaces between them can be simulated. In some cases, carefully designed laboratory tests can be used to define the responses to loading. In other cases, it is necessary to conceptualise and vary the behaviour and then compare with results from tests or instrumentation.

The simple example presented above was restricted to analysis in two dimensions. However, the software also allows for simulations in three dimensions. This capability enables simulation of the twisting that results during axial tension loading of multiple wire strand the writer observed in constant loading tests performed in the early 1980s and also observed and reported by Bawden at al. (1996) who designed an in situ pull out test arrangement to prevent rotation.

In summary, it is anticipated that the software will improve the understanding of reinforcement system responses to complex loadings produced by in situ soil and rock movements that cannot be simulated in laboratory experiments. The results from the simulations may be used to aid in design of reinforcement for given field conditions and to infer loads developed in reinforcement in response to measured ground movements.

ACKNOWLEDGEMENTS

The writer wishes to acknowledge the following organisations for their support over many years going back to the early 1980s: CSIRO, AMIRA International, Rock Technology Pty Ltd, WA School of Mines/Curtin University, CRCMining, and the many other supporting organisations including mining companies and reinforcement suppliers. The work described would not have been possible without the individual encouragement of long term colleagues and friends Chris Windsor, Glynn Cadby and Ernesto Villaescusa.

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