Improved Soil-Water Characteristic Curves and Permeability Functions for Unsaturated Soils

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ABSTRACT
Soil-water characteristic curve (SWCC) which represents the moisture-suction relation of soils is one of the important constitutive models needed to simulate the behavior of unsaturated soils. An effective SWCC model should be capable of calculating the moisture-suction variation for the entire range of degree of saturation. Applicability of popular SWCC models such as Brooks and Corey, van Genuchten, and Fredlund and Xing is limited, especially in low (<20%) degree of saturation range. In this study, all these models are modified, so that these models can be effectively used in entire range of degree of saturation. The Fredlund et al (1994) permeability function is also modified based on the modified Fredlund and Xing SWCC model. The applicability of the improved models is investigated by calibrating the SWCC of various types of soil and presented in this paper. Based on this study it can be concluded that the modified models are flexible enough to fit the experimental data for the entire range of degree of saturation.

RÉSUMÉ
sol-eau courbe caractéristique de qui représente le d'humidité et d'aspiration relation des est l'un des modèles importants nécessaires pour simuler le comportement des sols non saturés. Un modèle efficace devrait être capable de calculer la variation de l'humidité d'aspiration dans tout le gamme degré de saturation. L'applicabilité des sol-eau courbe caractéristique modèles tels que Brooks et Corey (B-C), van Genuchten (v-G), et Fredlund et Xing (F-X) est limitée, en particulier dans la fouchette de degré de saturation faible. Dans cette étude, tous ces modèles sont modifiés ou améliorés, de sorte que ces modèles peuvent être utilisés tout le gamme degré de saturation. Le Fredlund et al fonction de perméabilité est également modifié sur le amélioré modèle. L'applicabilité des modèles améliorés d'une enquête approfondie est en calibrant les courbe caractéristique sol-eau de divers types de sol et présentés dans le présent le papier. Basé sur cette étude, on peut conclure que les modèles modifiés sont suffisamment souples pour s'adapter aux données expérimentales pour dans tout le gamme du degré de saturation.

1 INTRODUCTION

The Soil Water Characteristic Curves (SWCC) is a relationship between the amount of water present in the soil (moisture) and the suction characteristics of the soil matrix. The amount of water present in the soil can be expressed in terms of degree of saturation (S), volumetric water content (Θ), or gravimetric water content (u). Many researchers have identified the factors which influence the shape of the SWCC and based on that, many mathematical SWCC models were developed. Gardner (1956), Brooks and Corey (1964), van Genuchten (1980), Kosugi (1994), and Fredlund and Xing (1994) are some of the models found in the literature. All these models confirm an inverse proportional relationship between S and suction (ψ). This can be explained with the fundamental meniscus theory as follows. When the S increases, the radius (R_s) of the meniscus will increase. When R_s increases, the pressure difference between the pore air pressure and the pore water pressure (suction) will decrease (see Eqn. 1).

$$\psi = p^g - p^l = \frac{2T_s}{R_s} \quad [1]$$

where ψ is the suction, p^g is pore gas pressure, p^l is pore liquid pressure, and T_s is surface tension.
The air-entry suction that is also known as bubbling pressure and pore size distribution are two basic parameters incorporated in most of the SWCC models. In models such as Brooks and Corey (B-C), van Genuchten (v-G), and Fredlund and Xing (F-X), these two parameters are represented by ***a*** and ***n***, respectively.
The Brooks and Corey model (Eqn. 2) is one of the basic SWCC models developed with two parameters. This model does not provide a continuous mathematical function for the entire range of S.

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \begin{cases} 1 & if \quad \psi < a \\ \left(\psi / a\right)^{-n} & if \quad \psi > a \end{cases} \quad [2]$$

where ***a*** and ***n*** are the fitting parameters. The parameter ***a*** is related to the air-entry suction of the soil and the ***n*** is related to the pore size distribution of the soil. ψ is suction, θ is volumetric water content, θ_r is residual water content, and θ_s is saturated water content.
The *v-G* model (Eqn. 3) provides a single equation for the entire range of S. This model has an additional fitting parameter ***m***, thereby making this model more flexible compared to the B-C model.

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + (a\psi)^n\right]^m} \quad [3]$$

where the fitting parameter ***m*** is related to residual water content. All the other parameters are same as in the *B-C* model.
The *F-X* model is presented in Eqn. 4. The ability of this model to predict the SWCC for entire range of S is

considered as the major advantage of this model. The *F-X* model assumes a maximum suction of 1,000,000 kPa at dry condition, while the *B-C* and the *v-G* models assume infinite value of maximum suction. The *F-X* model is rather similar to the v-G model other than the correction factor $C(\psi)$ and “ln-term” in the equation. The “ln-term” is very effective in keeping the SWCC without reaching zero normalized water content in low suction range, especially for sandy soils. Fredlund and Xing (1994) have also suggested another form of the model (Eqn. 5) which can be used if a residual water content is known.

$$\frac{\theta}{\theta_s} = \frac{C(\psi)}{\left[\ln\left(e + (\psi/a)^n\right)\right]^m} \qquad [4]$$

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[\ln\left(e + (\psi/a)^n\right)\right]^m} \qquad [5]$$

$$C(\psi) = 1 - \frac{\ln\left(1 + \psi/C_r\right)}{\ln\left(1 + 10^6/C_r\right)}$$

where C_r is a parameter related to residual water content and other parameters are same as in the v-G model.

The properties which affect the soil water characteristic curve also affect the permeability coefficients of pore fluids in unsaturated soil. Therefore, SWCCs can be effectively used to calculate permeability-suction relation, which is commonly referred as permeability function. Based on F-X SWCC model, a permeability function (Eqn. 6) is proposed by Fredlund et al. (1994).

$$K_r(\psi) = \frac{\displaystyle\int_{\ln\psi}^{b} \frac{\theta(e^y) - \theta(\psi)}{e^y}\,\theta'(e^y)\,dy}{\displaystyle\int_{\ln\psi_{aev}}^{b} \frac{\theta(e^y) - \theta_s}{e^y}\,\theta'(e^y)\,dy} \qquad [6]$$

where ψ is suction, $K_r(\psi)$ is the relative permeability at suction ψ, ψ_{aev} is the air-entry suction, y is a dummy variable of integration, b = ln(l,000,000), θ is volumetric water content given in Eqn. 4 and θ' is its derivative. ***a**, n, m* and C_r are fitting parameters of the F-X model (Eqn. 4).

The B-C, v-G, and F-X models are being widely used to calculate the moisture-suction relation of unsaturated soils. For the B-C and v-G models, a residual water content value has to be specified. However these two models calculate unrealistic suction when the normalized water content is zero or less, i.e. water content of the soil is less than or equal to the residual water content. In the F-X model, the maximum suction is assumed to be 1,000,000 kPa. Although there are thermodynamic concepts to back up this maximum suction, it is a concern to use a fixed value for all types of soils. In addition, when the actual maximum suction is low, usage of such larger maximum suction value might over predict shear strength in numerical simulations. Similar to the B-C and v-G models, the second form of the F-X model (Eqn. 5) also calculates an unrealistic suction when the normalized water content is zero or less. Therefore, to avoid an unrealistic suction value at zero normalized water content, the maximum suction value should be specified even with a residual water content specified. In addition, the fourth model parameter C_r in the F-X model is chosen from a wide range (1 to 1,000,000 kPa) and it creates difficulties in achieving a unique set of calibrated model parameters. Also, the C_r affects the initial portion of the curve when the value of C_r is relatively low and it is considered as another disadvantage (Leong and Rahardjo, 1997). The primary objective of this study is to increase the flexibility of the B-C and v-G models so that these models can predict realistic high suctions in low degree of saturations without causing numerical instabilities in finite element simulations.

It is very challenging to model the soil behavior from a fully dry condition to a fully saturated condition using a single fully coupled finite element computer code. The current state of the art suggests that there are three major difficulties in developing numerically stable simulation capability. They are: difficulties in dealing with multiple nodal/element variables in finite element formulation of porous media at these extreme conditions, difficulties in developing stress-strain behavior with appropriate stress state variables at these extreme conditions, and difficulties in accurately calculating the suction over the entire range of degree of saturation. The modified models can be incorporated in finite element simulation without introducing numerical instabilities arise from SWCC.

In this study, the B-C and v-G models are modified by incorporating correction factors. Also, the correction factor in the F-X model is modified to avoid the effects of additional fitting parameter C_r. Incorporating the maximum suction as part of the model increased its flexibility in fitting measured data of various soils over the full range of S. All three models are improved with the feature to specify both residual water content and maximum suction values. The capability of the improved models is verified by matching with the experimental data and prediction of original models. Based on the improved F-X model, the permeability function proposed by Fredlund et al. (1994) is modified and presented.

2 IMPROVED SWCC MODELS AND COMPARISONS

Although there are numerous SWCC models available in the literature, this study is intended to improve the popular B-C, v-G, and F-X models. The B-C and v-G models are modified primarily to make sure that these models no longer calculate high suction when the normalized water content is zero or less. And also the modified models have the feature to specify both residual water content and maximum suction values.

2.1 The Improved Brooks and Corey (I-B-C) Model

The improved Brooks and Corey (*I-B-C*) model is given in Equation 7. To preserve the advantage of the B-C model, no additional fitting parameter is introduced. Even though the maximum suction ψ_{max} is incorporated in the equation, it cannot be considered to be a fitting parameter, as the shape of the SWCC cannot be changed by adjusting the ψ_{max}. The *I-B-C* model does not provide a continuous mathematical function for the entire range of S.

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \begin{cases} 1 & \text{if} \quad \psi < a \\ \dfrac{C(\psi)}{(\psi / a)^n} & \text{if} \quad \psi > a \end{cases} \qquad [7]$$

$$C(\psi) = 1 - \sqrt{\psi / \psi_{max}}$$

where ψ_{max} is maximum suction and other parameters are same as in the *B-C* model.

2.1.1 Comparison of the B-C and the I-B-C Models

Capability of the improved B-C (*I-B-C*) model in predicting the moisture-suction relation is investigated and compared with the B-C model for four different soils. The comparison of B-C and I-B-C Models for Columbia sandy loam (data - Brooks & Corey 1964) is shown in Figure 1. The Figures 2 and 3 show the comparison for Madrid clay sand and Arlington soil, respectively. The Figures 4 shows the comparison for Indian head till (data - Vanapalli et al. 1999).

It should be noted that the experimental SWCC data are not available for the full range of S (0-100%). Based on the experimental data, the maximum suction of 1,000,000 kPa is chosen for all four soils. The residual water content is assumed to be zero for all four soils. As shown in these figures, the *I-B-C* model is capable of calculating the moisture-suction relation for full range of S, whereas the *B-C* model is not effective. The B-C, I-B-C models are not effective for sandy soils and it is evidently shown in Figure 1 as these models failed to keep the SWCC without reaching zero normalized water content in low suction range.

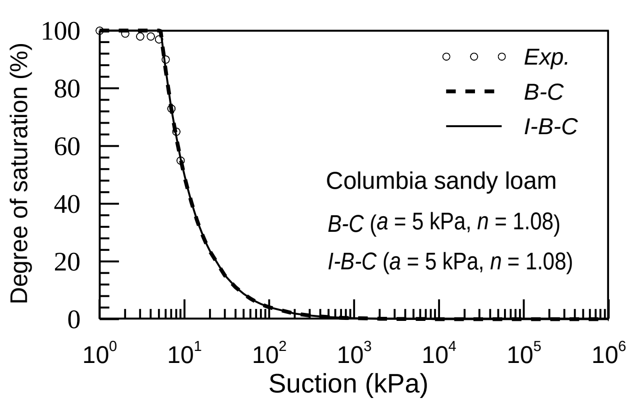

Figure 1. *B-C* and *I-B-C* SWCCs for Columbia sandy loam

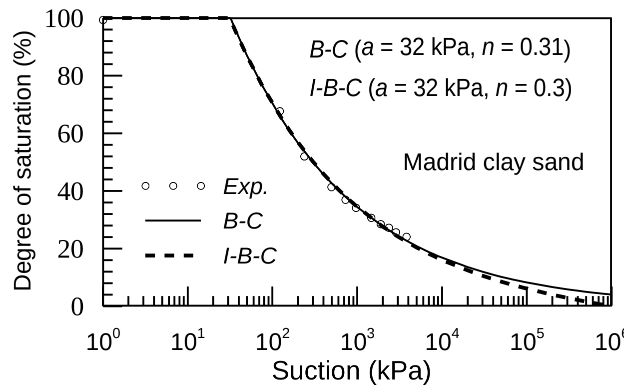

Figure 2. *B-C* and *I-B-C* SWCCs for Madrid clay sand

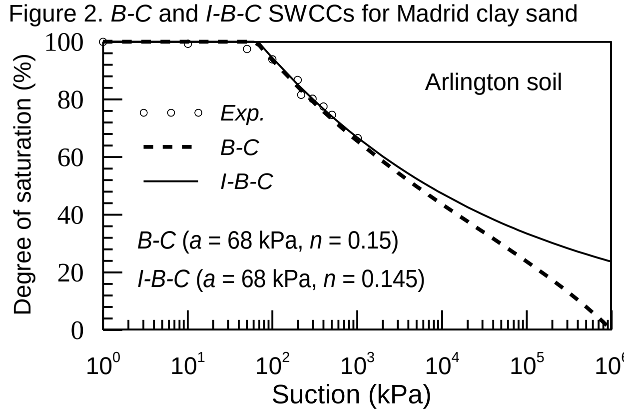

Figure 3. *B-C* and *I-B-C* SWCCs for Arlington soil

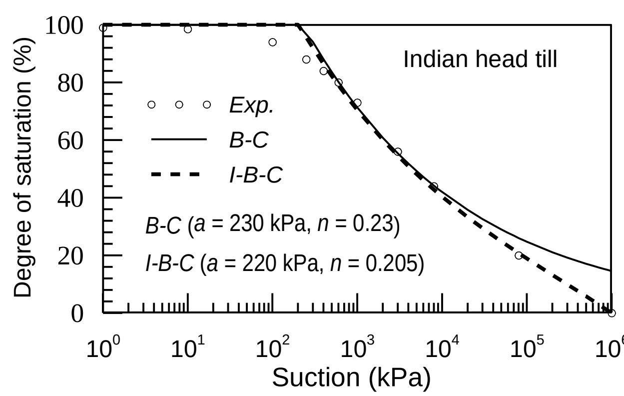

Figure 4. *B-C* and *I-B-C* SWCCs for Indian head till

2.2 The Improved van Genuchten (I-v-G) Model

The improved van Genuchten (*I-v-G*) model is given in Equation 8. Since the parameter ***a*** is related to the air-entry suction, the model is revised so that the parameter ***a*** has the unit of suction. The *I-v-G* model is developed with the feature to specify both residual water content and maximum suction value with no additional fitting parameter.

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{C(\psi)}{\left(1 + \left(\psi/a\right)^n\right)^m} \qquad [8]$$

$$C(\psi) = 1 - \left(\frac{m+1}{m + \psi_{max}/\psi}\right)^{0.5}$$

where ψ_{max} is maximum suction and other parameters are same as in the *v-G* model.

2.2.1 Predictive Capability of the I-v-G Model

Capability of the improved v-G (*I- v-G*) model in predicting the moisture-suction relation is presented for Columbia sandy loam, Madrid clay sand, Arlington soil, and Indian head till in figures 5 through 8, respectively. Similar to the *I-B-C* model, maximum suction of 1,000,000 kPa and residual water content of zero are used for all four soils. As shown in figures 5 through 8, the *I-v-G* model is capable of calculating the moisture-suction relation for full range of S, whereas the *v-G* model is not effective. As shown in Figures 5, the v-G, I-v-G models are also not suitable for sandy soils as these models also failed to keep the SWCC without reaching zero normalized water content in low suction range.

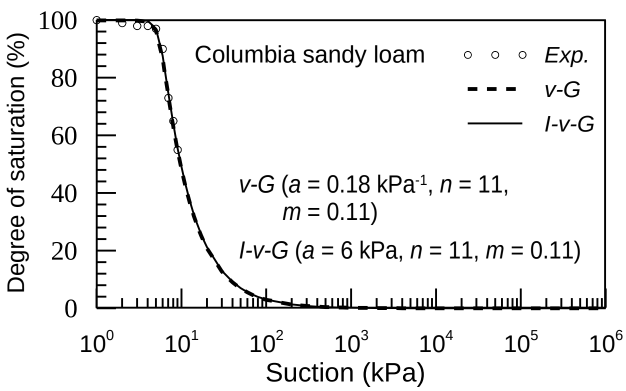

Figure 5. *v-G* and *I-v-G* SWCCs for Columbia sandy loam

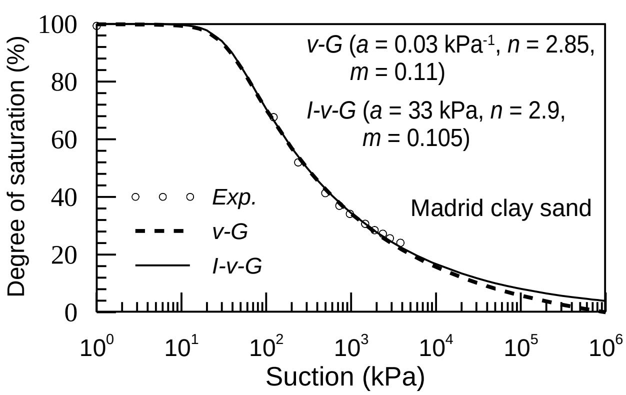

Figure 6. *v-G* and *I-v-G* SWCCs for Madrid clay sand

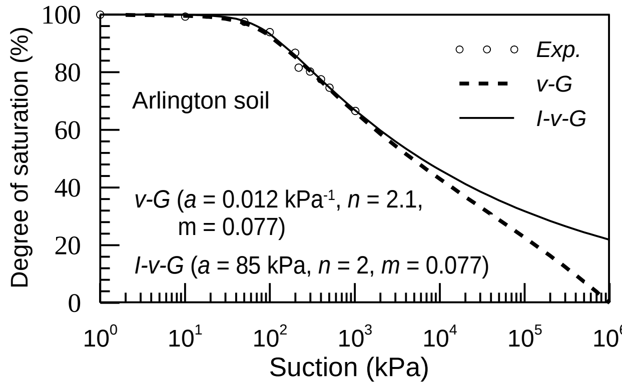

Figure 7. *v-G* and *I-v-G* SWCCs for Arlington soil

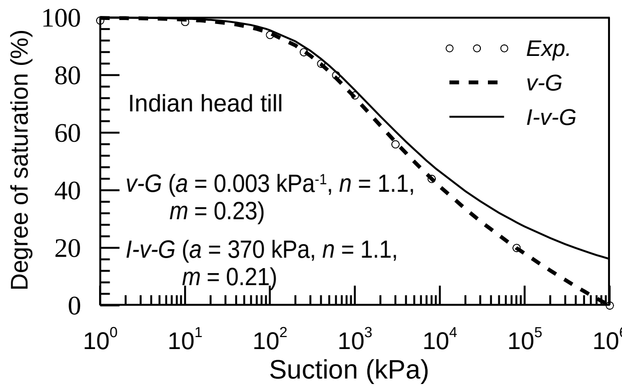

Figure 8. *v-G* and *I-v-G* SWCCs for Indian head till

2.3 The Improved Fredlund and Xing (I-F-X) Model

The improved Fredlund and Xing (*I-F-X*) model is given in Equation 9. The *I-F-X* model is developed with the feature to specify both residual water content and maximum suction value without the parameter C_r, i.e. with only three fitting parameters. Therefore, the effect of C_r in the initial portion of the *F-X* model (Leong and Rahardjo, 1997) is avoided in the *I-F-X* model.

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{C(\psi)}{\left(\ln\left(e + \left(\psi/a\right)^n\right)\right)^m} \qquad [9]$$

$$C(\psi) = 1 - \left(\frac{m+1}{m + \psi_{max}/\psi}\right)^{0.5}$$

where all the parameters are same as in the *I-v-G* model.

2.3.1 Predictive Capability of the I-F-X Model

The predictive capability of the *I-F-X* model in predicting the moisture-suction relation is presented in figures 9 through 12. Similar to the *I-B-C, I-v-G* models, 1,000,000 kPa maximum suction and zero residual water content are used. It can be noted that the *I-F-X* model is also effective in full range of S. However the *I-F-X* model can be considered better as it has only three fitting parameters, whereas the *F-X* model has four.

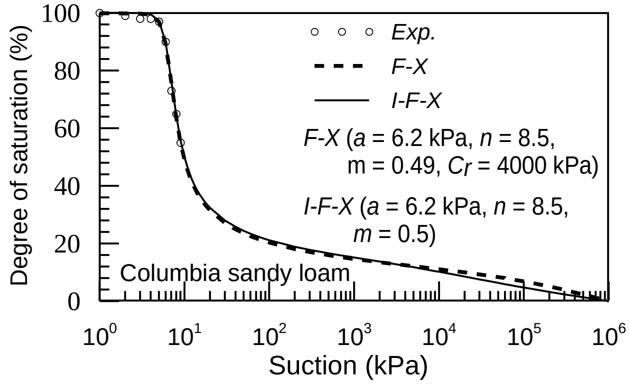

Figure 9. *F-X* and *I-F-X* SWCCs for Columbia sandy loam

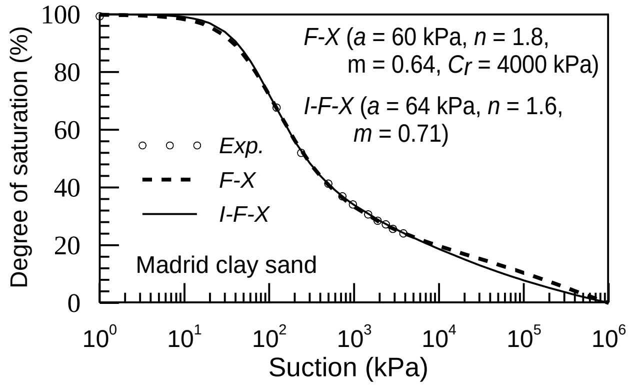

Figure 10. *F-X* and *I-F-X* SWCCs for Madrid clay sand

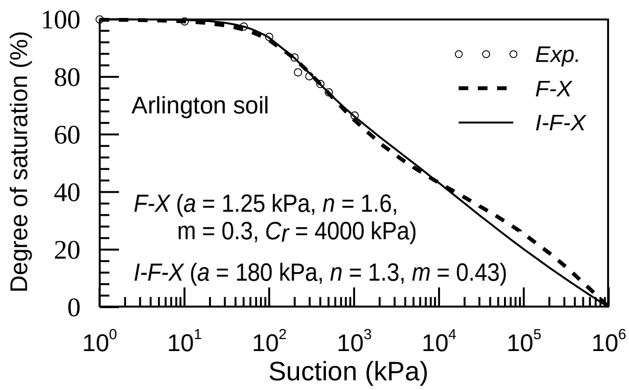

Figure 11. *F-X* and *I-F-X* SWCCs for Arlington soil

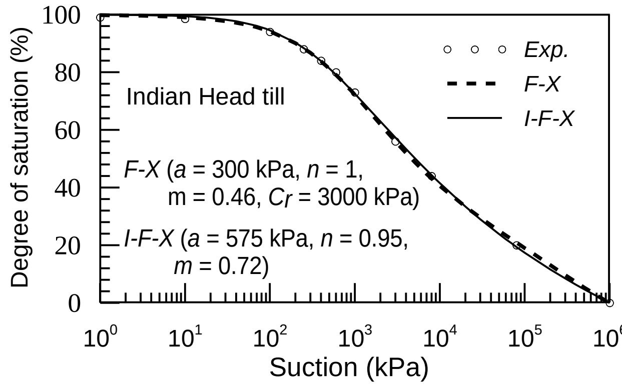

Figure 12. *F-X* and *I-F-X* SWCCs for Indian head till

3 MODIFIED PERMEABILITY FUNCTION AND COMPARISONS

Based on F-X SWCC model, a permeability function is proposed by Fredlund et al. (1994) and it is being widely used. Therefore, it is important to modify the Fredlund et al permeability function (F-All model) based on the I-F-X SWCC model. The F-All model is modified based on the I-F-X SWCC model, and presented as I-F-All model in Equation 10. The only difference between the F-All and I-F-All models, is the correction factor $C(\psi)$.

$$K_r(\psi) = \frac{\displaystyle\int_{\ln(\psi)}^{b} \frac{\theta(e^y) - \theta(\psi)}{e^y}\,\theta'(e^y)\,dy}{\displaystyle\int_{\ln(\psi_{aev})}^{b} \frac{\theta(e^y) - \theta_s}{e^y}\,\theta'(e^y)\,dy} \qquad [10]$$

The functions θ and $C(\psi)$ are given by

$$\theta = C(\psi)\frac{\theta_s}{\{\ln[e + (\psi/a)^n]\}^m} \text{ and}$$

$$C(\psi) = 1 - \left(\frac{m+1}{m + \psi_{max}/\psi}\right)^{0.5}$$

where ψ_{max} is maximum suction and other parameters are same as in the F-All model.

3.1 Predictive Capability of the I-F-All Model

The permeability coefficients of water in four different soils are predicted with F-All and I-F-All models and presented in Figures 13 through 16. Figure 13 illustrates the predictions for Superstition sand and the comparison with experimental data (from Richards, 1952). As shown in Figure 13, the F-All and I-F-All models show better match with the experimental data. However, because of the lack of experimental data, the accuracy of these two models in the higher suction range could not be verified.

The Figure 14 shows the comparison of predicted results and experimental data for Columbia sandy loam (experimental data from Brooks & Corey 1964). Similar to the Superstition sand, the predictions of F-All and I-F-All models match well with the experimental data in the lower suction range. As shown in Figure 15, similar predictions are obtained for the Touchet silt loam (experimental data from Brooks & Corey, 1964). Figure 16 shows the prediction and comparison for Yolo light clay (data from Moore 1939). As shown there, the difference between the experimental data and the predictions of F-All and I-F-All models increases as the suction increases. In addition, the prediction of F-All model slightly deviates from the prediction of I-F-All model at higher suction range.

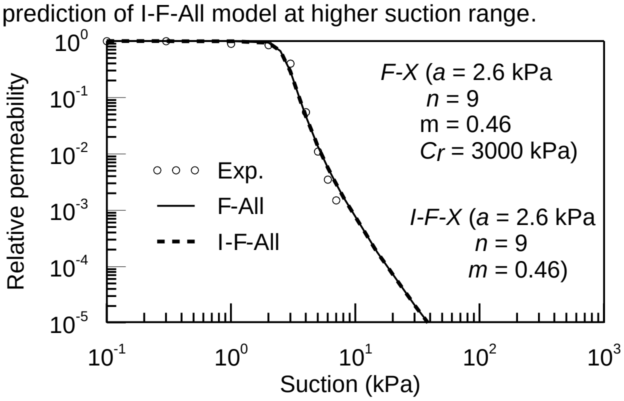

Figure 13. *F-All* and *I-F-All* models for Superstition sand

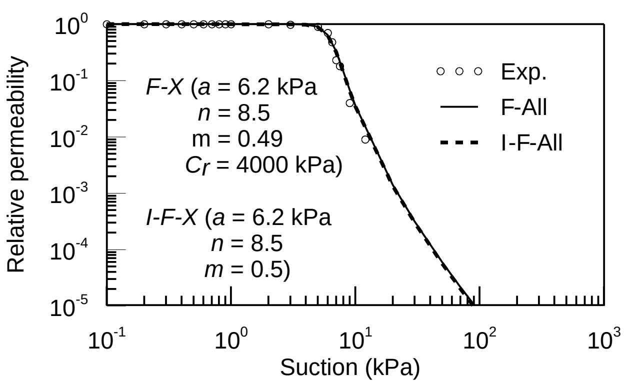

Figure 14. *F-All* and *I-F-All* models for Columbia sandy loam

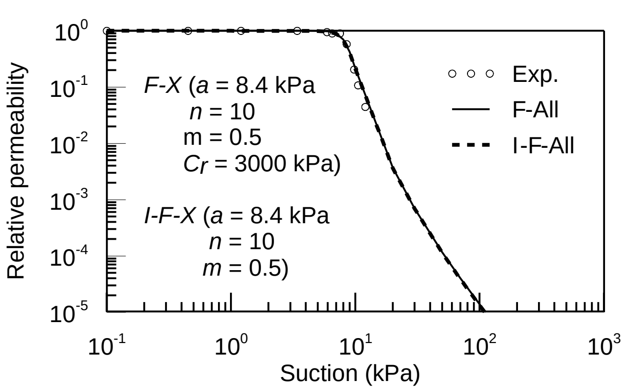

Figure 15. *F-All* and *I-F-All* models for Touchet silt loam

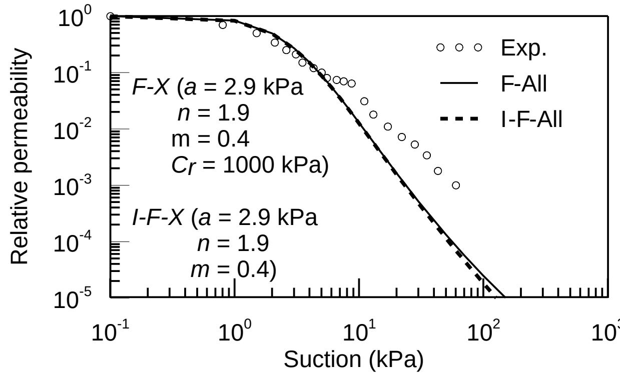

Figure 16. *F-All* and *I-F-All* models for Yolo light clay

4 CONCLUSION

The Brooks and Corey, van Genuchten, and Fredlund & Xing models are modified to capture the high suctions at low degree of saturations. Both maximum suction and residual water content can be used as input in these modified models. Since there is no data available to verify its capability in high suction range, the flexibility of these modified models has been verified by fitting experimental data for four different soils in high suction range and the predictions from original models.

REFERENCES

Brooks R.H., Corey A.T. 1964. Hydraulic properties of porous media. *Hydrology Paper*, Colorado State University. Fort Collins, 27(3).

Fredlund D.G., Xing A. 1994. Equations for the soil-water characteristic curve. *Canadian Geotechnical Journa,* 31: 533-546.

Fredlund D.G., Xing A., Huang S. 1994. Predicting the permeability function for unsaturated soils using the soil-water characteristic curve. *Canadian Geotechnical Journal,* 31: 533-546.

Kosugi, K. 1994. The parameter lognormal distribution model for soil water retention. *Water Resource Research*, 30: 891-901.

Leong, E. C., and Rahardjo, H. 1997. Review of soil-water characteristic curve equations. *Journal of Geotechnical and Geoenvironmental Engineering*, 123:1106-1117.

Moore R.E. 1939. Water conduction from shallow water tables. Hilgardia, 12: 383-426.

Richards L.A. 1952. Water conducting and retaining properties of soils in relation to irrigation. Proceedings, *International Symposium on Desert Research*, Jerusalem, 523-546.

van Genuchten M.Th. 1980. A closed form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, 44: 892-898.

Vanapalli, S. K., Pufahl, D. E., and Fredlund, D. G. 1999. The influence of soil structure and stress history on the soil-water characteristic of a compacted till. *Geotechnique*, 49(2): 143-159.