Evaluation of strain level and frequency effects on the dynamic properties of sand

Zahid Khan
Department of Civil Engineering – American University of Sharjah, UAE
Soheil Moayerian, Giovanni Cascante
Department of Civil Engineering – University of Waterloo, ON, Canada
Murray Grabinsky
Department of Civil Engineering – University of Toronto, Toronto, Canada

ABSTRACT
Dynamic properties of soils are critical for the design of structures subjected to vibrations. These properties can be measured from the transfer function between the excitation and the response using the resonant column device. The force function generated by a sinusoidal sweep excitation induces different shear strain levels at different frequencies. Consequently, the shape of the measured transfer function is distorted and differs from the theoretical transfer function for an equivalent single-degree-of-freedom system. The difference between the measured and assumed transfer functions becomes more pronounced with the increase in shear strain levels. This study presents a new methodology for the evaluation of dynamic properties from an improved transfer function. In this methodology, the soil specimen is excited using a fixed sine at the required strain level along with a simultaneous excitation of small amplitude random noise. The strain level induced by the fixed sine controls the resonant frequency and damping coefficient of the specimen whereas the random noise controls the shape of transfer function. The new methodology also shows a good potential for the evaluation of dynamic properties of soils as function of frequency in resonant column testing.

RÉSUMÉ
De nombreux problèmes géotechniques tels que la conception de systèmes résistants aux séismes et les vibrations de machines requièrent l'installation d'amortisseurs ou d'isolateurs pour contrôler l'amplitude des vibrations. Des remblais conçus avec une plus grande capacité à dissiper l'énergie peuvent fournir une approche plus économique pour contrôler des vibrations excessives. Cette étude présente une technique pour augmenter le taux d'amortissement d'un sable sans affecter sa rigidité de façon significative. L'augmentation du taux d'amortissement est évaluée en effectuant des essais à colonne résonnante sur le sol synthétique. Le taux d'amortissement du sable est augmenté en ajoutant une quantité contrôlée de matériau viscoélastique dans le sable. Les essais à colonne résonnante indiquent que le taux d'amortissement du sable peut être augmenté d'un factor 10 sans effet significatif sur le module de cisaillement. L'évaluation micromécanique des résultats montre une bonne corrélation entre la surface des particules en contact avec le mélange présent dans les pores et le taux d'amortissement du sable.

1 INTRODUCTION

The low and medium strain shear modulus and damping ratio of soils is commonly measured in resonant column (RC) device (ASTM D4015-92, 2000). Conventional RC testing is based on the determination of the resonant frequency of a soil specimen by exciting the specimen at different frequencies (frequency sweep). The shear wave velocity is determined by solution of the equation of motion for a column-mass system. The resonant frequency and the material damping ratio can be computed by curve fitting the measured transfer function (ratio of the applied torque and induced angle of twist) with the theoretical equations (e.g. Cascante et al. 2003). During a frequency sweep, the imposed shear strain levels are different at different frequencies of excitation. The effect of unequal strains in a typical frequency sweep RC tests have been reported in literature (e.g. Cascante et al. 1997; Khan et al. 2008). These studies indicate that the error in the measured damping ratio becomes larger at large strains because of the non-symmetrical shape of the measured transfer function. The non-symmetrical shape is due to unequal shear strains at different frequencies. Although the measured transfer function can be fitted within acceptable error with the theoretical transfer functions at low strain levels, the error increases with increase in shear strain levels (Khan et al. 2008). Moreover, the effect of frequency on the dynamic properties is difficult to evaluate in conventional RC tests.

Testing methodologies based on non-resonance (NR) approach uses single frequency excitation instead of frequency sweep to measure the dynamic properties (Khan et al. 2008; Lai et al. 2001; Rix and Meng 2005). These methodologies are based on the solution of the equation of motion governing the forced vibration of a continuous, homogeneous, and linear viscoelastic cylinder representing a soil specimen. These methods allow for the simultaneous determination of the shear wave velocity and material damping ratio at the same frequency of excitation. Since the dynamic properties can be determined at different frequencies, the non-resonance (NR) methods are considered suitable to investigate the frequency dependence of dynamic properties. The experimentally determined quantity of the complex shear modulus allows the simultaneous measurement of the shear wave velocity and damping ratio as a function of frequency. Complex shear modulus, however, forces one of the dynamic properties to grow linearly with frequency (Khan et al. 2008). NR results on two cohesive soils presented by Meza and Lai (2006) showed that the data is well described by the Kramers-Kronig relationships (Booij and Thoone, 1982); however, the large values of damping ratio as a function of frequency probably masked the limitation of NR method.

The main objective of this study is to present a simple testing technique that will allow the improved determination of the shape of transfer function and frequency dependent dynamic properties. The new technique (FN) is based on exciting the specimen with a main sinusoid as a carrier frequency followed by addition of relatively small amplitude random noise. The main frequency controls the imposed shear strain level and hence the resonant frequency, whereas the relatively small amplitude random noise excitation controls the symmetrical shape of the transfer function around resonance. The proposed methodology (FN method) has several advantages. First it allows the better estimate of shear strain in the soil column. Secondly, the errors in curve fitting are significantly reduced especially at large strain excitations due to symmetrical shape of the transfer function and thirdly, the dynamic properties can be evaluated as function frequency using conventional solutions to the equation of motion instead of using elastic visco-elastic correspondence principle.

Resonant column tests are performed on a sand specimen at different frequencies, and shear strain levels using the conventional (RC), non-resonance (NR), and the proposed (FN) methods. As expected, the dynamic properties of sand computed from FN method appear invariant with frequency within the studied bandwidth.

2 BACKGROUND

In a fixed-free resonant column (RC) configuration, the base of the specimen is assumed fixed and torsional loads are applied at the top of specimen. In the standard interpretation of resonant-column test, the shear modulus and damping ratio of the specimen are computed by solving the equation of motion of a single degree of freedom system (Richart et al. 1970). The torsional excitation should be perfectly perpendicular to the top surface to avoid the development of flexural modes in addition to torsional modes (Cascante et al. 1998).

The analytical solution for a resonant-column test is obtained using a visco-elastic Kelvin-Voigt model for a material characterized by weak energy dissipation (i.e. low material damping ratio).

$$\frac{d^2\theta}{dt^2} = \frac{G}{\rho}\frac{d^2\theta}{dz^2} + \frac{c}{\rho}\frac{d^3\theta}{dz^2 dt} \qquad [1]$$

The solution of Eq. 1 is obtained using the separation of variables method and appropriate boundary conditions which are i) zero rotation at the fixed end of the soil column and ii) at the top of the specimen $z = H$, the torque must be equal to the applied torsional excitation (Lai et al., 2001):

$$\frac{T_o}{\varphi(H)} = J_P \ \frac{\rho\,\omega^2 H}{\beta\tan(\beta)} - I_o\,\omega^2 \qquad [2]$$

where ω is the angular frequency, I_o is the mass polar moment of inertia of the driving plate, ρ is the mass density, Jp is the area polar moment of inertia, T_o is the applied torque, H is the height of the specimen, $\varphi(H)$ is the rotation angle at $z = H$, and β is given by (Hardin 1965)

$$\beta = \frac{\omega H}{V_S} \qquad [3]$$

At the resonant frequency (i.e. ω_o), the amplitude of twist (or rotation angle) approaches infinity for a zero damping material; thus, Eq. 2 becomes:

$$\frac{I}{I_o} = \beta\tan\beta \qquad [4]$$

where I is the mass polar moment of inertia of the specimen ($I = J_P\,\rho\,H$). Equation 4 is used in conventional RC testing to compute the shear wave velocity (V_S) of the material. The standard analysis of resonant column results is based on the approximation of a SDOF model when $I_o >> I$ (e.g., Richart et al. 1970).

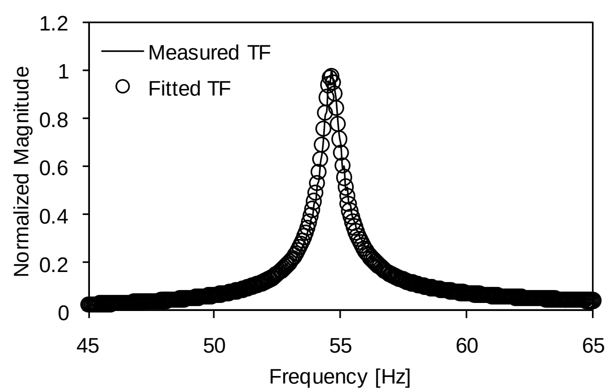

Figure 1. Transfer function from RC measurements fitted with theoretical TF ($\gamma = 1.2 \times 10^{-5}$).

The material damping ratio and resonant frequency is typically obtained by measuring the transfer function between induced current and resulting rotation of the top of the specimen during a frequency sweep. Figure 1 presents the typical measured transfer function during RC testing. The measured transfer function is curve fitted with a theoretical acceleration transfer function (TF(ω)) to obtain the resonant frequency and the damping ratio (e.g. Cascante et al. 2003).

$$TF(\omega) = \frac{-(\omega/\omega_o)^2\left[\dfrac{B1 r_m r_a}{J}\right]}{\left[(1-(\omega/\omega_o)^2 + (c_E r_m{}^2 + d_s)\dfrac{(\omega/\omega_o)i}{\omega_o J}\right]} \qquad [5]$$

where $B1$ (the effective product of the magnetic-field induction and the length of wire in the coils) is the magnetic force factor of the coils (N/A); J is the mass moment of inertia of the specimen and driving plate (kg•m^2); c_E is the damping coefficient that represents energy losses resulting from the eddy-current forces (N/(m/s)); d_S is the viscous damping coefficient that represents energy losses in the specimen (N•m^2/(m/s)); r_m is the distance from the centre of the specimen to the magnets (m); and r_a is the distance from the centre of the specimen to the accelerometer (m). The theoretical transfer function assumes that the shear strains induced by all the frequencies in a frequency sweep are constant. At large strains; however, the goodness of fit deteriorates and the damping ratio is over estimated (Figure 2).

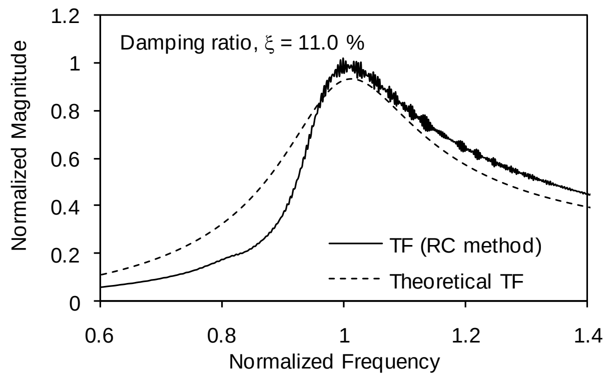

Figure 2. Transfer functions from RC measurements fitted with theoretical TF (fo = 42 Hz, $\gamma = 5.27 \times 10^{-4}$).

The current in the coil system of resonant column is controlled by either a power amplifier or current amplifier. Irrespective of the type of amplifier used in the RC tests, shear strains at different frequencies are difficult to control. The shear strain associated with a measured resonant frequency is typically computed from the peak-to-peak acceleration response of the specimen in time domain by

$$\gamma = \frac{d\ g\ 0.71\,V_{out}}{\pi^2\ 16\ r_a\ S\ H\ \ f_o\,10^{\left(\frac{Amp}{20}\right)}} \qquad [6]$$

where d is the diameter, g is acceleration due to gravity, V_{out} is the amplitude of response in volts, S is the sensitivity of the accelerometer, and *Amp* is the amplification in the filter amplifier. This equation can also be used to compute the shear strains at any frequency of excitation during equal strain testing.

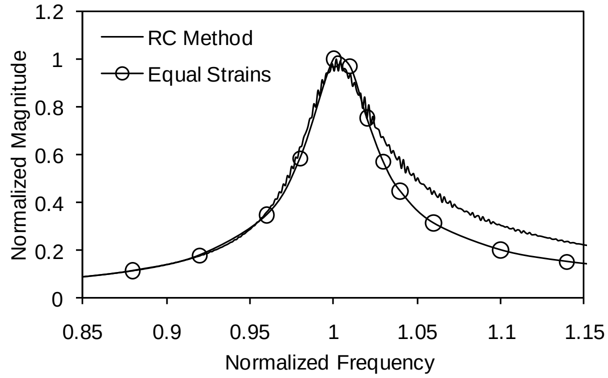

Figure 3. Transfer functions from equal strain and frequency sweep (RC) measurements (f_o = 51 Hz, $\gamma = 9.54 \times 10^{-5}$).

Khan et al. (2008) presented the effect of unequal strains in a transfer function on the computation of dynamic properties. They proposed a new procedure of constructing the equal strain transfer function. The procedure is based on exciting the specimen with sinusoids inducing similar strains in the specimen and measurement of discrete values of transfer function (TF). The resulting transfer function (Figure 3) has a symmetrical shape even at large strains that improves the error in curve fitting of theoretical TF. The method; however, is time consuming and induces undesirable number of cycles to the specimen in calculating the Transfer Function which is critical at mid to large strain levels in loose sands and saturated soils.

Non-resonance (NR) methods involving single frequency excitation are based on elastic-viscoelastic correspondence principle (Christensen 1971, Lai et al., 2001, Lai and Rix 1998) which replaces elastic shear modulus with the complex-valued shear modulus. Khan et al. (2008) presented another non-resonance model based on transfer function approach. The substitution of elastic shear modulus with complex shear modulus allows the simultaneous determination of dynamic properties but implicitly forces one of the property to be function of frequency even for a hypothetical response of an elastic material. Khan et al (2008) showed that the damping ratio of a hypothetical specimen with frequency independent damping (as assumed in RC testing) increases linearly with frequency.

3 EXPERIMENTAL SETUP AND EXPERIMENTAL PROGRAM

A modified, Stokoe-type resonant-torsional column apparatus is used in this study. The driving plate has a radius of 15 cm. The input signal to the driving coils is generated by a dynamic signal analyzer (HP-35670A) and amplified through a power amplifier (Bogen, GS-250). The response of the sample is acquired using an accelerometer (PCB 352A78) attached to the driving plate. The output signal from the accelerometer and the current through the coils are measured with a digital oscilloscope (HP-54645A) and recorded in the dynamic signal analyzer (HP-35670A). The mass polar moment of inertia ($MPMI$) of the driving plate ($Io = 6.9 \times 10^6$ g-mm^2) was measured by testing an aluminium (fo = 29.7 Hz) and a PVC probe (fo = 29.4 Hz). The calibration probes of lower resonant frequencies were selected to avoid the effects of base fixidity (Khan et al. 2008b).

RC tests were performed on a sand specimen at a confinement of σ_o = 50 kPa and 120 kPa. The tests were performed using conventional RC method, NR method (using equal strains), and the proposed methodology (FN) at different shear strain levels. The details of test setup for each method are presented in following sub-sections.

3.1 RC Method

A broad-band frequency sweep (100 Hz) was used to approximately locate the resonance before a narrow band frequency sweep (25 Hz) was used to measure the transfer function, the induced current through the coils, and the acceleration response of the specimen. The equation of the transfer function (Eq. 5) was used to curve fitt the measured TF to determine the resonance and damping ratio. The shear strain was computed from equation 6 after measuring the peak-to-peak response of the specimen from the oscilloscope. The procedure was repeated at different shear strain levels to obtain the variation of dynamic properties with shear strain level.

3.2 NR Method

The specimen was excited with a fixed sine function at the resonant frequency and the corresponding value of transfer function was recorded (TF). The amplitude of the fixed sine was adjusted until the TF value matched the TF value obtained from RC method. The TF (e.g. Fig. 3) was then reconstructed around resonance by changing the frequency of fixed sinusoids but keeping the shear strains constant by controlling the amplitudes. Only one equal strain transfer function is generated for a shear strain level. The dynamic properties were then evaluated with Eq. 2 (e.g. Lai et al. 2001) using the discrete measurements of the equal strain transfer function. This procedure was repeated for other shear strain levels.

3.3 FN Method

The new methodology (FN) is based on exciting the specimen at any frequency with a fixed sine using the desired shear strain level; which controls the position of resonant frequency in the tested frequency bandwidth. A single frequency excitation produces one peak in the TF; hence, a small amplitude random noise excitation is added to the main sinusoidal excitation (carrier) to measure the transfer function around resonance. This transfer function (figure 4) has a symmetrical shape around resonance even though the strains are not equal at other frequencies. The shape of the TF is important in evaluating dynamic properties as illustrated by large strain testing using RC method (Figure 2).

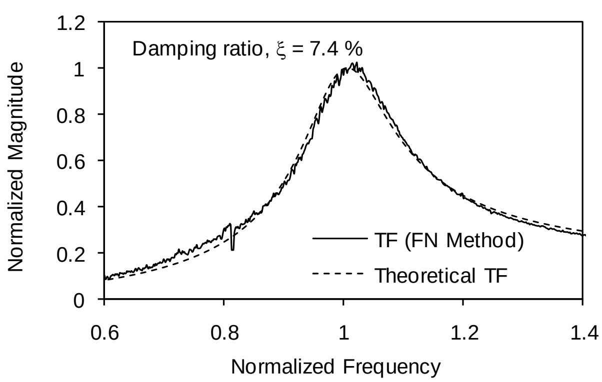

Figure 4. Transfer functions from FN measurements fitted with theoretical TF (fo = 42 Hz, $\gamma = 5.27 \times 10^{-4}$).

The effect of frequency is evaluated by changing the frequency of the carrier signal (shear strains kept constant). Since the shear strains are constant at all frequencies of the carrier signal, the position of transfer functions do not change for frequency independent behaviour. The dynamic properties are obtained by curve fitting to the measured transfer functions with Eq. 5. This procedure is repeated at other shear strain levels.

Finally the variation of dynamic properties with shear strain level is obtained by increasing the amplitude of the fixed sinusoid. Any frequency of the sinusoid can be used for the measurements as in the NR method. The use of the same excitation frequency at all shear strain levels eliminates the effect of frequency on the measurements if present.

4 RESULTS AND DISCUSSIONS

Dynamic properties of a dry-sand specimen were evaluated using conventional RC method, NR method and the proposed methodology (FN method). The new methodology is proposed to better evaluate the dynamic properties as function of frequency and shear strain level. Typical results and discussion are presented in the following sections.

The FN method produces transfer functions that have symmetrical shapes even at large strains that results in better fit of theoretical transfer function. Comparison of figures 2 and 4 (similar shear strain level) indicates a 33 % decrease in damping ratio when the transfer function is evaluated by FN method. The proposed method also shows great potential for the evaluation of dynamic properties at different frequencies. The transfer functions obtained by exciting the specimen at different fixed sinusoids have identical shape and resonance.

Figure 5 presents the comparison of transfer functions obtained from RC, equal strain, and FN methods at an isotropic confinement of σ_o = 120 kPa. Although many carrier frequencies were used to evaluate the frequency dependence in FN method, TFs for only two sinusoids are presented in the figure for clarity. Figure 5 indicates excellent matching among the transfer functions obtained from different methods at low strain levels.

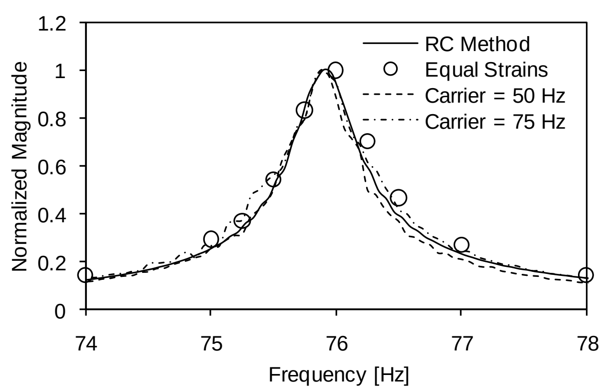

Figure 5. Comparison of transfer functions from RC, equal strains, and FN measurements (γ = 9 x 10^{-6}).

Figure 6 shows typical results from the NR and FN methods for the variation of dynamic properties with frequency for the transfer functions presented in figure 5. These results are in agreement with previous results that show a practically frequency-independent damping ratio and wave velocity for sands at low strain levels (Hardin and Black 1966; Iwasaki et al. 1978; Bolton and Wilson 1989; Kim and Stokoe 1995). The NR method; however, forces shear wave velocity to increase with frequency as noted earlier (viscoelastic model).

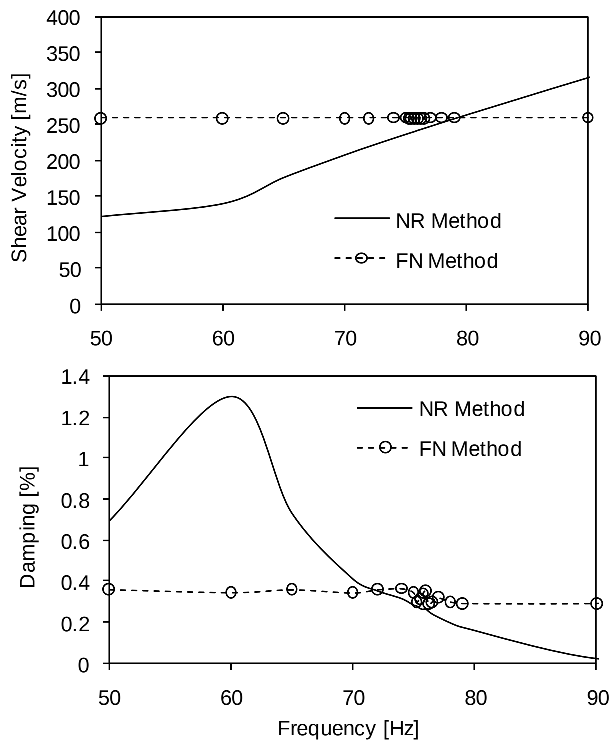

Figure 6. Evaluation of dynamic properties as function of frequency from NR and FN method (σ_o = 120 kPa, γ = 9 x 10^{-6}).

NR method is sensitive to the phase difference between the applied torque and resulting rotation; these small phase differences produce significant errors in damping even with small participation of flexural mode. This is evident from very large damping value at about 60 Hz.

The comparison of dynamic properties from the NR and FN methods for sand specimen at σ_o = 50 kPa is presented in figure 7 (γ = 9.5 x 10^{-6}). The variation of dynamic properties is similar to the results presented in figure 6. The slight decrease in shear wave velocity with frequency is insignificant but requires further investigation. The damping ratio; however, decreases in the FN method when the specimen is excited with a carrier frequency close to resonant frequency. A similar but smaller decrease is also evident in the measurements at σ_o = 120 kPa (figure 6). Although the exact mechanism is still being investigated, the significant reduction in the net current (excitation) going through the coils because of the EMF damping could generate inaccurate measurements close to resonance (EMF damping increases for lower frequencies). The jump in damping ratio at resonance is expected to decrease with increase in shear strain level due to relatively larger damping ratios and the reduction of EMF damping with strain level (Cascante et al. 2005).

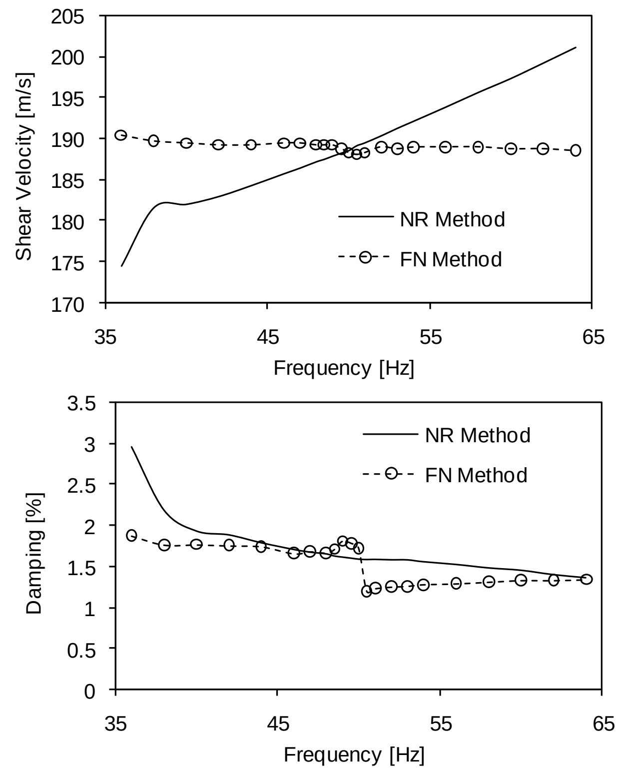

Figure 7. Evaluation of dynamic properties as function of frequency from NR and FN method (σ_o = 50 kPa, γ = 9.5 x 10^{-6}).

The variation of dynamic properties with shear strain level is presented in figure 8 for the RC, NR, and FN methods at σ_o = 50 kPa. The results indicate that the variation of shear modulus is similar for all the methods. These findings are in agreement with literature (Khan et al. 2008). The damping ratio on the other hand shows more variation among the methods. The difference increases with the increase in shear strain level. As noted earlier (Figure 2), the damping ratio from RC method is overestimated because of the distorted shape of transfer function due to unequal strains. At low shear strain levels, the damping ratios from FN and RC measurements are in good agreement whereas, the NR method underestimates the damping ratios by 19 %. At shear strain level of 0.1 %, difference between FN and NR measurements is 20 % whereas the difference between NR and RC measurements is 45 %. The damping ratios measured from NR method at large strains are more reliable because the larger phase difference can be measured with greater accuracy. Although the shape of

TF is symmetrical, FN method does not produce enough shear strains at frequencies other than the carrier frequency and hence the damping ratios are underestimated at large strains. The damping ratios measured from FN method are however better than measured in RC method in terms of accuracy.

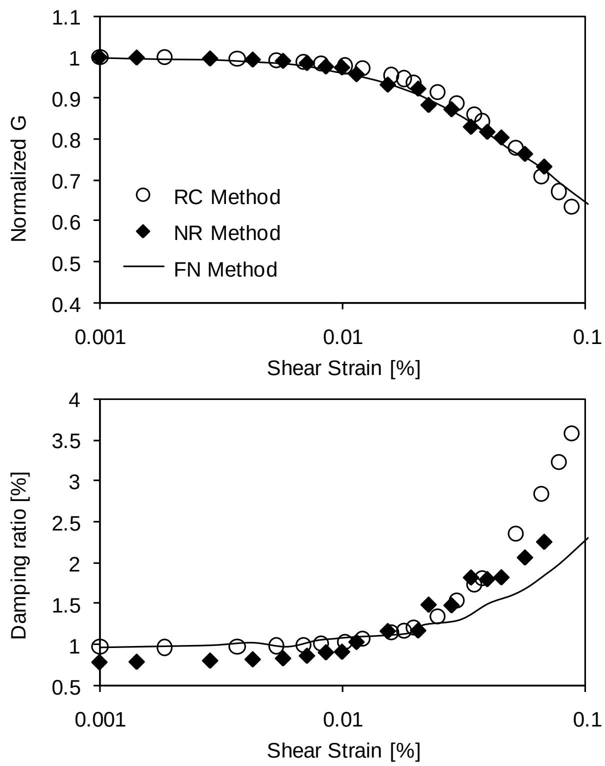

Figure 8. Evaluation of dynamic properties as function of shear strain from RC, NR and FN methods (σ_o = 50 kPa).

5 CONCLUSIONS

Resonant column tests were performed on a sand specimen at two isotropic confinements of 50 kPa and 120 kPa. The dynamic properties were evaluated by conventional RC, NR, and newly proposed FN methods. Dynamic properties were evaluated as a function of frequency and shear strain levels. The main conclusions from the study are:

The damping ratios from RC measurements are overestimated at large strains. A reconstruction of TF using equal strains provides better estimate of damping ratio.

The variation of dynamic properties with frequency is better estimated by newly proposed FN method compared to NR method; which forces one property to change with frequency. NR method is also sensitive to slight variations in phase difference especially at low strain levels.

The variation of dynamic properties with shear strain level is better characterized by using either NR or FN methods. The degradation of shear modulus is practically similar from RC, NR, and FN measurements. Damping ratio is overestimated by RC method due to distorted shape of transfer function at large strains. The FN method slightly underestimates the damping ratios at large strains due to smaller than required strains at frequencies different from carrier frequency.

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REFERENCES

ASTM 2000. Standard Test Methods for Modulus and Damping of Soils by the Resonant-Column Method. *American Society for Testing and Materials*, Annual Book of Standards, D4015-92.

Bolton, M. D., and Wilson, J. M. R. 1989. An Experimental and Theoretical Comparison between Static and Dynamic Torsional Soil Tests. *Géotechnique*, 39(4), 585-599.

Booij, H. C., and Thoone, G. P. C. M. 1982. Generalization of Kramers-Krönig Transforms and Some Approximations of Relations between Viscoelastic Quantities. *Rheologica Acta*, 21, 15-24.

Cascante, G. and Santamarina, J. C. 1997. Low Strain Measurements Using Random Noise Excitation, *Geotechnical Testing Journal*, 20(1): 29-39.

Cascante, G., Santamarina, J.C., and Yassir, N., 1998. Flexural Excitation in a Standard Resonant Column Device. *Canadian Geotechnical Journal*, 35(3): 488-490.

Cascante, G., Vanderkooy, J., and Chung, W., 2003. Difference between current and voltage measurement in resonant column testing. *Canadian Geotechnical Journal*, 40(4): 806-820.

Cascante, G., Vanderkooy, J., and Chung, W., 2005. A new mathematical model for resonant-column measurements including eddy-current effects. Canadian Geotechnical Journal, 42(1): 121-135.

Christensen, R. M., 1971. Theory of Viscoelasticity – An Introduction. Academic Press, New York.

Hardin, B.O., 1965. The Nature of Damping in Sands. *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 91, No. SM1, pp. 63-97.

Hardin, B. O., and Black, W. L. 1966. Sand Stiffness under Various Triaxial Stresses. *Journal of the Soil Mechanics and Foundations Division*, Proceedings of the ASCE, 92(SM2), 27-42.

Iwasaki, T., Tatsuoka, F., and Takagi, Y. 1978. Shear Modulus of Sands under Cyclic Torsional Shear Loading. *Soils and Foundations*, 18(1), 39-56.

Khan, Z. H., Cascante, G., El Naggar, M.H., and Lai, C. 2008. Measurement of frequency-dependent dynamic properties of soils using the Resonant-Column device. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 134(9): 1319-1326.

Khan, Z. H., Cascante, G., and El Naggar, M.H., 2008b. Linearity of the first torsional mode of vibration and base fixidity in resonant column. ASTM, *Geotechnical Testing Journal*, 31(1): 587 - 606.

Kim, D.-S., and Stokoe, K. H. 1995. Deformational Characteristics of Soils at Small to Medium Strains. *Earthquake Geotechnical Engineering*, Tokyo, Japan, 89-94.

Lai, C. G., and Rix, G. J. 1998. Simultaneous inversion of Rayleigh phase velocity and attenuation for near surface site characterization. Report, National Science Foundation and U.S. Geological Survey, Georgia Institute of Technology.

Lai, C. G., Pallara, O., Lo Presti, D. C. and Turco, E. 2001. Low-strain stiffness and Material Damping Ratio Coupling in Soils. *Advanced Laboratory Stress-strain Testing of Geomaterials*, Tatsuoka, T., Shibuya, S., and Kuwano, R. Eds, Balkema, Lisse, pp: 265-274.

Meza-Fajarado, C.K., and Lai, C.G. 2006. Exact causal relationships between attenuation and phase velocity in linear dissipative media. *First European Conference on Earthquake Engineering and Seismology* (a joint event of the 13th ECEE & 30th General Assembly of the ESC) Geneva, Switzerland.

Richart F. E., Hall J. R. and Woods R. D. 1970. Vibrations of soils and foundations. Prentice Hall, Englewood Cliffs, pp: 414.

Rix, G. J., and Meng, J. 2005. A non resonance method for measuring dynamic soil properties., *Geotechnical Testing Journal*, 28(1), pp.1-8.