Prediction of moisture-density characteristics of compacted fill using mixture theory

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ABSTRACT
This study presents a discussion of three prominent mixture models and assesses their predictive capabilities for several fly ash-modified soil mixtures. This study specifically focuses on the ability of the mixture models to predict moisture-density characteristics of compacted fill containing varying percentages of fly ash. The results of this study show that the behaviour of the mixture is a function of the fly ash component and the fractions of each component contained in the total mixture. This particular focus is in response to greater emphasis being placed on incorporating sustainability into geotechnical engineering. Specifically, it is hoped that the results of this study will be used to facilitate greater utilization of fly ash in earthwork construction.

RESUMEN
Este estudio presenta una discusión sobre tres importantes modelos de mezclas y se establecen sus capacidades predictivas para varias mezclas de suelo modificado con cenizas de carbón. Específicamente, este estudio se centra en la capacidad de los modelos de mezclas para predecir características de humedad-densidad de rellenos compactados con diferentes porcentajes de cenizas. Los resultados de este estudio muestran que el comportamiento de las mezclas es una función del contenido de cenizas y de la fracción de cada componente en el total de la mezcla. Este enfoque es una respuesta del gran énfasis puesto en incorporar la sostenibilidad en la ingeniería geotécnica. Específicamente, se espera que los resultados de este estudio sean usados para facilitar un mayor uso de las cenizas en construcción de obras geotécnicas.

1 INTRODUCTION

It has been observed that a wide range of theories in soil mechanics are response-based and mainly involve the basic soil types; clays and sands. Most constitutive models for soils are also applicable to pure sands and clays. However, these concepts usually fall short in addressing the problems involving soil mixtures. Soil mixtures by definition are materials consisting of two or more different soil types. As a result, it becomes difficult to predict the effects of soil constituent variations on soil mixtures. This is because variations in the soil components can alter the mixture in such a way that it behaves as an entirely different material (Tien et al., 2004; Vallejo and Mawbry, 2000; Kumar and Wood, 1997). There is therefore the need to improve the fundamental understanding of the behaviour of the constituents of soil mixtures when they are combined and how the constituent properties influence the overall behaviour of the soil mixture.

In particular, soil mixtures containing fly ash are of interest due to possible engineering applications. Fly ash is a by-product of the coal combustion processes found in many industrial plants around the world. Fly ash is categorized into two classes; Class C and Class F. These classes refer mostly to the chemical composition of the particles in fly ash, more specifically calcium oxide concentration. Class C is typically used as an admixture in concrete while Class F is typically considered waste. A thorough understanding of how both classes of fly ash in a soil mixture behave could decrease fly ash waste while fulfilling engineering needs. For example, soil improvement techniques are needed to develop areas of the world where soil is too weak for its intended purposes. Fly ash, and more specifically Class F fly ash, could be an effective way to treat soil to increase strength of soil.

2 MIXTURE THEORY IN SOILS

The fundamental flaw of assuming soil is ideal such as pure clays or uniform sands is many natural soils consist of coarse granular particles in a matrix of clay paste or vice versa, fine particles in a matrix of granular particles. Thus, soil mixtures are, in general, heterogeneous on the microscopic level. The properties of natural mixtures are expected to be intermediate between the individual properties of the different constituent materials. The estimate of these properties can be done using micromechanics models. A micromechanics model predicts the properties of the composites from the properties and volume fractions of the individual constituents (Tien et al., 2004).

The formulation of the mixture theory model is based on the basic principles of the law of thermodynamic and

the laws of conservation of energy and mass. From the mixture theory model it is observed that the concept is primarily based on the properties of the individual constituents and their respective volume or gravimetric fractions. The theory can therefore be used to predict any property in any mixture regardless of the nature of the materials as long as the properties of the individual constituents are known. Therefore, a soil mixture containing fly ash can use these same theories to predict behavior.

2.1 Insitu Soil Mixtures

Therefore, soil mixtures do not have to be solely natural soils. In engineering, admixtures are often added to natural soils to transform properties of poor soil into a usable product. For example, fly ash may be added as filler to clays in order to improve their mechanical behaviour, creating a man-made soil mixture in the field. According to mixture theory the behaviour of soil mixtures may be estimated from the constituent materials of the mixture. Because of the difficulty in mixing, the properties of in-situ treated soils vary widely depending on the level of mixing. Thus, there is little knowledge of the effects of these insitu mixtures. The mechanical behaviour of man-made mixtures, in geotechnical engineering applications, needs to be clarified through more research.

A two-phase mixture, also referred to as a binary mixture, consists of two different materials. The primary material is termed the matrix and the secondary material is called the inclusion, which is usually randomly distributed in the mixture.

2.2 Effect of Inclusions on Mixture Behaviour

The behavior of soil mixtures is expected to be influenced by both the particle size and other index properties of the materials present. Void ratio is an index used to characterize the degree of packing of soils with different fines content or particle sizes. The effective void ratio of a soil mixture can be determined by the volume fractions and void ratios of the individual constituents. This is given by Equation 1:

$$e_{eff} = e_c f_c + e_f f_f \qquad [1]$$

In Equation 1 e_{eff} is the effective void ratio and the subscripts c, and f denotes coarse and fine particles, respectively. The shape and surface roughness of particles affects the porosity and the effective packing void ratio of multi-sized particles. The maximum and minimum void ratios of spherical particles are lower in packing than that of angular shaped particles. The fractions of individual constituents in mixtures also tend to influence the properties of the mixture, but the optimum percentage required to significantly alter the mixture properties is the question. This is important to achieve the desired engineering properties for mixtures such as fly ash-modified soils, which are used in this research.

It was also observed that, in binary mixtures, a minimum porosity or void ratio is reached where maximum or minimum engineering properties of the mixture are realized. This minimum porosity or void ratio is dependent on the shape of the materials constituting the binary mixture and, according to a common rule-of-thumb, usually occurs within 20% to 40% of the inclusion in the mixture. This phenomenon is usually realized regardless of whether the inclusion is coarse and the matrix is fine or vice versa as demonstrated by research done by Santamarina (2001), Vallejo (2000), and Kumar and Wood (1997).

3 MIXTURE THEORY MODELS

Due to varying behaviors of the individual materials it can be difficult to predict the exact behavior of a mixture. Therefore, researchers have found different models to predict the combined effect of a soil mixture. Earlier work on the subject of mixture theory is shown by Viogt's (1889) work. He developed a model yielding the upper bound of the rule of mixtures. The models have since been modified to yield better and more accurate predictions. Some of the modifications include Omine et al. (1998), and Braem et al. (1987).

3.1 Viogt's Mixture Theory Model

Voigt (1889) developed a model on the assumption that all elements constituting the mixture are subjected to the same uniform strain. Equation 2 shows the calculation of the bulk modulus, K, for the mixture.

$$K_{mix} = f_i K_i + (1 - f_i) K_m \qquad [2]$$

In Equation 2, the subscripts *i*, *m*, and *mix* represent the inclusion material, matrix material, and soil mixture, respectively and *f* is the volume content of the inclusion.

The bulk modulus describes volumetric elasticity and the relation in Equation 2 can be used in determining other elastic moduli of the mixture such as young's modulus and the shear modulus. Voigt's approximation gives the upper bound of the elastic moduli of a mixture. This model gives fairly accurate predictions of mixture behavior when the difference between the elastic moduli of the two materials is very small.

3.2 Omine's Mixture Theory Model

Omine et. al. (1998) developed a relation that was an improvement upon earlier work done by Voigt (1889). The formulations of Omine's work were based on the principle that the stress experienced by the mixture, $\sigma_{mix,}$ is based on the weighted averages of the stresses experienced by the individual constituents, σ_i and σ_m. This

weighted average can be obtained from the relations shown in Equation 3.

$$\sigma_{mix} = f_i\sigma_i + (1 - f_i)\sigma_m \qquad [3]$$

In Equation 3, the subscripts *i*, *m*, and *mix* represent the inclusion material, matrix material, and soil mixture, respectively and *f* is the volume content of the inclusion. A similar assumption was made for strains, ε, as well. This is in accordance with Voigt's equation. The resulting incremental stress-strain relationship was given in Equation 4.

$$\varepsilon_{mix} = \frac{f_i b C_i + (1 - f_i) C_m}{(b-1) f_i + 1} \sigma_{mix} \qquad [4]$$

In Equation 4, the subscripts *i*, *m*, and *mix* represent the inclusion material, matrix material, and soil mixture, respectively and *f* is the volume content of the inclusion. Also in equation 4, C is the coefficient related to the material properties, and *b* is a stress distribution parameter which represents the average stress ratio of inclusion to matrix. This stress distribution parameter is given in Equation 5.

$$b = \left(\frac{E_i}{E_m}\right)^{1/2} \qquad [5]$$

Equation 5 uses a ratio of the elastic modulus, E, parameter for the matrix and inclusion materials. The subscripts *m* and *i* represent these materials, respectively. This concept of the stress and strains of the solution is used to obtain the elastic moduli of two-phase mixtures from specific stress conditions. Young's modulus under one-dimensional stress conditions is given by Equation 6.

$$E_{mix} = \frac{(b-1) f_i + 1}{\frac{f_i b}{E_i} + \frac{(1 - f_i)}{E_m}} \qquad [6]$$

Similar relation applies to both bulk (K) and shear (G) moduli.

3.3 Braem's Mixture Theory Model

The Braem et al. (1987) developed a mixture model based on the linear mixing of the log of the elastic modulus, E, of the matrix and inclusion of the composite. This model was also a modification of the rule of mixtures earlier proposed by Voigt (1889). Braem's model is given in Equation 7.

$$E_c = E_m \left(\frac{E_i}{E_m}\right)^x \qquad [7]$$

In Equation 7, the subscripts *c*, *m*, and *i* denotes composite, matrix, and inclusion materials, respectively. The parameter *x* is the volume fraction of the inclusion material.

4 FLY ASH-MODIFIED SOILS

In engineering, situations arise where soil improvement becomes necessary to increase the load-carrying capacity of the foundation soil. These improvements are typically done through ground improvement techniques such as mixing in stabilizing agents or adding geosynthetics as reinforcement. However, for this study, the focus is ground improvement through the use of admixtures. Admixing techniques in soils are effective and relatively easy in soil improvements (Prabakar et al., 2004). In particular, fly ash has been used in various applications including soil modification. It is relatively cheap to use fly ash for soil improvement since it is a coal combustion by-product and is typically available at a low cost.

With the mixture theory models developed by Voigt (1889), Omine et al. (1998), and Braem et al. (1987), a simple predictive model that will aid in accurately predicting mixture properties of fly ash-modified soils is sought. This model will be used in conjunction with other properties or phenomena where necessary to better predict the engineering properties of fly ash-modified soils. This will help engineers in accurately predicting the properties of soil mixtures based on the constituents before actually preparing them.

Most of the work reported in the literature on fly ash modified soils involves high calcium Class C fly ash, lime, or cement in addition to the fly ash. This research is focused on fly ash modified soils that used mainly Class F and low calcium oxide Class C fly ash with no lime or cement admixtures. These types of fly ash modified soils have been the focus of recent research by Prabakar et al. (2004), Kumar and Sharma (2004), and Misra (2000). Prabakar et al. and Kumar and Sharma concentrated on Class F fly ash, while Misra used Class C fly ash.

Prabakar et al. (2004) conducted a study on the influence of Class F fly ash on strength behavior of soils. In their study, three different soil samples and one fly ash were used. The soil samples used were classified as low plasticity clay (CL), low plasticity organic silty clay (OL), and inorganic silt (MH). The major consideration was the effect of fly ash on strength behavior of different soil types. All tests were performed at optimum moisture content (OMC) conditions. The percentage of fly ash

varied from 0 to 46 percent. The investigation revealed a decrease in maximum dry density (MDD) and an increase in OMC with increasing fly ash content in all the soil types.

Prabakar et al. (2004) inferred that the reduction in MDD with increasing fly ash content may have been due to a decrease in specific gravity of the mixtures as the fly ash content increases. With regards to shear strength parameters, it was observed that cohesion increased with increasing fly ash content. This trend was observed in the CL and OL soil types with the exception of the MH soil type. In MH soil, cohesion decreased with increasing fly ash content with no specific trend observed. Generally, the angle of internal friction also increased with increasing amount of fly ash in all soil types, with MH experiencing the biggest increase.

Researchers Kumar and Sharma (2004) studied the effects of fly ash on engineering properties of expansive soils. The soil used was classified as high plasticity clay (CH) with a liquid limit (LL) of 80 and plasticity index (PI) of 52. The soil had a free swelling index (FSI) of 250%. The fly ash used had a calcium oxide (CaO) content of 2.21% and the sum of silica, alumina, and ferric oxide was 90.44%. According to the chemical composition of the fly ash, it is classified as Class F in accordance to ASTM C 618. All tests conducted on the fly ash, and fly ash-expansive soil blends conformed to ASTM standards. The effects of fly ash on consistency, compaction, hydraulic conductivity, and shear strength on the expansive soil were evaluated. The investigation considered fly ash-soil mixtures with 0, 5, 10, 15, and 20 percent fly ash contents on a dry weight basis.

Lastly, Misra (2000) researched the effects of low calcium oxide Class C fly ash on soil improvement as it relates to design, construction, and environmental issues. Eight different clay soils were used. These included four natural clays from around the Kansas City, Missouri area and four clays synthetically made by blending a small proportion of bentonite and kaolinite together. These synthetic clays had properties easily duplicated providing standard clays to clearly evaluate the effect of the fly ash.

5 MIXTURE THEORY MODEL ASSESSMENT

The predictive accuracy of the models discussed in Section 3, as applied to fly ash-modified soils presented in Section 4, were analyzed by using them in predicting and comparing some of the properties of the fly ash soil mixtures based on the known properties of the individual constituents. The properties considered include optimum moisture content (OMC), maximum dry density (MDD), cohesion intercept, internal friction angle (Φ), and California bearing ratio (CBR).

5.1 Moisture-Density Parameters

Soil compaction is important in geotechnical engineering for many reasons. Compactive effort is used to render the soil configuration into a denser material, which increases strength and stability. As a result of compaction, compressibility and hydraulic conductivity of a soil mass is reduced. Usually, the maximum properties are realized at the optimum moisture content. At optimum moisture content, void ratio is minimum leading to the maximum dry density in the soil. The improved properties of the soil due to compaction make it suitable for geotechnical applications such as embankments and subgrades. Mixture theory models have been used in predicting and comparing moisture-density relationships from Prabakar et al. (2004), Kumar and Sharma (2004), and Misra (2000).

Figure 1 presents the maximum dry density of soil mixtures made from different soil types and Class F fly ash. These data are plotted as a function of their respective optimum moisture content, and then compared with predicted maximum dry density from mixture theory models. It can be observed in Figure 1 that the maximum dry density decreases with increasing optimum moisture content although the mixtures are different in each case with respect to fly ash content, which ranges from 0 to 46 percent depending on the research. It appears that the soil type had no effect on the trend between the dry densities and the moisture content. The mixture theory models predictions followed similar trends as the actual and they predict closely to the actual data as well. The results in Figure 1 show a similar trend with all the data from Prabakar (2001) and Kumar and Sharma (2004).

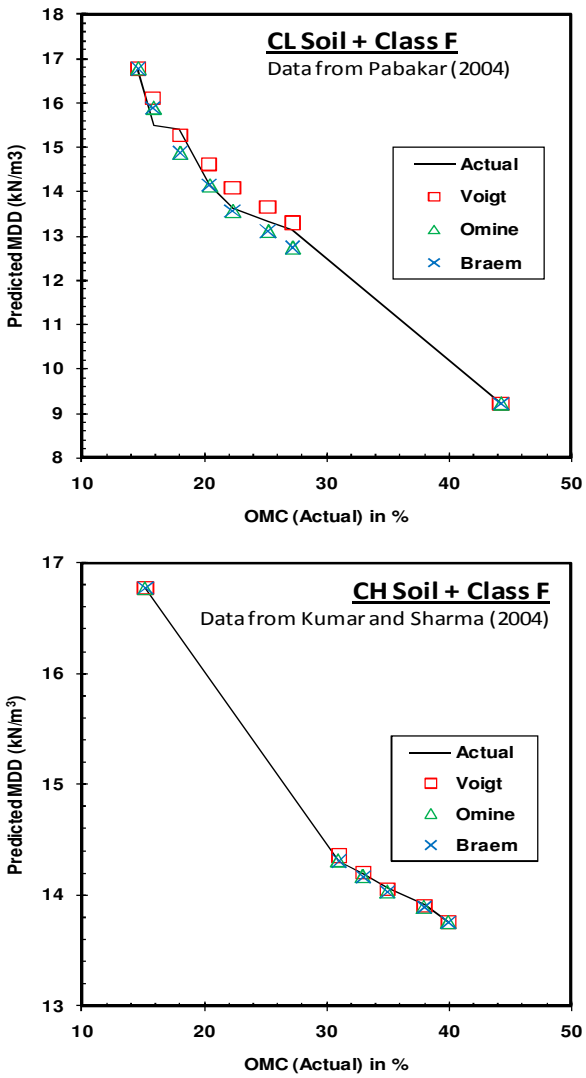

Figure 1. Comparing Actual and Predicted Maximum Dry Density (MDD) of Class F Fly Ash Mixtures

Figure 2 presents a comparison between predicted values from given mixture theory models and the actual data observed by Misra (2000). Here a similar relation between maximum dry density and optimum moisture content with two different clay types (CL and CH) and Class C fly ash is shown. It can be seen in Figure 2 that the trend observed in maximum dry density with respect to optimum moisture content is reversed as compared to those in the Class F mixtures in Figure 1. This could be due to the chemical composition of the fly ashes. The models predicted closely to the actual in the CH soil mixture, but were not the case in all the data points in the CL soil mixture.

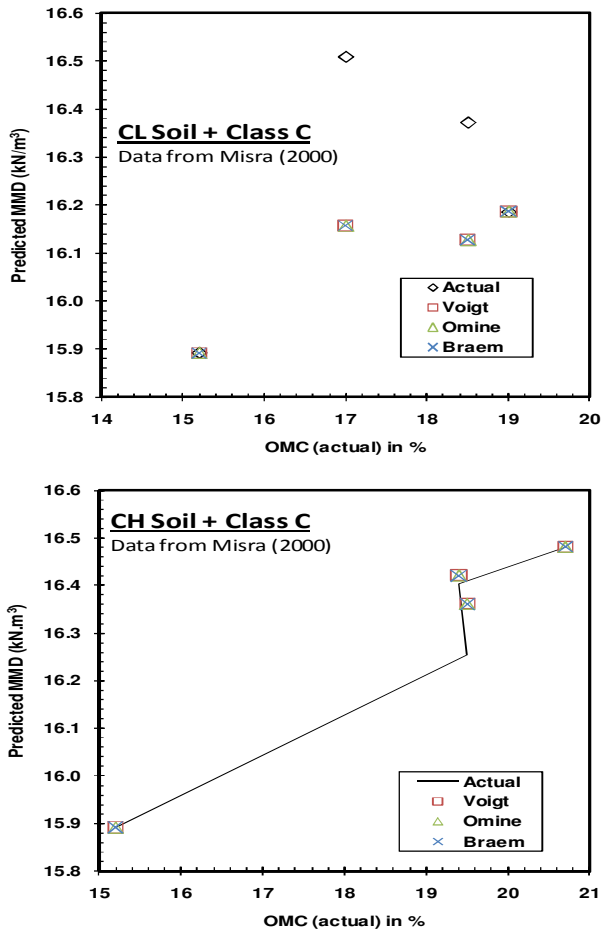

Figure 2. Comparing Actual and Predicted Maximum Dry Density (MDD) of Class C Fly Ash Mixtures

Figure 3 relates the actual and predicted moisture-density parameters to index properties. According to Figure 3, the optimum moisture content increases with increasing liquid limit (LL) and plasticity index (PI). The models proposed by Braem et al. (1987) and Omine (1998) predicted closely to each other and both predicted better with respect to the actual than that of Voigt (1889). In Figure 3, it is observed that maximum dry density decreases with increasing PI. All the models predicted well with respect to the actual data and relatively closer to each other as well.

Figure 4 shows a summary comparison of moisture-density parameters for the three different soil types used by Prabakar (2004) in his investigation of fly ash contribution to strength parameters. The soil types used were clay of low to medium plasticity (CL), an organic silty-clay of low plasticity (OL), and inorganic silts (MH). The fly ash used in the mixtures is classified as Class F. From the figure, optimum moisture content and maximum dry density predictions with the models gives a fairly good correlation between predicted and actual results with all the data. The data showed that regardless of the type of soil or fly ash the models can fairly predict moisture-density parameters well.

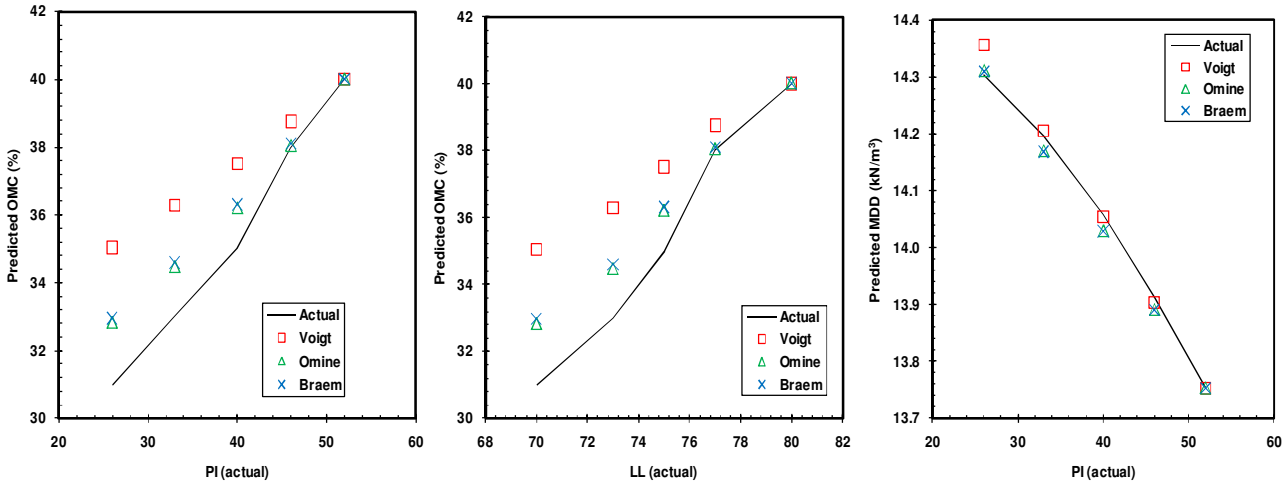

Figure 3. Relating Consistency Limits to Moisture-Density Parameters and Comparing Actual to Predicted of a CH soil and Class F fly ash. (Data from Kumar and Sharma, 2004).

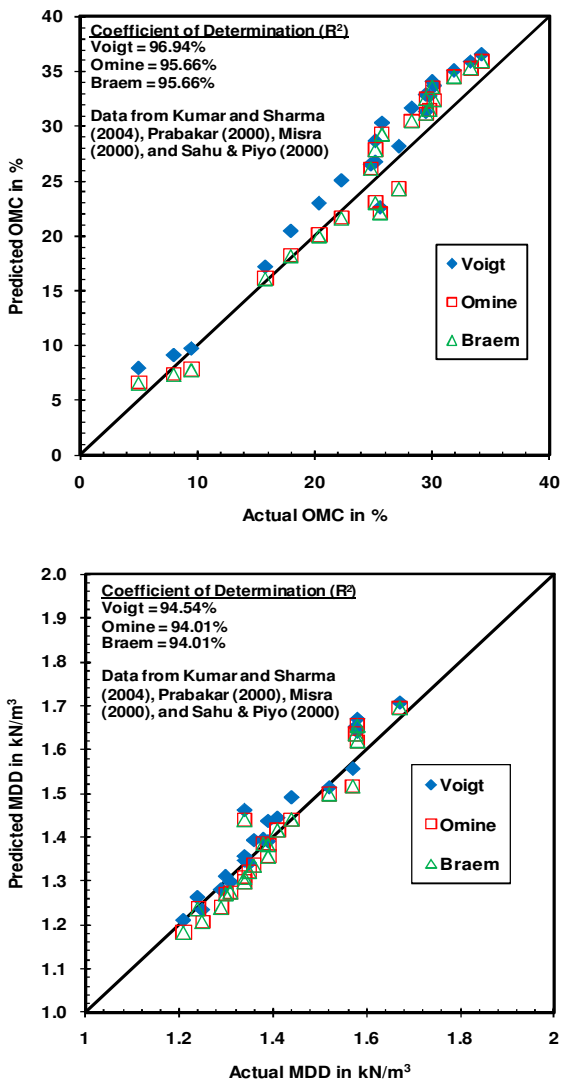

Figure 4. Comparisons of Predicted and Actual Maximum Dry Density and Optimum Moisure.

5.2 California Bearing Ratio Parameter

The California bearing ratio (CBR) of the soil mixtures were also underpredicted by all the models according to Figure 5. Figure 5 compares the three different soil types used by Prabakar (2004) in his research of fly ash modified soils. The soil type of which predictions were very close to the actual data is the inorganic silts. This could be attributed to the fact that there were minimal or no reactions between the silts particles and the fly ash particles. Fly ashes consist mainly of silt sized particles and therefore in the absence of chemical or physicochemical interactions, they replace similar sized particles in the inorganic silt-fly ash mixtures. This could, therefore, be the reason why the predictions in the inorganic silt mixtures are better than that of the clay soil mixtures. The skewness in the trends observed in the models predictions need to be modified to yield well correlated results comparable to the actual data. This could be rectified by analyzing the physicochemical behavior of the soil and the fly ash interactions.

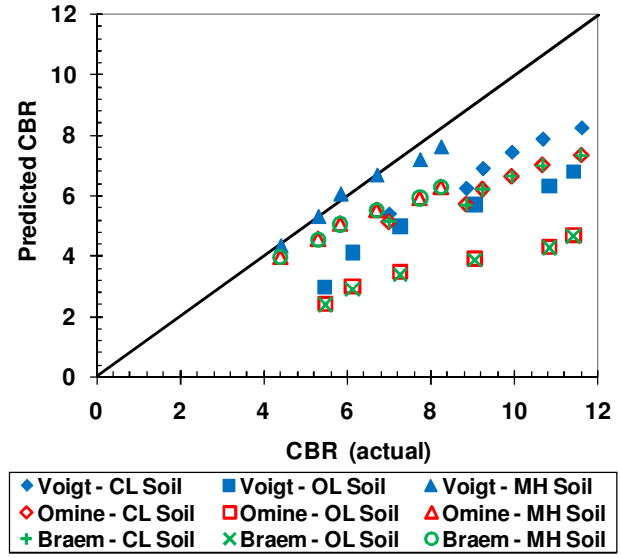

Figure 5. Comparisons of Predicted and Actual CBR.

5.3 Strength Parameters

Using the Prabakar (2004) soil types, the correlation between predicted and actual data, although skewed, was very good in the strength parameters considered. It can be seen in Figure 6 that predictions with all the models were very precise and were strongly correlated with respect to cohesion in both CL and OL soils.

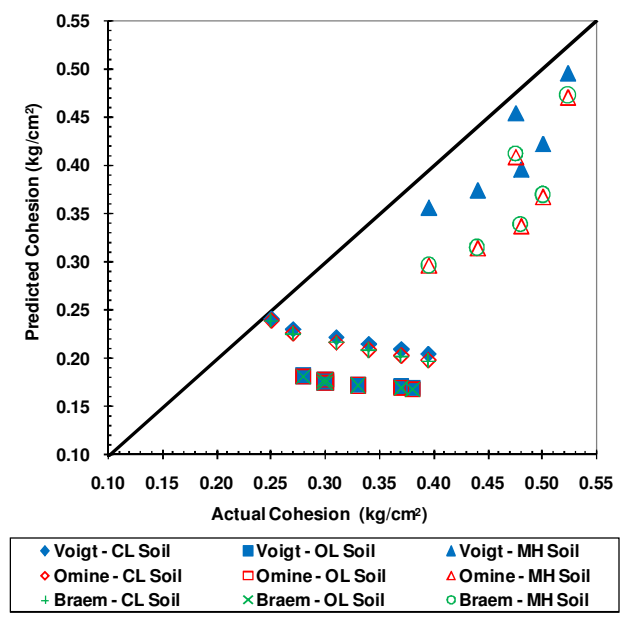

Figure 6. Comparisons of Predicted and Actual Cohesion. (Data from Prabakar, 2004).

Omine (1998) and Braem et al. (1987) models predicted very close to each other in the inorganic silts (MH) compared to that of Voigt (1889). However, Voigt seem to be more accurate in that soil. The coefficient of determination is summarized in Table 1. It can be seen that, in general, Voigt's model predicts better than the other models.

Table 1. Determination Coefficient (R^2) in percent (%).

CBR			
Model	CL	OL	MH
Voigt	97.2	93.6	97.0
Omine	97.0	96.2	98.5
Braem	97.0	96.2	98.5
Cohesion			
Model	CL	OL	MH
Voigt	97.3	87.2	73.2
Omine	96.4	87.2	68.9
Braem	96.4	87.2	68.9
Friction Angle			
Model	CL	OL	MH
Voigt	8.3	95.1	96.5
Omine	8.3	94.7	96.5
Braem	8.3	94.7	96.5

The trend in cohesion of the clayey soils was found to be different from that of the inorganic silt. Depending on the chemical compositions of the soil and the fly ash constituting the mixture, relatively different types of chemical interactions can take place. The difference in trends among the soil types as seen in Figure 6 could be due to different chemical interactions that takes place between the different soil types and the fly ash. In Figure 7, a similar trend was observed with friction angle in all the soil types as well.

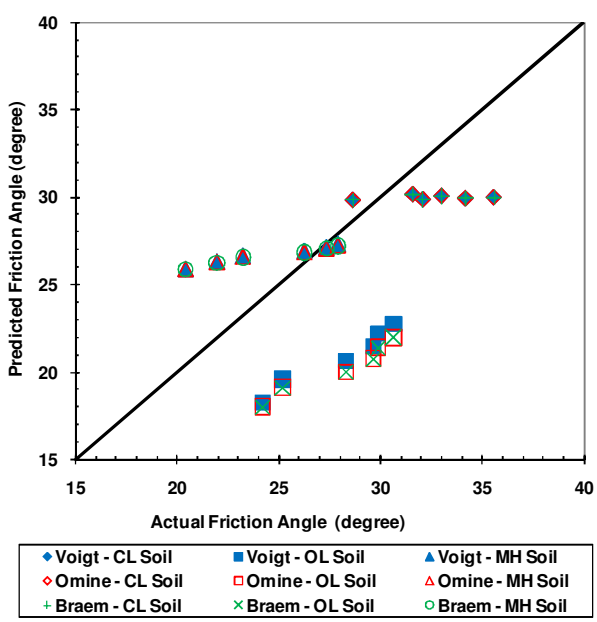

Figure 7. Comparisons of Predicted and Actual Friction Angle. (Data from Prabakar, 2004).

This further explains why strength predictions might not be accurate using properties from the individual constituents alone. Unlike the predictions in cohesion, all the model predictions in friction angle were very close to each other in all the soil types. With the exception of friction angles in the inorganic silts, which all the models overpredict, the rest of the strength parameters in all the soil types are underpredicted by the models.

6 CONCLUSIONS

The following conclusions can be made from the above analysis:

- It was observed that the mixture theory models can predict some of the geotechnical properties well. In certain cases, the models either overpredict or underpredict the actual results. This could be due to other factors that the models do not take into consideration such as the influence of chemical composition of the materials forming the mixtures and chemical reactions between particles.
- In terms of moisture-density parameters, the models' predictability was found to be independent of the soil type. Voigt's model predicted better in the case of maximum dry density than that of the other two models. There was a good trend in relating moisture-density parameters to consistency limits.
- Most data reported in literature, depending on the focus of the research, has concentrated on the effect of fly ash on either consistency limits or moisture-density parameters with very few investigating the effects of fly ash on strength properties. Lack of data on investigating all the engineering properties mentioned above on the same mixtures make it difficult to have a good assessment of the models. The models rely basically on individual properties constituting the mixtures to predict the mixture properties. Since most of the study in literature is usually based on one of the engineering properties and in most cases either the properties of the soil or the fly ash alone is not reported, it becomes difficult applying the mixture theory models to predict the mixture properties.
- In situations where the individual properties are not available, it will be difficult and probably inaccurate to extrapolate the properties from the mixtures. It would also not be a good idea in using data from one researcher and compare or predict other engineering properties from other researchers since the soil types and the fly ashes used may not be the same, as well as the method used in obtaining the results. These data gaps call for detailed laboratory experiments on mixtures to aid in assessing the predictive accuracy of the models on soil mixtures.

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