Unsaturated soil-pile interaction analysis using an improved simplified finite element formulation

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ABSTRACT
The dynamic behavior of a steel pipe pile in unsaturated soil is simulated using an improved simplified finite element model, which incorporates the Rayleigh damping model into the formulation. In the modeling, the stress-strain behavior of the soil was modeled using an elastoplastic constitutive model for unsaturated soil based on a bounding surface concept. The structural elements were modeled by Timoshenko beam elements using a linear elastic model. The dimensions of the model are constrained with the intention of performing a geotechnical centrifuge test on the model in the future. The response of the pile and the soil were investigated at three initial degrees of saturation. The results showed that the coupled soil-pile interaction is not largely affected by the range of initial degrees of saturation used in this study.

RÉSUMÉ
Le comportement dynamique d'un tas de tuyaux en acier dans des sols non saturés est simulé par un modèle simplifié et amélioré des éléments finis, qui intègre le modèle de Rayleigh d'amortissement dans la formulation. Dans la modélisation, le comportement contrainte-déformation du sol a été modélisé en utilisant un modèle élastoplastique pour les sols non saturés sur la base de sélection concept surface. Les éléments structurels ont été modélisés par des éléments de poutre de Timoshenko en utilisant un modèle élastique linéaire. Les dimensions du modèle sont limités avec l'intention d'effectuer un test centrifugeuse géotechnique sur le modèle à l'avenir. La réponse de la pile et le sol ont été étudiés à trois degrés initial de saturation. Les résultats ont montré que l'interaction sol-pieu couplé n'est pas largement affectée par la gamme des degrés de saturation initial utilisé dans cette étude.

1 INTRODUCTION

A pile foundation is an integral part of many civil engineering structures such as highway bridges, high rise buildings, and towers. The behavior of the superstructure is influenced not only by the properties of the structural foundation, and soil but also by the interaction among these components. The soil-pile-superstructure is a complex system and the complexity is further increased when the soil-pile system is subjected to dynamic loads and/or the soil is in an unsaturated state. Better understanding of the dynamics of unsaturated soil and the coupling between the unsaturated soil and the pile are required to understand the unsaturated soil-pile interaction.

Unsaturated soils are a three phase porous material. The bulk phases consist of a solid, liquid, and gas. There are also three interfaces solid-liquid, solid-gas and liquid-gas. The mechanical behaviour of unsaturated soil is governed by the bulk phases, interfaces, and the interaction among the different bulk phases and interfaces. Among the three interfaces, the liquid-gas interface, also known as the contractile skin, plays a critical role. The contractile skin maintains the pressure balance between the air and liquid phases. The shape of the contractile skin (radius) changes with the amount of moisture in a given soil resulting in change in pressure difference between air and liquid phase. This pressure difference is referred to as matric suction. Matric suction is one of the two stress state variables widely used to describe the shear strength and volume change characteristics of unsaturated soils (Fredlund and Morgenstern, 1977). The shear strength of soil increases with matric suction. Therefore the change in soil strength due to the change in moisture content must be taken into account for proper analyses. The amount of water present in the soil, in the form of degree of saturation (DOS) or water content is directly related to matric suction via the use of the soil water characteristic curve (SWCC).

A typical analysis procedure for a pile foundation usually involves determining the free-field response for the project site with a design earthquake loading. The acceleration design response spectrum (ADRS) of the predicted free-field response is used to calculate the base shear and the bending moment for the pile. This does not take into account that the soil reaction is dependent on the pile movement and that the pile movement is dependent on the soil response. Thus, a fully coupled analysis is needed to understand these responses.

Various studies have been performed on the influence of unsaturated soils on pile behavior. The axial capacity of a pile was studied by Georgiadis et al. (2003), in which they determined that the ultimate pile load increases as the degree of saturation decreases. The analysis also showed an excessive settlement due to collapse exhibited by the unsaturated soil under the tip of the pile. This settlement could not be recognized with saturated finite element analyses (Georgiadis et al., 2003). Three dimensional finite element analysis was performed by Weaver and Grandi (2009), to assess the applicability of p-y curves that incorporate friction and apparent cohesion. Results show that the curves compare well and could be used in assessing lateral pile behavior in unsaturated soils. Due to the differences in saturated and unsaturated soils, the effect of unsaturated soils on the soil-pile behvaior should be investigated under various loading and environmental conditions.

Because of difficulties in conducting an advanced soil-structure interaction experiment under a controlled environment, numerical methods such as the finite element method can be used to obtain initial understanding on the unsaturated soil-pile interaction. The primary objective of this study is to improve the simplified finite element formulation by incorporating external damping at the governing equation level and to use the improved model to investigate the dynamic behavior of a steel pipe pile in unsaturated soil. In the modeling, the stress-strain behavior of the soil was modeled using an elastoplastic constitutive model for unsaturated soil. The structural elements (pile and superstructure) were modeled by Timoshenko beam elements using a linear elastic model. The dimensions of the model are constrained with the intention of performing a geotechnical centrifuge test on the model in the near future. The response of the pile and the soil with three different initial degrees of saturation are presented and compared.

2 SUMMARY OF COUPLED GOVERNING EQUATIONS FOR UNSATURATED SOILS

The governing equations of the dynamics of unsaturated soils are summarized in this section. The governing equations for the dynamics of unsaturated soils are derived using fundamental laws such as mass balance, momentum balance, energy balance and laws of thermodynamics. In the case of unsaturated soil that consists of three bulk phases, two independent mass balance equations and three momentum balance equations can be derived by considering the motion of a representative soil element. A detailed explanation of the governing equations and the formulations is in Ravichandran and Muraleetharan (2009).

2.1 Mass balance equation for the liquid phase:

The final form of the mass balance equation for the liquid phase is given in equation [1]. It should be noted that the mass balance equation for the solid phase is incorporated in this equation to eliminate the time derivative of the porosity of the liquid phase.

$$\left(\frac{\partial \eta^l}{\partial \varepsilon_v}\right)\dot{u}^s_{i,i} + \eta^l \dot{u}^l_{i,i} + \left(\frac{\eta^l}{\Gamma^l} - \frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^l + \left(\frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^g = 0 \qquad [1]$$

where u^s is the displacement of the solid phase, u^l is the displacement of the liquid phase, Γ^l is the bulk modulus of the liquid phase, ε_v is the volumetric stain, η^l is the volume fraction of the liquid phase given by $\eta^l = V^l / V^T$, V^l is the volume of liquid, V^T is the total volume p^l is the liquid pressure, p^g is the gas pressure and ψ is the matric suction given by $\psi = p^g - p^l$.

2.2 Mass balance equation for the gas phase:

Similar to the liquid phase, the mass balance for the gas phase can be expressed as:

$$\left(1 - \eta - \frac{\partial \eta^l}{\partial \varepsilon_v}\right)\dot{u}^s_{i,i} + \eta^g \dot{u}^g_{i,i} + \left(\frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^l + \left(\frac{\eta^g}{\Gamma^g} - \frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^g = 0 \qquad [2]$$

where u^g is the displacement of the gas phase and Γ^g is the bulk modulus of the gas phase, η^g is the volume fraction of the gas phase and η is the total porosity of the soil.

2.3 Linear momentum balance for the mixture:

$$\eta^s \rho^s \ddot{u}^s_i + \eta^l \rho^l \ddot{u}^l_i + \eta^g \rho^g \ddot{u}^g_i - \sigma_{ij,i} - \rho g_j = 0 \qquad [3]$$

2.4 Linear momentum balance for the liquid:

$$\rho^l \ddot{u}^l_j - \left(\hat{k}^l_{ij}\eta^l\right)\dot{u}^s_i + \left(\hat{k}^l_{ij}\eta^l\right)\dot{u}^l_i + \left(\delta_{ij} p^l\right)_{,i} - \rho^l g_j = 0 \qquad [4]$$

2.5 Linear momentum balance for the gas:

$$\rho^g \ddot{u}^g_j - \left(\hat{k}^g_{ij}\eta^g\right)\dot{u}^s_i + \left(\hat{k}^g_{ij}\eta^g\right)\dot{u}^g_i + \left(\delta_{ij} p^g\right)_{,i} - \rho^g g_j = 0 \qquad [5]$$

where σ_{ij} is the total stress tensor, g_j is the gravitational acceleration vector, $\hat{k}^l_{ij}$ is the inverted permeability tensor of the liquid phase (i.e., in 1-D $\hat{k} = 1/k$, where k = coefficient of permeability of liquid), $\hat{k}^g_{ij}$ is the inverted permeability tensor of the gas phase, and δ_{ij} is the Kronecker delta. These five equations (1-5) have five unknowns: solid displacement ($\mathbf{u}^\mathbf{s}$), liquid displacement ($\mathbf{u}^l$), gas displacement ($\mathbf{u}^g$), liquid pressure (p^l) and gas pressure (p^g). However, p^l and p^g in the momentum balance equations can be eliminated using the mass balance equations, thusly yielding a displacement formulation ($\mathbf{u^s} - \mathbf{u^l} - \mathbf{u^g}$) with solid, liquid and gas displacements as the primary unknowns. The corresponding finite element equations can be written in the following matrix form by considering the solid, liquid and gas displacements as the primary nodal unknowns

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K_p}\mathbf{u} + \mathbf{f_I} = \mathbf{f_E} \qquad [6]$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K_p}$ is the pore fluid stiffness matrix, $\mathbf{f_I}$ is the internal force vector, and $\mathbf{f_E}$ is the external force vector, $\mathbf{u}$ is the generalized displacement vector and $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the corresponding velocity and acceleration vectors that will include solid, liquid and gas components. In general, the finite element equation for the dynamics of saturated or unsaturated soil consists of two stiffness matrices: a pore fluid stiffness matrix ($\mathbf{K_p}$) and a solid stiffness matrix ($\mathbf{K}_s$). The solid stiffness matrix is usually written as an internal force vector ($\mathbf{f_I}$) as shown in equation 6 and given by $\mathbf{f_I} = \mathbf{K_s u^s}$. Due to numerical instability and lengthy computational times the use of the complete formulation ($\mathbf{u^s} - \mathbf{u^l} - \mathbf{u^g}$ formulation) is limited. Thus, a numerically stable formulation with less compromise in the actual physics of the problem must be developed for use by practicing engineers and researchers. One such formulation is the simplified finite element formulation. The simplified formulation is developed by neglecting the relative accelerations and velocities of the liquid and gas phases as shown in equations 7 through 9. Simulation of an unsaturated soil embankment showed that the simplified formulation is approximately 36 times more computationally efficient than the complete formulation. It is obvious that this computational efficiency is dependent on the problem. However, the number of nodal unknowns can give an idea of the computational time requirement of these two formulations. In 2D the complete formulation has 6 nodal unknowns and the simplified formulation has 2 nodal unknowns. Neglecting the relative velocities and accelerations will result in an undrained condition in each element. The dynamic problems of unsaturated soils can be approximated as an undrained problem due to the very low permeability of unsaturated soil as compared to the corresponding saturated state and short shaking period.

$$\rho \ddot{u}_j - \sigma_{ij,i} - \rho g_j = 0 \qquad [7]$$

$$\left(\eta^l + \frac{\partial \eta^l}{\partial \varepsilon_v}\right)\dot{u}_{i,i} + \left(\frac{\eta^l}{\Gamma^l} - \frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^l + \left(\frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^g = 0 \qquad [8]$$

$$\left(1 - \eta - \frac{\partial \eta^l}{\partial \varepsilon_v}\right)\dot{u}_{i,i} + \eta^g \dot{w}^g_{i,i} + \left(\frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^l + \left(\frac{\eta^g}{\Gamma^g} - \frac{\partial \eta^l}{\partial \psi}\right)\dot{p}^g = 0 \qquad [9]$$

Even though the relative movement of fluids is neglected, pore liquid and pore gas pressures can be computed using mass balance equations 8 and 9 for the purposes of considering the suction effect in unsaturated soil. In the simplified formulation, the degree of saturation is directly related to the volumetric deformation of the solid skeleton and not to the flow of fluids, as seen in equations 8 and 9. When the pore liquid and pore air pressure changes due to volumetric deformation of the solid skeleton, the degree of saturation changes, thusly altering the matric suction and unsaturated soil behavior.

In this case, only the momentum balance equation (equation 7) will be solved considering the solid displacement as the primary nodal unknown. The corresponding finite element equations for the simplified governing equations and boundary conditions are expressed in the matrix form below.

$$\mathbf{M}\ddot{\mathbf{u}}^s + \mathbf{K_p}\mathbf{u}^s + \mathbf{f_I} = \mathbf{f_E} \qquad [10]$$

From a comparison of the equations 6 and 10, it is apparent that the damping matrix does not appear naturally in the simplified formulation at the governing equation level. This limits the application of the simplified formulation for dynamic problems. One of the methods to improve this formulation is the incorporation of external viscous damping in the form of Rayleigh damping as explained in the next section.

3 THE INCOPORATION OF EXTERNAL DAMPING IN THE SIMPLIFIED FORMULATION

Since an external damping must be applied to the simplified formulation to obtain more reasonable results. The Rayleigh damping model was incorporated into the formulation.

In this model, the damping is considered propositional to both the mass and the stiffness of the system. The damping matrix for the finite element formulation is calculated using equation 11.

$$\mathbf{C^R} = \boldsymbol{\alpha}^\mathbf{R}\mathbf{M} + \boldsymbol{\beta}^\mathbf{R}\mathbf{K} \qquad [11]$$

Where $\mathbf{C^R}$ is Rayleigh damping matrix, $\mathbf{M}$ is mass matrix, $\mathbf{K}$ is the stiffness matrix, the α^R and β^R are mass and stiffness related Rayleigh damping coefficients, respectively. α^R and β^R are given by equations 13 and 14 respectively.

$$\alpha^R = \xi_{tar}\left(\frac{4\pi}{T}\right)\left(\frac{n}{n+1}\right) \qquad [12]$$

$$\beta^R = \frac{\xi_{tar} T}{\pi(n+1)} \qquad [13]$$

where ε_{tar} is the target damping, $\boldsymbol{n}$ is an odd integer (1, 3, 5 or 7) and $\boldsymbol{T}$ is the fundamental period of the soil deposit given by:

$$T = \frac{4H}{V_{s,avg}} \qquad [14]$$

where H is the depth of the soil deposit and $V_{s,avg}$ is the average shear wave velocity.

The spatially discrete governing equations for the improved-simplified formulation that includes Rayleigh damping can be written in matrix form as follows:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C^R}\dot{\mathbf{u}} + \mathbf{K_p}\mathbf{u} + \mathbf{f_I} = \mathbf{f_E} \qquad [15]$$

The finite element formulation was derived using four-node quadrilateral isoparametric elements. Solid skeleton displacements in x and y directions were considered as the nodal unknowns. The time integration was performed using Hilber-Hughes-Taylor α-method together with a predictor corrector algorithm proposed by Hughes and Pister (1978).

4 SOIL-PILE INTERACTION ANALYSIS

A PP14 x 0.375 pile was modeled with a service state load of 90 kips in unsaturated Minco silt. The finite element mesh for the model is shown in Figure 1. The base motion applied to the model is shown in Figure 2 (El Centro earthquake acceleration time history). This is a preliminary study for the centrifuge tests planned to be performed in the near future.

Through the preliminary study the placement of instruments for the pile and the soil can be determined for the centrifuge test. Displacement measurements for the pile can be used to select linear variable differential transducers (LVDTs) with the proper range. Also preliminary tests can be used to determine the best location to place the pile in the box.

In this preliminary test a dynamic test is modeled other preliminary tests will also be performed for static pile tests.

The stress strain behavior of the solid skeleton is modeled using an elastoplastic constitutive model for unsaturated soil based on the bounding surface concept. The bounding surface model was developed by Dafalias and Herrman (1986). This model was later modified by Muraleetharan and Nedunuri (1998) to incorporate the suction related behavior of unsaturated soils. The parameters in Table 1 for the material model are calibrated from laboratory tests on Minco silt (Vinayagam, 2002). The corresponding suction related parameters for each DOS are listed in Table 2. The in situ soil stresses were also calculated for the elastoplastic model, a lateral earth pressure coefficient of 0.5 was assumed.

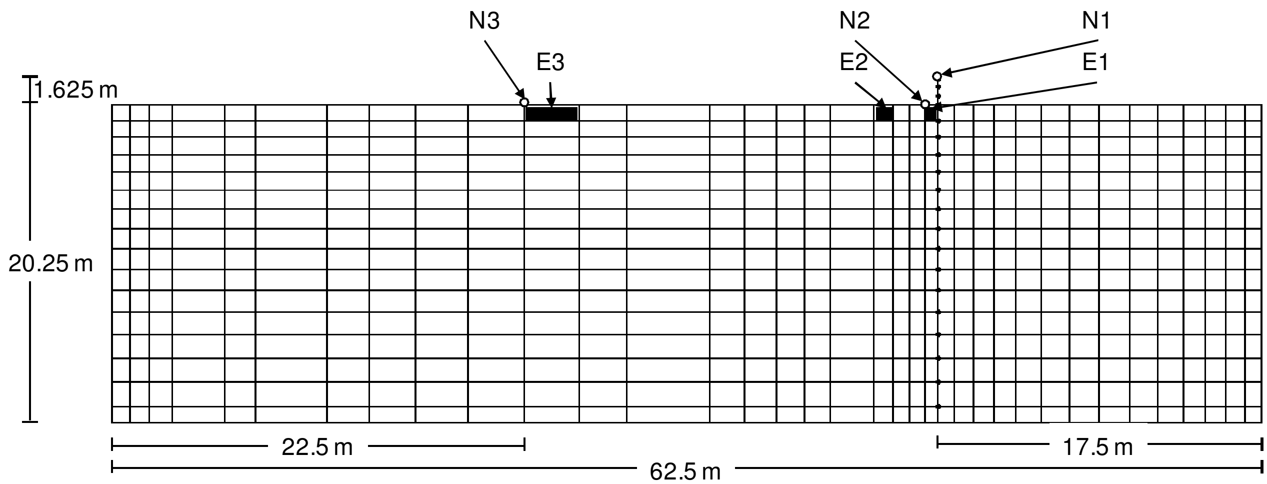

Figure 1: Finite Element Mesh, with Location of Nodes and Elements Discussed

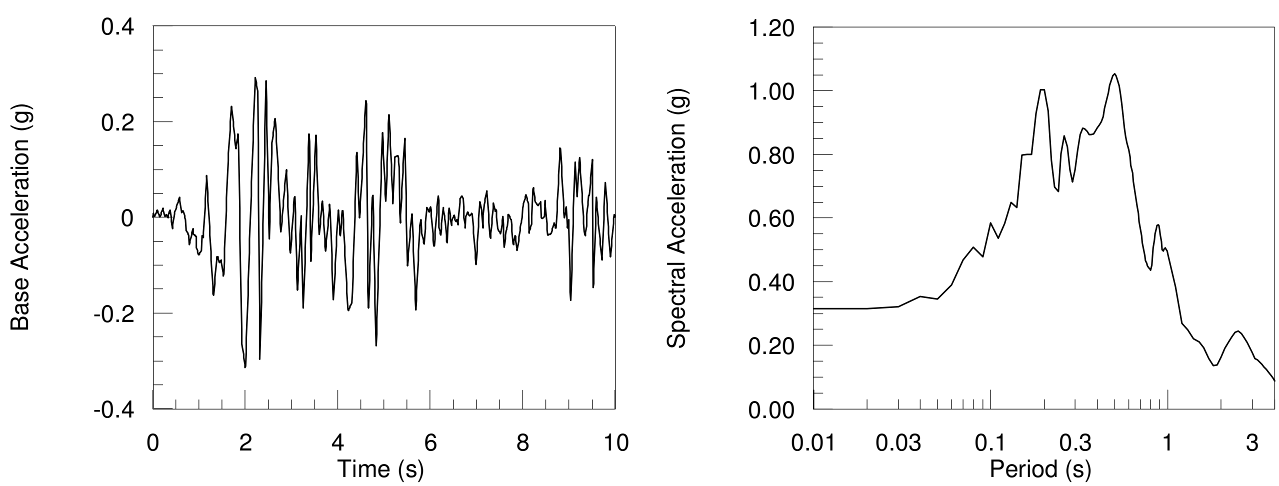

Figure 2: Time History of Applied Base Motion and Spectral Acceleration of Applied Base Motion

The SWCC proposed by van Genuchten (1980), was used to model the relationship between DOS and suction. The parameters used for Minco silt are listed in Table 3.

The unsaturated soil is represented by the equations described before and the pile is represented by Timoshenko beam theory. The pile is modeled with three components the concentrated mass on top of the pile (service state load), the pier (portion of the pile above the surface of the soil layer), the foundation (portion of the pile in the soil layer). These structural elements are assumed to behave elastically. The structural properties and parameters are listed in Table 4. The mass on top of the pile is modeled with a single element of very high density. The pile is modeled with nodes connected to the solid skeleton nodes, forcing the pile and soil to move together. For example there are no special interface elements between the soil and the pile to capture the opening and closing of gaps or relative movement in the vertical direction.

The Rayleigh damping coefficients for the three DOS of the soil and the three components of the pile are listed in Table 5. ξ_{tar} and ***n*** were not calibrated and were assumed to be 5% and 5, respectively. These are recommended values for site response analysis using nonlinear site response analysis tools (Park and Hashash, 2009). The predicted responses using the model are discussed in the next section.

Table 1. Bounding Surface Based Elastoplastic Model Parameters for Minco Silt

Parameter	Value
Slope of the isotropic consolidation line on e - λn p' plot, λ	0.02
Slope of an elastic rebound line on e - λn p' plot, κ	0.002
Slope of the critical state line in q - p' space , M_c (compression)	1.00
Ratio of extension to compression value of M (M_e / M_c)	1.00
Value of parameter defining the ellipse1 in compression (R_C)	2.60
Value of parameter defining the hyperbola in compression (A_C)	0.10
Parameter defining the ellipse 2 (tension zone) (T)	0.05
Projection center parameter (C)	0.00
Elastic nucleus parameter (S)	1.00
Ratio of triaxial extension to compression value of R (R_e / R_c)	1.00
Ratio of triaxial extension to compression value of A (A_e / A_c)	1.00
Hardening parameter (m)	0.02
Shape hardening parameter in triaxial compression (h_c)	2.00
Ratio of triaxial extension to compression value of h (h_e/h_c)	1.00
Hardening parameter on I-Axis (h_o)	2.00

Table 2. Suction Related Parameters

Parameter	DOS 58%	DOS 43%	DOS 28%
μ	50	80	140
B	0.12	0.12	0.12
N	1.526	1.66	2.017
A	0.27	0.27	0.27
r	1.57	1.57	1.57
β	0.0133	0.0133	0.0133

Table 3. van Genuchten SWCC Parameters

Parameter	Value
a	0.172
n	1.5
m	0.333
Irreducible Saturation	0.001

Table 4. Properties of Structure

Parameter	Value
Mass on top of the pier (Mg)	40.81
Cross sectional area of the pile and pier (m)	0.010356
Cross sectional area of mass (m)	0.099355
Length of pile (m)	19.25
Length of pier (m)	1.625
1st beam moment of inertia of pile and pier (m^4)	1.55×10^{-4}
1st beam moment of inertia of mass (m^4)	7.85×10^{-4}
Young's modulus (GPa)	200
Poisson's Ratio	.32
Density of pile and pier (Mg/m^3)	7.850
Density of mass on top of pier (Mg/m^3)	821.522

Table 5: Rayleigh Damping Coefficients

Parameter	α	β
Mass on top of the pile	79.13424	0.0000176
Pile	21.02706	0.0000661
Pier	359.7963	0.0000039
Soil DOS 28 %	1.118209	0.0012420
Soil DOS 43 %	1.093451	0.0012700
Soil DOS 58%	1.070267	0.0012980

5 RESULTS AND DISCUSSION

Initial degrees of saturation of 28%, 43%, and 58% were used to investigate the effect of DOS on the coupled performance of piles. The responses at nodes N1, N2, and N3 and elements E1, E2, and E3 are discussed.

The horizontal displacement histories for nodes N1, N2, and N3 are shown in Figure 3 with a DOS of 43%. The displacement at the end of the time history was greatest for the node on the soil near the pile. Figure 3 also shows the horizontal displacement history for node N1 for the three degrees of saturation. The results show that the horizontal displacement of the top of the pile is not largely affected by the range of initial degrees of saturation used in this study during initial shaking and begins to show differences after about six seconds into the shaking for the problems analysed. Though the differences are minor, the DOS with the greatest displacement is the DOS of 28%, a complete understanding of the effect of the force, stiffness and cyclic load is needed to further understand this result.

The acceleration response spectrums for the three nodes for a DOS of 43% are shown in Figure 4. The stiffer pile has a smaller spectral acceleration compared to the less stiff soil. The soil response at node N3 has the greatest spectral acceleration. The acceleration response spectrums for node N1 for the three degrees of saturation do not have a significant difference.

The incremental change of the degree of saturation for the three elements with a DOS of 43% is shown in Figure 5. Since the pile is modeled with nodes connected to the solid skeleton nodes, this prevented the element from decreasing in area (volumetric strain) like elements E2 and E3. The response actually caused element E1 to increase in area, due to no flow in the simplified formulation the volume of the water remains constant and the total volume increased resulting in the volume of the voids increasing and the degree of saturation decreasing. Figure 5 also shows the incremental suction at element E3 for all three degrees of saturation. As DOS increases the incremental change in degree of saturation also increases.

The incremental matric suction for the three elements with a DOS of 43% is shown in Figure 6. Element E1 is once again being influenced by the nodes connected to the pile. Figure 6 also shows the incremental suction at element 3 for all three degrees of saturation. As the DOS decreases the incremental matric suction decreases increasingly.

From the results of Figures 5 and 6 and the SWCC (semi-log relationship with suction plotted on the logarithmic scale) it is reasonable that a small increase in DOS can result in a large decrease in matric suction, for degrees of saturation that are initially low. It is also reasonable that a larger increase in DOS can result in a smaller decrease in matric suction, for degrees of saturation that are initially high.

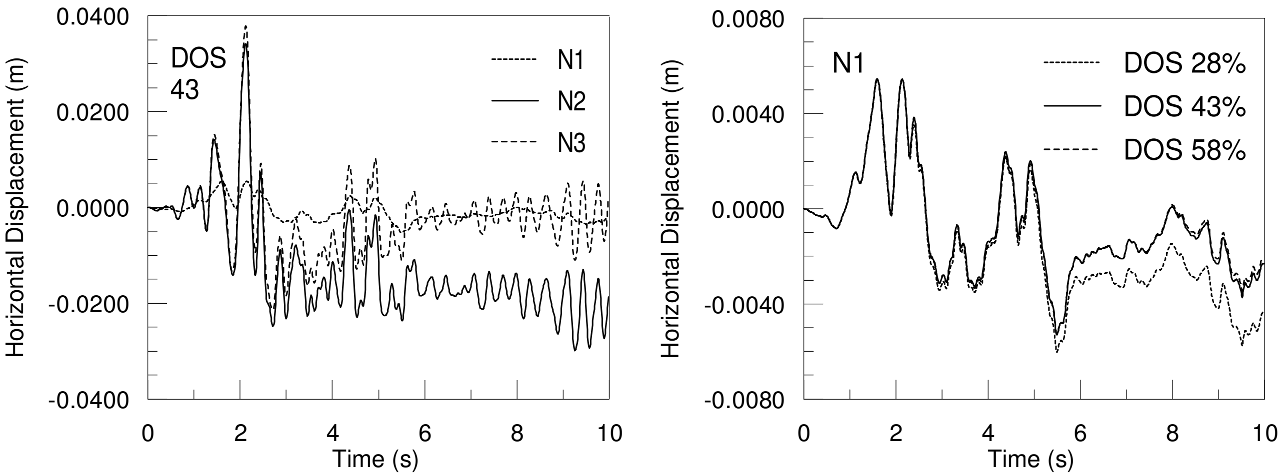

Figure 3: Comparison of Horizontal Displacement Histories for Different Nodes and DOS

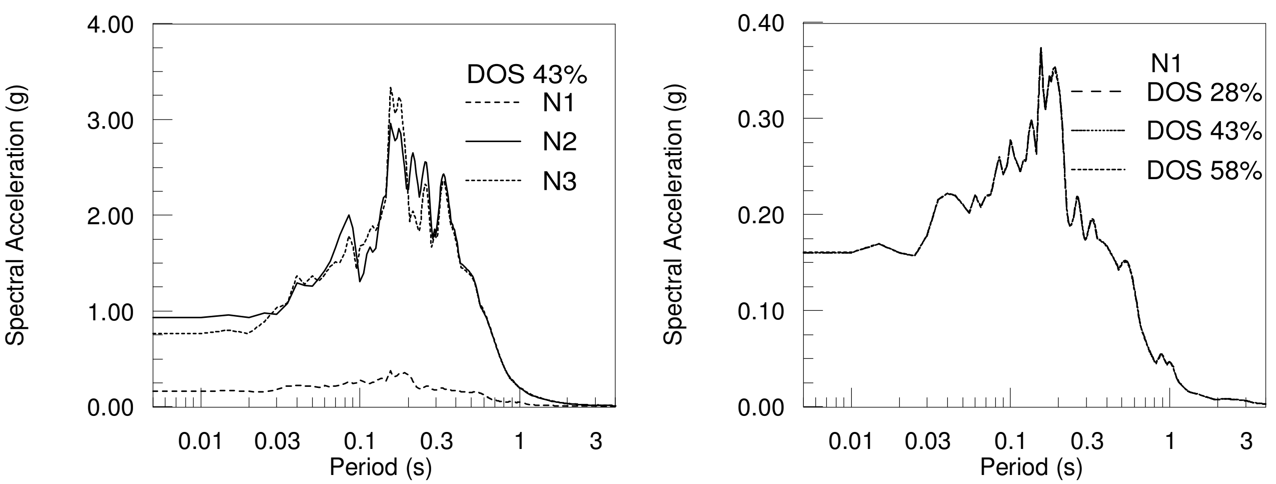

Figure 4: Acceleration Response Spectrums for Different Nodes and DOS

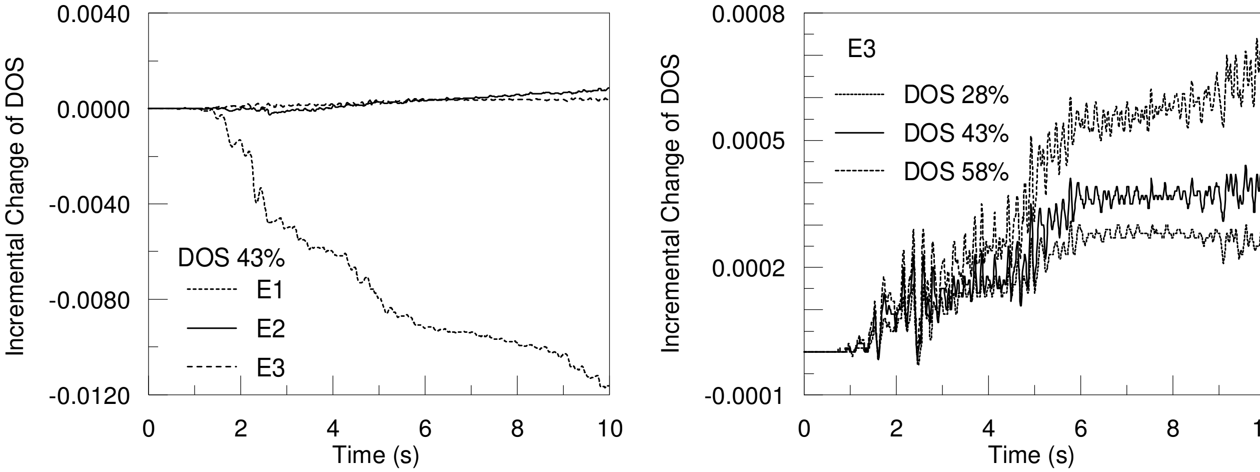

Figure 5: Incremental Change of DOS for Different Nodes and DOS

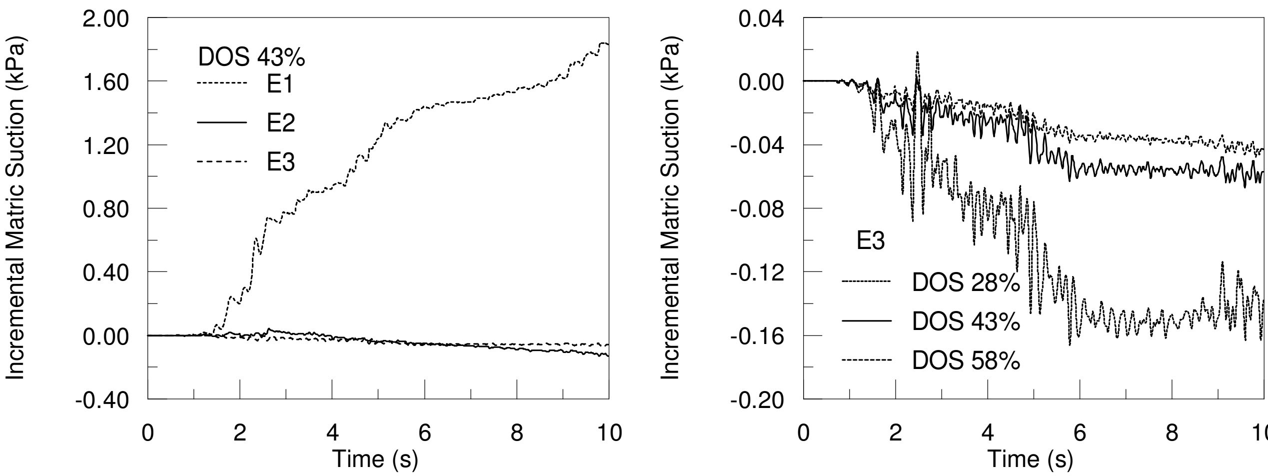

Figure 7: Incremental Change of Matric Suction for Different Nodes and DOS

6 CONCLUSION

The soil-pile system is analysed in a coupled manner using a simplified finite element formulation. The simulation results show that the free field response that is typically used in the design of piles is significantly different from the response very close to the pile. Also, the initial degree of saturation seems to have insignificant influence on the displacement response of the pile during the initial shaking and starts to influence the horizontal displacement after about six seconds into the shaking for the problems analysed. The proposed finite element model needs to be validated with centrifuge test results and the validated model calibrated before it is used for further study to gain a better understanding of soil-pile interaction in unsaturated soils with earthquake loading. With the current soil profile and degrees of saturation used, the measured horizontal displacement differences would be insignificant to a structural engineer, but the trend of the differences after six seconds must be investigated with wider ranges of saturation and profiles to determine if it is necessary for engineers to consider the effects of unsaturated soil structure interaction.

The improved simplified finite element model, which incorporates the Rayleigh damping model into the formulation is a useful tool that is simple enough to be used by practicing engineers effectively not only for understanding the effect of degree of saturation on the soil and structures but also the interaction between soil and structures in a coupled manner.

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