Analytical method to calculate lateral pressures on-, and mechanic elements of- shaft linings

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ABSTRACT
From several years ago, the method proposed by Prater (1977) to calculate the lateral pressures acting over shaft linings has been in use. Unfortunately, some algebraic errors have been found in his analytical development. Every step of that development was checked and corrected if errors were found, to find the equations that describe the pressure distribution in accordance to the suppositions made by Prater in his model. The procedure was combined with an analytical method proposed to study the soil-tunnel interaction problem (Romo, 1984). This method includes the phenomenon of interaction lining-soil. The combination of the two methods is made under the hypothesis that the mechanical elements of the shaft lining can be calculated with the analytical solution of the problem of interaction soil-lining of the tunnel, considering the pressures calculated with Prater's (modified) model.

RESUMEN
Desde hace varios años se ha utilizado el método propuesto por Prater (1977) para el cálculo de las presiones laterales sobre la superficie de lumbreras. Sin embargo, se han encontrado algunos errores algebraicos en su desarrollo analítico. Se procedió a revisar cada paso de dicho desarrollo y a corregir los errores encontrados, para así llegar a las ecuaciones que describen la distribución de presiones de las suposiciones hechas por Prater en su modelo. Se combinó el procedimiento con un método analítico que permite calcular los elementos mecánicos del recubrimiento de túneles (Romo, 1984). Este método incluye el fenómeno de interacción revestimiento-suelo. La combinación de los dos métodos se hace bajo la hipótesis de que los elementos mecánicos del revestimiento de la lumbrera se pueden calcular con la solución analítica del problema de interacción suelo-revestimiento del túnel, considerando las presiones calculadas con el modelo (modificado) de Prater.

1 INTRODUCTION

Prater's method (1977) for determining the pressure distribution acting over shaft linings has been used for a long time in engineering practice. Its use is very convenient because it requires little information about the soil and the lining to get to useful results for design purposes. In his publication, Prater assumed a Mohr-Coulomb failure criterion, with a conic failure surface around the lining. Although Figures presented in his article about the behavior of the different parameters involved in the use of the method are consistent with the analytical procedure that he proposes, various errors were found in the final formulae of the procedure. As pointed out later, the graphics presented in the original paper are correct, but equations are incorrect. From Prater's assumptions, the analytical procedure was repeated searching to fix the errors. The corrected equations are presented in this paper.

A method to compute the mechanical elements in the lining is also presented. It was put together with Prater's to calculate stresses and bending moments in shaft linings. The method has the particularity of considering of the relative stiffnesses of the soil and the lining.

A calculation methodology based in the combination of both methods to obtain lateral pressures and mechanical elements in the shaft lining is also presented in this paper.

2 CALCULATION OF THE PRESSURE BY PRATER'S METHOD (CORRECTED)

Based in Berezantzev's (1958) results about the application of the limit equilibrium method with plane strain conditions, Prater (1977) proposed a conic-shaped failure surface since it approximates to Berezantzev's results (op. cit). Once the conic surface was proposed, Prater calculated the pressure exerted over the shaft assuming a Mohr-Coulomb failure criterion, considering just purely frictional materials. The inclination angle of the failure surface proposed by Prater, α (see figure 1), as it happens in slopes, it is strongly related with the friction angle φ of the soil surrounding the shaft.

2.1 About the corrections made

It is important to point out that despite the errors in Prater's equations, the graphics presented in diverse references, both in international magazines and in textbooks, are correct. The correction made in this paper is in equations that generate the above mentioned graphics. With an error in equations, it was impossible to automate the calculations of Prater's method. Nevertheless, those designs made with the graphics presented originally by Prater are free of errors conforming to this theory.

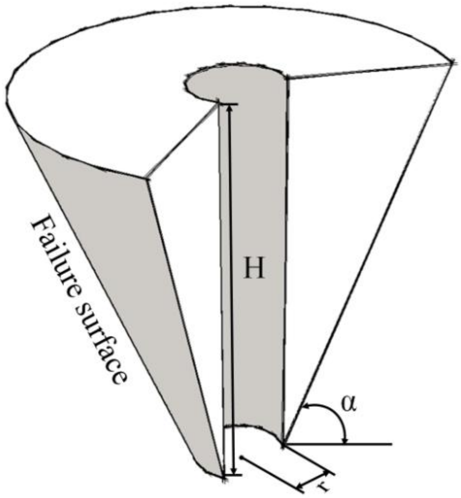

Figure 1. Faiulure surface around the shaft proposed by Prater (1977)

2.2 Calculation algorithm

The method for calculating the pressure distribution consists in as series of calculations along the extension of the shaft, all of them iterative, to define the variable inclination α of the failure surface through the shaft depth. The complete shaft depth h is traversed varying it from top to bottom.

To calculate the pressure distribution over the shaft lining, the following steps have to be taken:

First, β is computed with equation (1). The coefficient β is a fraction of the friction angle that is inside the limits of -φ and φ, for the active and passive pressure conditions respectively.

$$\beta = -\phi \tag{1}$$

In practice, it is usual to first excavate portions of the shaft and then the linings are placed in position, what lets the soil expand and act over the lining and not the opposite. That practical case guarantees the existence of active pressure, expressed in equation (1) in terms of coefficient β.

Next the coefficient of soil pressure is calculated with equation (2).

$$\lambda = 1 - \sin\phi \tag{2}$$

Before continuing the calculations, a resolution dh of the analytical model, a differential of the total depth H, must be defined in distance units. This resolution is the principal responsible of the accuracy of the calculated pressure distribution. In his paper, Prater did not mention any recommendation about this parameter highly influential in the results of the model. In parametric analysis made about Prater's model, it was found that using dh = 0.1m or less is a good choice for the analysis. Values of dh much bigger than 0.1m produce pressure distributions with maximum pressures that are a lot smaller than those produced by a dh=0.1 m, moving away from the safer side. For values smaller than dh=0.1m, when dh tends to 0, there is no significant change in the values of the maximum pressure nor in the distribution generated by the model.

The quantity of calculations to be made depends of dh because H/dh is the total number of calculation steps used in the method.

For the first depth h=dh, the value of n is computed with equation (3), in which r is the shaft radius and h is the depth of actual calculations.

$$n = \frac{r}{h} \tag{3}$$

Then, with equations (4) and (5), the failure surface inclination α must be found.

$$n = \frac{1}{3y\tan\alpha} * \left[\sin 2(\alpha+\beta) - 2\lambda\tan\alpha\cos^2(\alpha+\beta) - y\right] \tag{4}$$

In which:

$$y = \sin 2\alpha - \sin 2(\alpha+\beta) \tag{5}$$

It is necessary to propose an α such that the value of n, computed with equation (4) is equal to the one calculated with equation (3).

The graphic in Figure 2 was made approximating n to infinity, condition that exists near the surface. It is clear in the graphic that the values of α and φ are lineally related for the condition mentioned before. With that graphic, or with equation (6), the initial value of α for the first approximation of the first depth h=dh can be obtained and have a quicker convergence of the iterative method.

$$\alpha = \frac{\phi}{2} + 45 \tag{6}$$

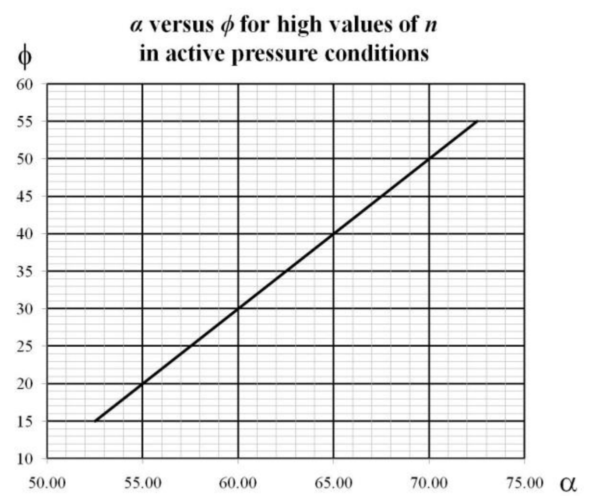

Figure 2. Lineal relationship between *α* and φ for *n*→ ∞.

For the active pressure condition, the value of α will be slightly bigger than the one obtained with equation (6). In parametric analysis and in the example of this paper, the Newton-Raphson method was applied in equations (4) and (5) to speed up the model's convergence.

Then the coefficient of horizontal pressure k_r is calculated with the next expression:

$$k_r = \frac{1}{n \tan\alpha}\left[\tan(\alpha+\beta)\left(\frac{1}{3\tan\alpha}+n\right)-\frac{\lambda}{3}\right] \quad (7)$$

Aftewards, the accumulated horizontal pressure E is calculated as a fraction of the vertical soil pressure using it's volumetric weight γ.

$$E = \frac{k_r \gamma h^2}{2} \quad (8)$$

Then, the pressure at depth h can be computed with equation (9).

$$P_i = \frac{E_i - E_{i-1}}{dh} \quad (9)$$

For the first depth (h=dh), E_{i-1} is equal to 0. Subsequently, the depth of calculation is increased ($h_i=h_{i-1}$+dh), and the steps that include equations (3) to (9) are followed. Beginning with the second depth, the inclination of the failure surface α used to start the iterations in equations (3) and (4) is the value of α obtained in the previous depth.

2.3 Model's behavior and design considerations

To illustrate the model's behavior, an example is presented in which: γ=2 t/m^3, φ=30º, r=2 m and H=20 m.

Since the shaft is embedded in a homogeneous soil stratum, the values of β and λ are constant: β=-30º and λ=0.5. In Figure 3 the distributions of the parameters that change with depth can be seen.

In Figure 3, it can be observed that the inclination of the failure surface α increases with depth, while the pressure coefficient k_r decreases. In the pressure distribution a maximum pressure P_{max} can be noticed at h_{pmax} depth. In this example P_{max}=2.65 t/m^2 y h_{pmax}=8.4 m. The accumulated horizontal pressure E increases up to h_{cr}, the depth where the soil pressure becomes null according to the analytical model, in this case h_{cr}=17.18m.

There are cases where h_{cr} will not be present because P_{max} will move closer to the bottom of the shaft. In other cases, h_{pmax} will be equal to the total depth of the shaft. These behaviors are a consequence of the periodical nature of the equations of the model and of the fact that such period is variable. Also, the location of P_{max}, h_{cr}, does not depend on the soil's volumetric weight but it is strongly influenced by the friction angle φ, while the maximum pressure depends on γ.

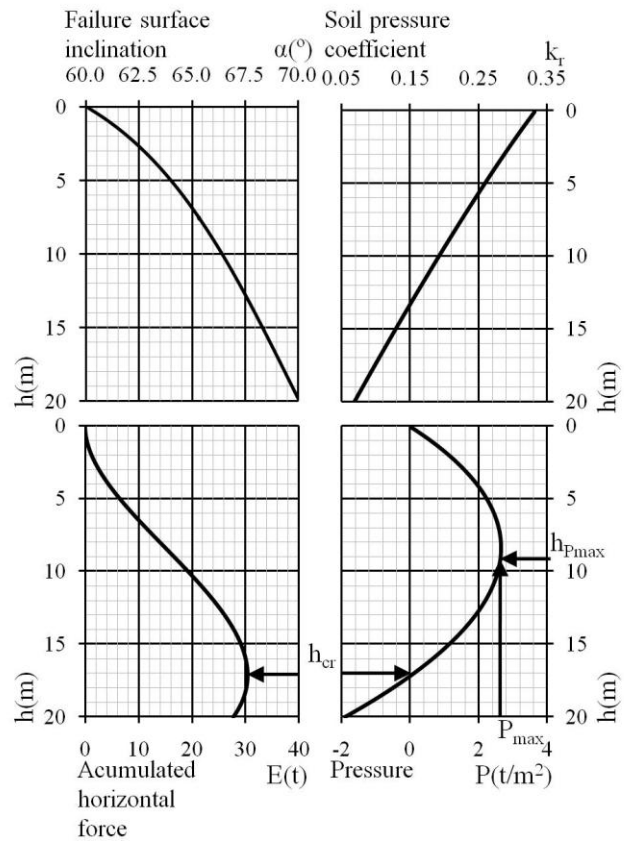

Figure 3. Variation of different parameters needed to obtain the pressure distribution in the example problem.

It is important to mention that, although the method can predict very low or even negative pressures near the bottom of the shaft, it is recommended to use the maximum pressure P_{max} for design purposes.

Regarding the value of the maximum pressure calculated by Prater's method, there are comparative studies between different analytical methods published in the international technical literature (*e.g.*, Tobar y Meguid, 2010). They show that Prater's calculations yield values that are larger than the ones obtained with the methods by Berezantzev (1958) and Terzaghi (1943), being the last one of the most used. Nevertheless, the maximum pressure computed by Prater is approximately 30% less than the maximum value calculated in Cheng and Hu's method (2005). Prater's method is a modification of Berezantzev's (1958), which uses a variable coefficient λ in place of an assumed value of 1, as is the case in Berezantzev's method. In this manner, Cheng and Hu obtained a range of values that theoretically can take the lateral pressures acting over a shaft, being the lower limit the one that corresponds to λ = 1 (Berezantzev's case), and the upper limit the value obtained with λ=k_0. Given that Prater's method, as Cheng and Hu's, is also obtained from Berezantzev's, the results obtained are in the same order of magnitude that can be obtained with the simplified methods available at the moment. It would be interesting to make a numerical-experimental investigation to test the effectiveness of these analytical methods.

3 CALCULATION OF STRESSES IN THE SHAFT LINING

Once the pressure distribution is obtained with Prater's method, the stresses acting in the shaft lining (Romo, 1984) can be computed.

This method considers explicitly the relative stiffnesses of the lining and the soil mass. Moreover, it takes into account the potential sliding between the lining and the soil mass that surrounds it, and the way the loads are applied over the shaft, according to different constructive methods. Taking into account these interactions, one can find a stress distribution that will vary from the one that is obtained with methods that suppose totally rigid supports and the ones that suppose totally flexible supports.

In researches on the behavior of tunnel linings, Peck (1969) demonstrated that a support idealized as totally flexible in an anisotropic stress field will deform until the acting stresses in the lining are uniform, *i.e.*, there will be no bending moments (see Figure 4a). Meanwhile, a rigid lining will not change its shape and consequently, the original anisotropic stress field is preserved. Accordingly, the lining is subject to bending moments (See figure 4b). Given that there are no essential differences between the tunnel and shaft linings along their cross section, the concepts developed by Peck can be extrapolated to the shaft problem.

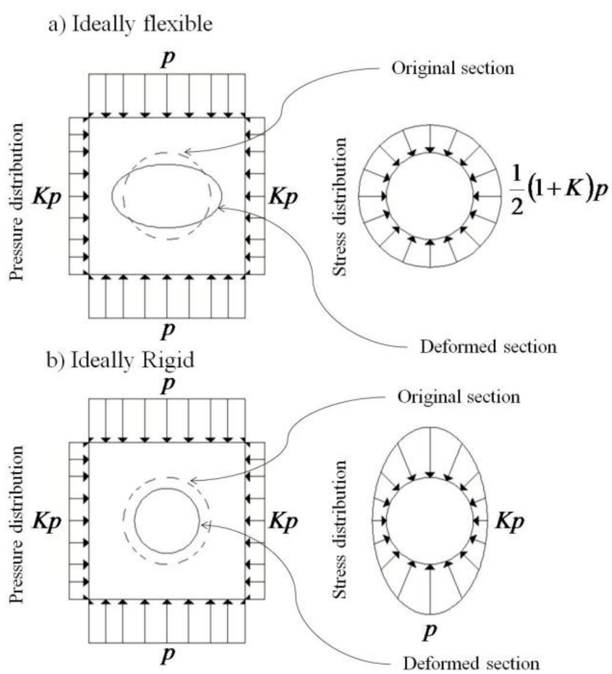

Figure 4. Stress distribution over ideally rigid and ideally flexible linings subjected to anisotropic stress fields.

3.1 Method's considerations

Even though it was developed originally to calculate acting stresses over tunnel linings, the similarity between the two geotechnical system, tunnel and shaft, at practical and theoretical levels, allows its application to the case of shafts.

The mass of soil is considered elastic, isotropic, homogeneous and infinite, subjected to a initial load equal to the stress in the soil at the central line of the shaft. The shaft lining is treated as an elastic shell in which both the circumferential and the flexion strains are taken into account. It is considered that the strains occur only in perpendicular planes to the shaft axis, in planar sections.

The method makes two distinctions of possible processes of load application, intimately related to the different constructive methods of tunnels. To adapt the tunnel model to be a shaft one, it will be necessary to analyze them. The load can be external, meaning that the tunnel has been excavated and lined before the loads corresponding to the free-field conditions are applied. This condition is similar to the one occurring in the construction of shallow sewage tunnels, where a box is first excavated to put in place the pipe that is then loaded with the compacted filling soil, and in tunnels exposed to loads caused by explosions (see Figure 5a). The other condition supposes that the tunnel is excavated and supported after the loads corresponding to the free-filed stresses have been applied (see Figure 5b).

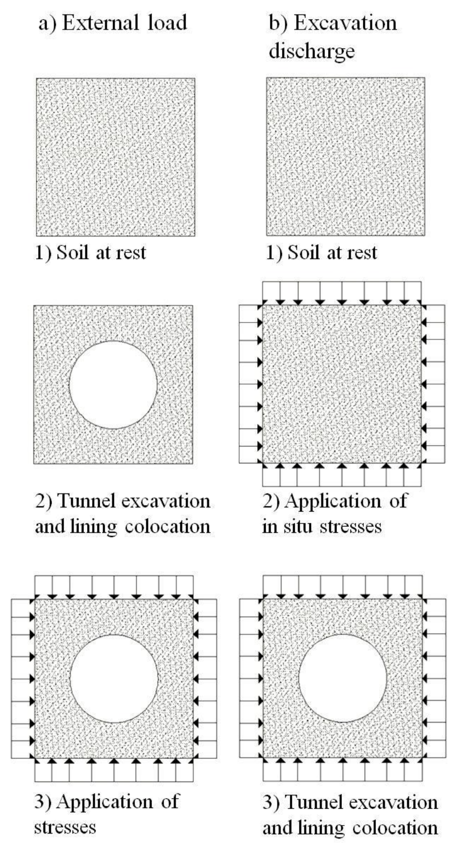

Figure 5. External load and excavation discharge processes.

It is evident that the condition of load by excavation is the one that is the one alike to the most usual methods in shaft construction; therefore it will be the condition that will be adapted for the calculation of stresses in shaft linings.

On the other hand, the method includes two conditions of sliding between the soil and the lining: one in which there is complete transmission of the shearing stresses, without relative movement between the support and the soil, and other in which relative displacements between the two are allowed, *i.e.*, without transmission of shear stresses. Of these two conditions, the one that is the closest to Prater's model is the one that does not consider shear stresses at the soil – shaft interface.

3.2 Dimensionless factors of stiffness

The system's stiffness is given by two kinds of stiffnesses. One is the longitudinal strain stiffness, which is the measure of uniform pressure acting over the lining required to cause a diametric unitary strain in the lining without changing its original shape. The second one involves the bending stiffness and is a measure of the magnitude of the non-uniform pressure surrounding the lining required to make occur a diametric unitary strain that results in a change of the initial geometric shape of the cross section, in other words: it takes the shape of an oval.

The Compressibility factor, C*, is a measure of the stiffness to extension of the medium relative to the one of the lining. It can be calculated with equation (10), where E is the elasticity modulus and v is the Poisson's ratio, both of the soil, while E_s, A_s, v_s and R are the elasticity modulus, the sectional area and the Poisson's ratio of the lining section, respectively.

$$C^* = \frac{ER(1-\nu_s^2)}{E_s A_s (1-\nu^2)} \qquad (10)$$

An analogous factor F*, that takes into account the relative bending stiffnesses of the soil-lining system. It can be obtained with the following expression:

$$F^* = \frac{ER^3(1-\nu_s^2)}{E_s I_s (1-\nu^2)} \qquad (11)$$

F* will depend largely on the construction system. The joints between the arch lining-segments will determine, in great measure, the bending stiffness of a cross section. In general, F* will be greater than 10, even for the most rigid linings.

These factors will serve to consider the whole system stiffness and thus determine the stresses to which it will be subject.

3.3 Equations to compute the stresses

For the calculation of the distinct acting stresses, the following equations are used. As mentioned before, equations are applied for the condition of relative displacements at the soil-shaft interface.

The following are equations proposed originally by Romo (1984) for the calculation of the mechanical elements, referred to Figure 6.

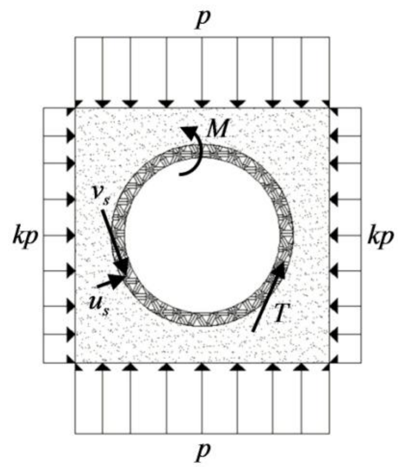

Figure 6. Notation for Romo's method.

Axial force, T:

$$\frac{T}{pR} = \frac{1}{2}(1+k)(1-a_o^*) + \frac{1}{2}(1-k)(1-2a_2^*)\cos 2\theta \qquad (12)$$

Bending moment, M:

$$\frac{M}{pR^2} = \frac{1}{2}(1-k)(1-2a_2^*)\cos 2\theta \qquad (13)$$

Radial displacement of the support, u_s:

$$\frac{u_s E_s}{pR(1+\nu_s)} = \frac{1}{2}(1+k)a_o^* - (1-k)* \left[(5-6\nu_s)a_2^* - (1-\nu_s)\right]\cos 2\theta \qquad (14)$$

Tangential displacement of the support, v_s:

$$\frac{v_s E_s}{pR(1+\nu_s)} = \frac{1}{2}(1-k)* \left[(5-6\nu_s)a_2^* - (1-\nu_s)\right]\sin 2\theta \qquad (15)$$

In which:

$$a_o^* = \frac{C^* F^* (1-\nu_s)}{C^* + F^* + C^* F^* (1-\nu_s)} \qquad (16)$$

And:

$$a_2^* = \frac{(F^* + 6)(1-\nu_s)}{2F^*(1-\nu_s) + 6(5-6\nu_s)} \quad (17)$$

The coefficient k that appears in equations (12) to (17) corresponds to the ratio between the horizontal and vertical stresses. For the case of tunnels in homogeneous and isotropic materials, the value of k is constant through the tunnel's axis and its value is different than 1 for geomaterials. Nevertheless, for the shaft case this condition is different, as commented in section 3.4.

3.4 Adaptation to the shaft problem

The most significant change between the conditions found in a tunnel and in a shaft is the orientation of the axis of the orifice in respect of the horizontal and vertical static stresses.

In Romo's model, the factor k is the coefficient of horizontal pressure, that represents the relationship kept between the horizontal and the vertical stresses, and it is employed to define the anisotropic pressure field. This pressure difference gives origin to bending moments that act in the cross section of the tunnel.

In the other hand, when orienting the axis of the tunnel in vertical direction, an isotropic pressure field is generated in the plane of the cross section of the shaft. This happens because the vertical stress acts in the same line of the shaft axis, thus the horizontal stress is the only one acting on the lining cross section.

As consequence of the state of isotropic pressures, the value of k that appears in equations (10) to (15) is equal to 1 for the static case. This implies that bending moments acting in the plane of the cross section of the shaft are not generated, nor the tangential displacements of the support.

Equations for the static case of the shaft will be:

Axial force, T:

$$\frac{T}{pR} = (1 - a_o^*) + (1 - 2a_2^*)\cos 2\theta \quad (18)$$

Radial displacement of the support, u_s:

$$\frac{u_s E_s}{pR(1+\nu_s)} = a_o^* \quad (19)$$

3.5 Seismic considerations

The exposed method can also be used for the seismic case, in an approximate way with the use of seismic coefficients. Under the action of the seismic load, the value of the coefficient k in shafts will be different from 1 since, for the seismic condition, it will represent the ratio between the orthogonal components of the earthquake acting in the plane of the cross section of the shaft. According to the building regulations of Mexico's Federal District (2004), the ratio between horizontal orthogonal components will be considered equal to 0.30.

4 CONCLUSIONS

With the corrections made to Prater's method, it is possible to make software to speed up the calculation of the pressure distribution acting over shafts.

The presented methodology is simple to apply, because, in addition of its accessible algebraic structure, it requires few input parameters to obtain both the lateral pressure distribution and the mechanical elements in shaft linings.

Even as Prater's method has proven to yield values inside the order of magnitude obtained with the simplified analytical methods that are available at the moment, and the method of tunnel lining analysis (Romo, 1984) has been shallowly compared with some field measurements, it is justifiable to make experimental and numerical researches to find the approximation of the methodology here presented against the reality. Even more, having field measurements would be an excellent way of comparing the results obtained with the proposed methodology.

5 REFERENCES

Berezantzev V.G. (1958). "Earth pressure on the cylindrical retainin walls". *Brussels conference on earth pressure problems* II pp 21-27

Cheng Y.M, Hu Y.Y. (2005). "Active earth pressure on circular shaft lining obtained by simplified slip line solution with general tangential stress coefficient", *Chinese Journal of Geotechnical Engineering,* 27(1), 110-115

Peck R.B. (1969). "State-of-the-Art Report: Deep Excavations and Tunneling in Soft Ground. Proceedings, Seventh International Conference on Soil Mechanics, México. Pp 225-290

Prater E.G. (1977). "An examination of some theories of Herat pressure on shaft linings", *Canadian Geotechnical Journal, 14(1), 91-106.*

Reglamento de Construcciones del Distrito Federal (2004). Normas Técnicas Complementarias para diseño por sismo. Gobierno del Distrito Federal.

Romo M P. (1984). "Diseño del recubrimiento de túneles", Memorias de la XII Reunión Nacional de Mecánica de Suelos, Querétaro, Qro. Nov 14-18

Tobar T., Seguid M. (2010). "Comparative evaluation of methods to determine the Herat pressure distribution on cylindrical shafts: A review", *Tunneling and Underground Space Technology*, 25: 188-197

Terzaghi K. (1943). Theoretical Soil Mechanics. John Wiley & Sons, New York.