Reliability of shafts constructed by the flotation technique

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ABSTRACT
In this paper a reliability analysis of shaft constructed using the flotation technique is presented. Ultimate states are evaluated, such as: nucleus stability, excavation stability, bottom uplift failure and floating of finished structure. In order to assess the reliability of the flotation construction system, the point estimate method of Rosenblueth is used. Finally some conclusions are presented.

RESUMEN
En este artículo se presenta un análisis de confiabilidad de lumbreras construidas por el método de flotación en el que se evalúan los principales estados límites de falla, tales como: falla del núcleo central, falla general por cortante, falla de fondo por subpresión y falla por flotación. Se evalúa la confiabilidad de este sistema constructivo mediante el método de estimación puntual de Rosenblueth. Finalmente se dan unas conclusiones respecto a lumbreras flotadas.

1 INTRODUCTION

Shafts are vertical or inclined accesses that enable all the auxiliary operations in the construction of a tunnel. Shafts can be subdivided into two groups depending on the material in which they are built: 1) shafts built in soft clay and soft silts 2) shafts built in firm soil or rock. This paper deals with shafts of the first type. In 1969, engineers Jorge Cravioto and Abel Villareal patented a method for constructing shafts by the flotation technique in soft soil. This ingenious invention was designed to control potential failure mechanisms that affect deep excavations in very soft soils including the possibility of failure by extrusion in the joints of cast in place wall panels, collapse of the excavation's walls as well as shear and uplift failure of the excavation´s bottom.

2 GENERAL CONSTRUCTION PROCEDURE OF SHAFTS USING THE FLOTATION TECHNIQUE

2.1 Guide walls construction

Reinforced concrete guide walls are constructed by forming two concentric rings constituted by a polygonal trench of 10, 12, or more sides. In the vertex, drills of 45 cm of diameter are made with machine all along the depth of the shaft, figure 1 (a).

2.2 Perimeter trench excavation

The circular holes remain filled with bentonite slurry. Digging with clam is performed until the bottom of the ground between two alternating perforations, keeping it casted since the beginning with bentonite slurry until the annular excavation void is complete, figure 1 (b).

2.3 Central nucleus excavation

The soil nucleus is excavated. The stability of the excavation is maintained by keeping it filled with re-circulated bentonite slurry continuously subjected to a strict density control, figure 1 (c).

2.4 Flotation tank placement, reinforced concrete assembly and casting of the bottom slab and of a first stretch of wall

In the upper part of the excavation, a cylindrical floating steel structure (inverted tank) is placed. This tank works as air chamber as well as base for the construction of the concrete structure of the shaft. The bottom slab is casted with a first stretch of the perimeter wall. The structure is secured and held at level, using steel beams, figure 1 (d).

2.5 Casting and immersion of the floating structure in stages

After the concrete of the structural stretch recently casted hardens, air is injected into the tank until a floating condition is reached. Then the sructure is separated from the beams and is partially immersed. The casting and immersion cycles are repeated until reaching the planned depth. When the buoyancy tends to be greater than the overall weight built impeding immersion, the structure is ballasted with the necessary volume of water, figure 1 (e).

2.6 Filling with mortar

Finally, a mortar that replaces the bentonite is injected, from the bottom up, in the steel tank and in the space between the wall of the shaft and the walls of the excavation. Then, the ballast is removed, figure 1 (f). For especially critical conditions (Auvinet *et al*., 2010) a mortar perimeter wall screen is built before starting the excavation of the trench perimeter. In some instances two walls are built: one of mortar and another of self-setting slurry are built. The mortar wall-screen pretends to confine the walls of the excavation in order to prevent detachment of non-cohesive materials that could hinder the construction procedure.

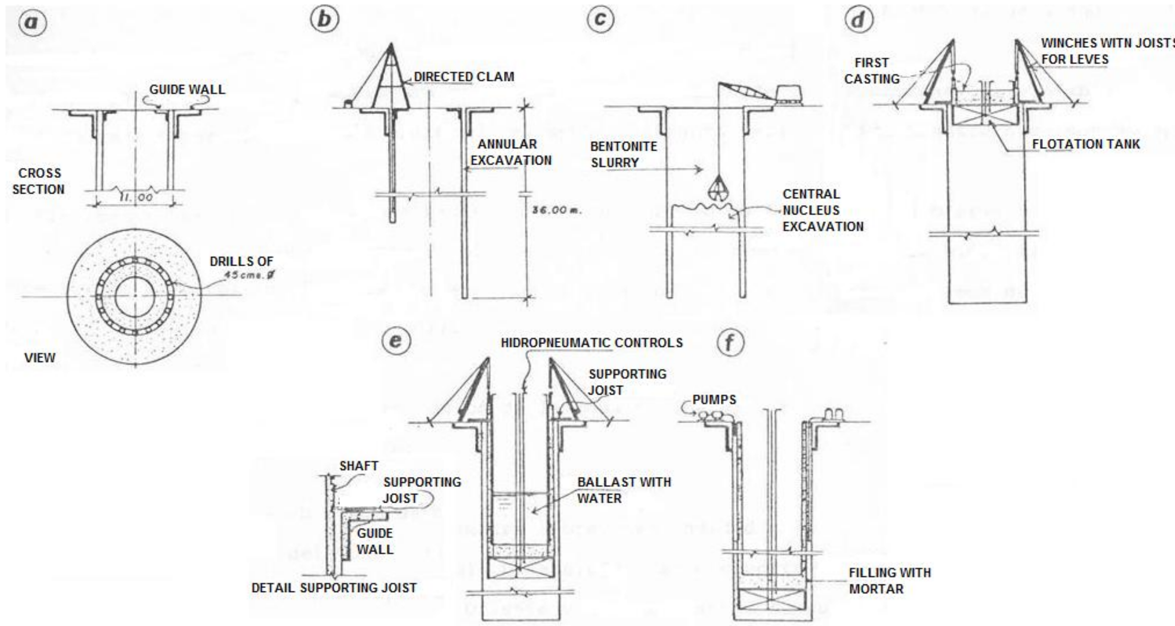

Figure 1. General construction procedure of a floated shaft (Moreno, 1991)

The second wall is used to seal the joints between panels of the wall of mortar and prevent the formation of cracks in the ground during the casting of the panels.

3 ASSESSMENT OF LIMIT STATES OF FAILURE FOR FLOATING SHAFTS

3.1 Failure of the guide walls due instability of the soil at the surface

The stability of the guide walls depends on the characteristics of the soil at the surface. Attention should be placed on the presence of faults, organic soils and other undesirable factors such as natural cracking. The stability of the walls can be evaluated by the Finite Element Method (FEM) performing an analysis in two dimensions (2D) and three-dimensional (3D), depending on the characteristics of the problem. When using FEM the safety factor is obtained by reducing values of the parameters used in the stability analysis until failure is reached. The ratio between the actual values of such parameters and critical values is equal to the factor of safety FS.

3.1.1 2D axisymmetric Finite Element Method

To assess the movement of the guide walls when subjected to the weight of the excavation equipment, the guide wall is considered as an axisymmetric body subjected to local load. Typical results of this analysis are presented in figure 2 (Auvinet *et al*., 2010).

3.1.2 3D Finite Element Method

Stability of the guide wall can also be assessed by using 3D finite element method, figure 3.

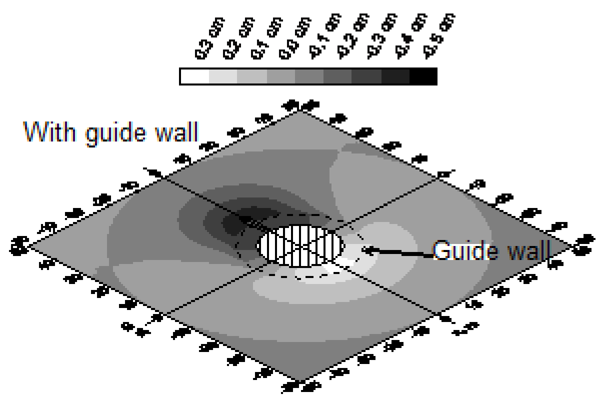

Figure 2. Vertical displacements at the ground surface (Auvinet *et al.*, 2010)

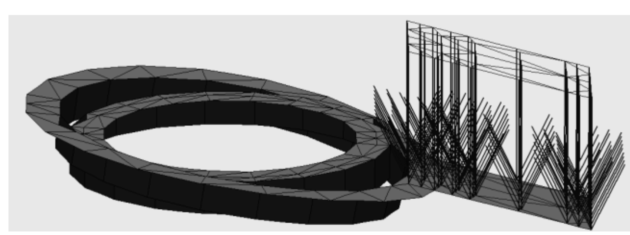

Figure 3. 3D Modeling of guide walls behavior (Auvinet *et al.*, 2010)

3.2 Soil fracturing

The fracture is the loss of continuity between two parts of the body and it involves the generation of a crack and its propagation until a general failure or a new equilibrium is reached.). In the case of cast panels used for constructing the perimeter screens, cracks in the soil induced by hydraulic fracturing have been observed, figure 4.

Figure 4. Soil cracking Induced by slurry pressure when constructing a *cast in situ* wall panel (Shaft 3, Río de los Remedios Tunnel)

3.3 Failure of the central nucleus

The soil nucleus left in place when excavating the circular perimetral trench is a long cylinder confined laterally by bentonite slurry. The stability of this mass of soil under its own weight can be checked by a simple assessment of shear stresses developed in this solid. The stability of the nucleus can be evaluated by analytical method, 2D FEM (axisymmetric) or 3D FEM.

3.3.1 Analytical method

Stability of the nucleus is verified by comparing the shear stress generated by decreasing the horizontal stress $\tau = \frac{(\sigma_1 - \sigma_3)}{2}$ against the shear strength of the soil under undrained conditions, C_u.

where:

$\sigma_3 = \gamma_L S$ (γ_L = Volumetric weight of the slurry and S = height of slurry)

$\sigma_1 = \gamma_s Z$ (γ_s = Volumetric weight of the soil and Z = considerated depth)

Figure 5 shows a diagram of the central nucleus. The factor of safety against shear failure of the soil in the central nucleus is simply:

$$FS_N = \frac{C_U}{\tau} = \frac{C_U}{(\sigma_1 - \sigma_3)/2} = \frac{2C_U}{(\gamma * Z - \gamma_L * S)} \quad (1)$$

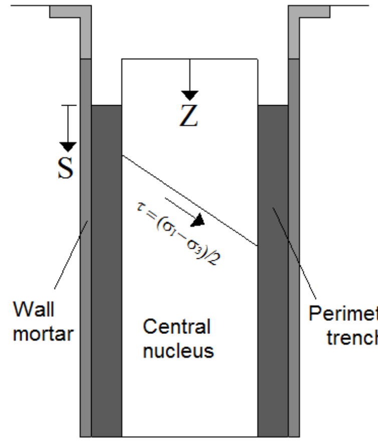

Figure 5. Stability of the central nucleus

3.3.2 Finite Element Method (FEM) 3D

For a more detailed analysis of the stability of the central nucleus, a 3D finite element model can be used, figure 6 (Auvinet *et al.*, 2010).

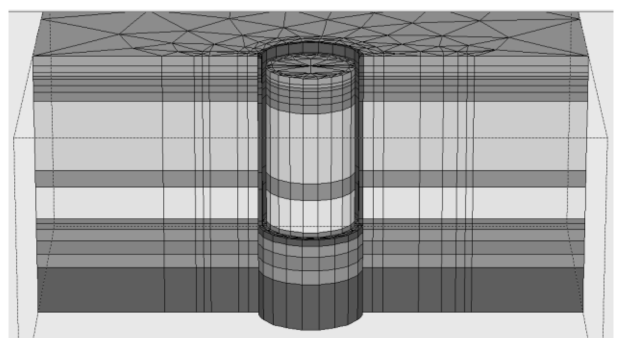

Figure 6. 3D modeling of nucleus

3.4 Shear failure of the bottom of the excavation

When analyzing the general stability of the excavation, it must be taken into account that, in the absence of a perimeter wall-screen (figure 7), the most critical failure mechanism is usually that of the wall, on the contrary, in the presence of such a wall the critical mechanism of failure is that of the bottom, figure 8.

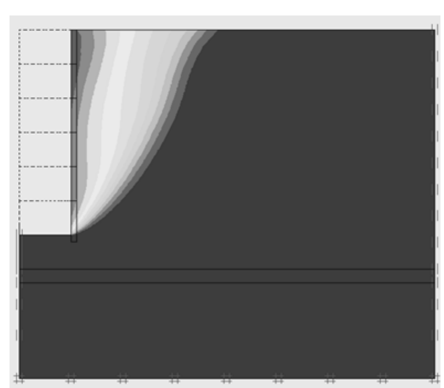

Figure 7. Potentially critical failure mechanism of excavation without a wall-screen. 2D axisymmetric FEM

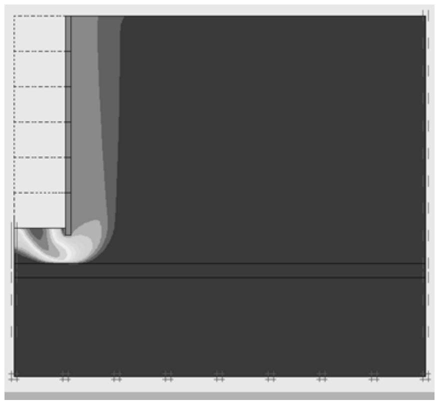

Figure 8. Potentially critical shear failure of the excavation´s bottom with perimeter wall-screen. 2D axisymmetric FEM

3.5 Bottom uplift failure

The existence of hydraulic pressure in permeable layers located directly below the bottom of the excavation can cause a bottom uplift failure, figure 9.

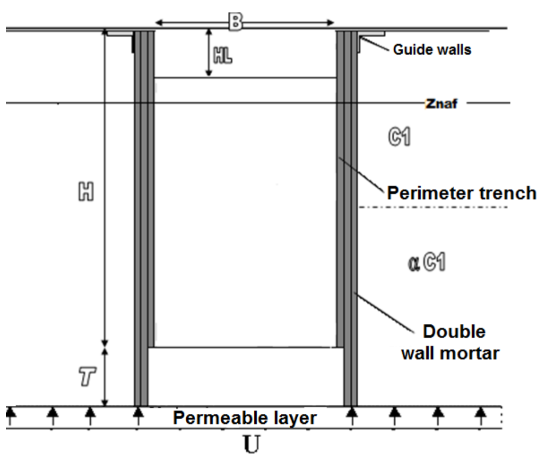

Figure 9. Bottom uplift analysis

The safety factor against bottom uplift failure (FS_S) can be calculated using the limit equilibrium method. The following expression is obtained:

$$FS_S = \frac{(H - H_L)\gamma_L + (T)\gamma_S + \left[\dfrac{\pi.B.T.\alpha C_1}{\dfrac{\pi B^2}{4}}\right]}{(H + T - Znaf)\gamma_w} \qquad (2)$$

where:
B = diameter of shaft
T = distance between base of shaft and permeable layer
C_1 = cohesion of clay
α = constant representing increasing resistance with depth
γ_S = soil volumetric weight
γ_L = slurry volumetric weight
γ_w = water volumetric weight
H = depth of shaft
H_L = depth of slurry
$Znaf$ = depth of groundwater level.
FS_S = factor of safety

When the excavation remains open during a significant time, the presence of slurry can induce, in the permeable layer, a higher pressure than the hydrostatic pressure due to its greater volumetric weight, figure 10. Dropping suddenly the level of the slurry in the excavation can then induce a bottom uplift failure, figure 11.

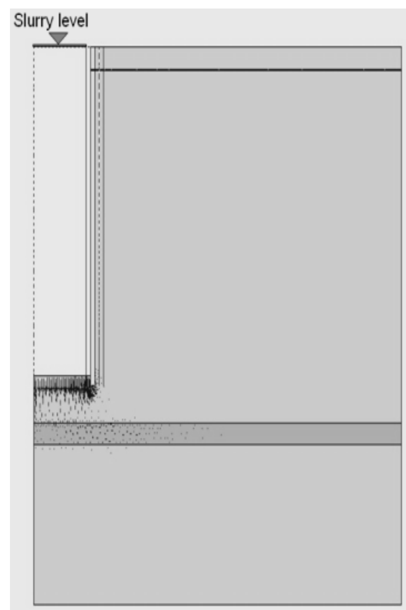

Figure 10. Flow into the hard layer induced by the mud (Auvinet *et al*., 2010)

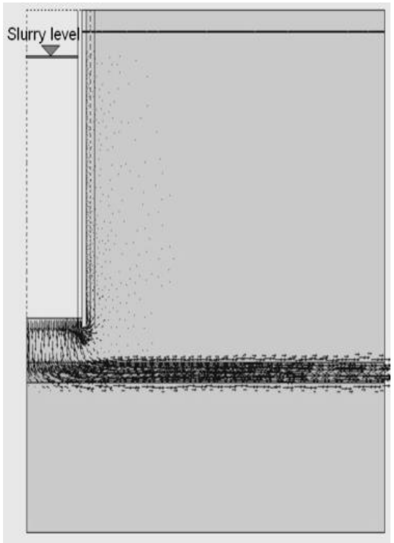

Figure 11. Flow from the hard layer to the bottom due to the mud abatement (Auvinet *et al.* 2010)

3.6 Failure by floating

This phenomenon corresponds to the emergence of the completed shaft above the surface of the ground due to the Archimedes pressure (The vertical upward pressure on a body partially or totally submerged in a liquid is equal to the weight of the displaced liquid volume). This pressure can be enough to cause the flotation of the shaft. This effect may occur immediately when the shaft is empty or it may manifest itself over time. To avoid this condition, it must be checked that the Archimedes pressure is balanced by the weight of the shaft, taking also into account adhesion between the structure of the shaft and the soil.

Adapting the model to be used, figure 12, the factor of safety against general flotation, is:

$$FS_F = \frac{W_L + (C_1 * 18m + \alpha C_1 * 11.5m)\pi B}{(H - Znaf)\gamma_W \pi \frac{B^2}{4}} \tag{3}$$

where:
W_L = weight of the shaft
C_1 = cohesion of the upper clay series
B = diameter of shaft
H = depth of shaft
α = constant representing increasing resistance with depth
$Znaf$ = depth of groundwater level
γ_W = water volumetric weight
FS_F = factor of safety

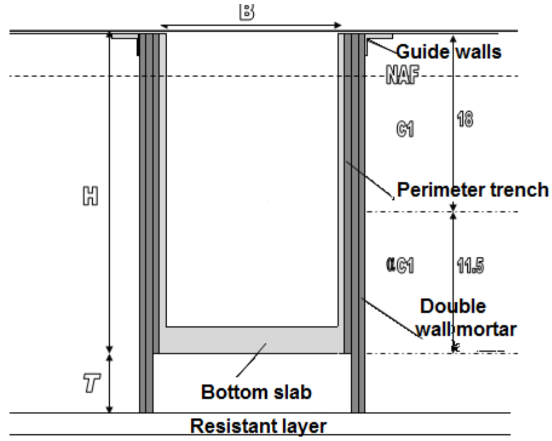

Figure 12. Floating of finished structure

4 RELIABILITY ANALYSIS

4.1 Introduction

In the previous stability analyses, the presence of many uncertainties and the need for a reliability study should be recognized. Reliability is the probability that a system carries out its function properly, for a proposed period and set operating conditions (Kaufmann, 1977). It is defined as the complement to unity of the likelihood of system failure. Reliability of an engineering work can be assessed by defining both control variables and random variables. The first are parameters on which the engineer can exerts enough control. The second ones refer to the parameters on which the geotechnical can not exert proper control.

A simplified reliability analysis can be carried out by calculating the reliability index *β*, defined as:

$$\beta = \frac{E(FS) - 1}{\sigma_{FS}} \tag{4}$$

where the so-called first moments: mathematical expectation $E(FS)$ and standard deviation σ_{FS} of the safety factor FS, can be calculated by the punctual estimation method proposed by Rosenblueth (1975).

4.2 Methodology for applying Rosenblueth's punctual estimation method

Rosenblueth developed a technique for estimating the first moments of a continuous function of random variables, from the first moments of each variable. The method consists in replacing the continuous probability density or the random parameters of the problem by a discrete distribution with the same moments that the continuous density. Figure 13 shows the outline of the principle of the point estimate method.

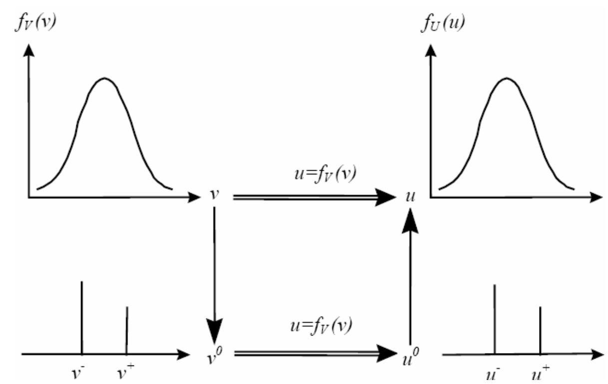

Figure 13. Principle of Rosenblueth's punctual estimation method

The steps toward the implementation of the method are described next for the particular case of four random variables:

a) Mathematical expectation of the safety factor

$$E\left[FS\right] = P_{++++}FS_1 + P_{+++-}FS_2 + \ldots\ldots\ldots P_{----}FS_{16} \qquad (5)$$

where:

$E\left[FS\right]$: expected value of safety factor

P: probability

FS_1, $FS2, \ldots\ldots FS_{16}$: safety factors for the possible combinations of random variables

If the random variables are independent, the value of p is

$$P = \frac{1}{2^4} = \frac{1}{16} = 0.0625 \qquad (6)$$

b) Variance of the safety factor σ_{FS}^2 is:

$$Var\left[FS\right] = P_{++++}\left[FS_1 - E\left[FS\right]\right] + P_{+++-}\left[FS_2 - E\left[FS\right]\right] + \ldots\ldots + P_{----}\left[FS_{16} - E\left[FS\right]\right] \qquad (7)$$

The minimum value of β must be defined by taking into account the consequences of failure. However, in civil engineering it is frequently accepted that the value of β must be at least 3, which corresponds to a probability of failure less than 0.00135.

5 APPLICATION

The above type of analysis can be applied to shafts. The failure limit states taken into account in the analysis are those of the central nucleus and of the excavation bottom. Both are assessed by the finite element method (FEM), using the commercial software Plaxis 2D in axisymmetric condition. The bottom uplift and floating of the shaft are also taken into account using the limit equilibrium method. The failure mechanisms of the guide all and soil cracking are not considered in this analysis.

5.1 Geomechanical model of the shaft

Figure 14 shows the axisymmetric modeling of the shaft used to evaluate the limit states of failure for the central nucleus and general failure of the bottom. Table 1 indicates the values for the material properties of the different layers. Dimension units are meters.

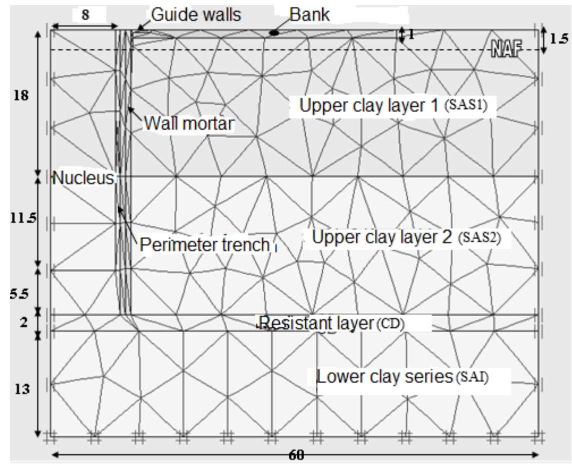

Figure 14. 2D axisymmetric modeling of the shaft by Plaxis, for assessing central nucleus and shear failure of the bottom

For failure by bottom uplift of the excavation (Figure 9) FS is calculated using formula 2 and the failure by flotation, (Figure 12) is assessed using expression 3.

Table 1. Values used in modeling with Plaxis

Material	γ,kN/m^3	E, MPa	ν	C, kPa	ϕ, º
Bank	14.0	20	0.33	0	40
SAS1	12.0	5	0.46	15	0
SAS2	12.0	5	0.46	30	0
CD	18.0	50	0.33	0	40
SAI	12.5	10	0.49	40	0
Wall -Mortal	16.0	5,000	0.25	1000	0
Guide walls	24.0	20,000	0.20	1000	40

5.2 Random variables and control variables

To perform reliability analysis of shafts built with flotation technique it is necessary to define these two types of variables.

For the problem at hand the control variables are:

a) The dimensions of the shaft (diameter, depth, depth of mortar screen and weight of the shaft).
b) The distance between the base of the shaft and the permeable layer.

c) The volumetric weight of water and volumetric weight of the slurry.

The random variables are:

a) The volumetric weight of the soil " γ"
b) The shear strength "C1"
c) The undrained elasticity modulus E
d) The level of slurry "HL"

Tables 2 through 5 provide the expected value and standard deviation of the considered random variables and their punctual values to consider in Rosenblueth's method.

Table 2. Point values for central nucleus

Variable	μ	σ	V_+	V_-
γ_{Soil} (kN/m^3)	12.0	0.3	12.3	11.7
C_1 (kPa)	15.5	0.5	16.0	15.0
E (MPa)	5.0	0.25	5.25	4.75
H_L (m)	0.10	0.10	0.20	0.00

Table 3. Point values for general shear failure

Variable	μ	σ	V_+	V_-
γ_{Soil} (kN/m^3)	12.0	0.3	12.3	11.7
C_1 (kPa)	15.5	0.5	16.0	15.0
E (MPa)	5.0	0.25	5.25	4.75
H_L (m)	1.0	1.0	2.0	0.0

Table 4. Point values for bottom uplift failure

Variable	μ	σ	V_+	V_-
γ_{Soil} (kN/m^3)	12.0	0.3	12.3	11.7
C_1 (kPa)	15.5	0.5	16.0	15.0
H_L (m)	1.0	1.0	2.0	0.0

Table 5. Point values for failure by flotation

Variable	μ	σ	V_+	V_-
C_1 (kPa)	15.5	0.5	16.0	15.0

5.3 Results

5.3.1 Reliability indices and probabilities of failure for each mechanism analyzed

Table 6 shows the reliability indices and failure probabilities for each mechanism analyzed.

Table 6. Reliability index "β" and failure probabilities

Limit state	Reliability index "β"	Failure probability
Nucleus central	0.789	0.2148
General shear failure (Wall –mortar)	4.122	0.00000317
General shear failure (double wall–mortar)	7.402	0.000000019
Hydrostatic-pressure water	3.838	0.0000115
Mud hydrostatic uplift	2.05	0.02275
Flotación adhesion 1	26.41	0.0000000050
Flotación adhesion 2	25.207	0.0000000070
Flotación adhesion 3	22.310	0.0000000095

The results of the analysis presented in table 6, show that the limit state of failure of central nucleus and bottom uplift involve a high probability, with β less than 3.

However, the failure of the central nucleus does not present serious consequences, because even if it becomes unstable during excavation, the excavation can proceed. The probability of failure of the bottom by uplift is significant, especially taking into account the possibility that the slurry may induce overpressure in the hard layer if the excavation remains open during a long time.

With the dual screen system founded on the hard layer, the probability of failure by shear stress of the excavation bottom is low.

The probability of failure by flotation is very low, even considering a 50% reduction in C1. This is due to that Archimedes' pressure is low as a result of the shaft diameter (16 m).

5.3.2 Assessing system reliability

Considering that the failure of the central nucleus has a high probability but without serious consequences, the system reliability will be controlled by the shear failure of the bottom, the failure by uplift and the failure by floating considering as serial mechanisms (occurrence of only one of these failure mechanisms is sufficient for the shaft to be fail)

In order to evaluate such probability Poincare's theorem (equation 8) can be used. The probability associated with the event of the equation 8 (system reliability) can be calculated from the probabilities of each individual event (reliability in a particular way) and their intersections.

$$P(\bar{E}) = P(\bar{E_1, E_2, ..., E_K}) = \sum_i P(\bar{E_i}) - \sum_{i \neq j} P(\bar{E_i, E_j}) + \sum_{i \neq j \neq 1} P(\bar{E_i, E_j, E_1}) - ... \quad (8)$$

For simplicity, only the first term in Poincaré's equation which represents an upper limit of the probability of failure of the system can be considered. Table 7 shows the corresponding probability of failure of the system for different construction conditions and different index and mechanical properties.

Table 7. System failure probabilities for different conditions

System failure	Probability of failure
Wall-mortar, pressure water and adhesion 1	0.00001467
Wall-mortar, pressure water and adhesion 2	0.00001467
Wall-mortar, pressure water and adhesion 3	0.00001467
Wall-mortar, pressure mud and adhesion 1	0.02275317
Wall-mortar, pressure mud and adhesion 2	0.02275317
Wall-mortar, pressure mud and adhesion 3	0.02275379
Double wall-mortar, pressure water and adhesion 1	0.00001152
Double wall-mortar, pressure water and adhesion 2	0.00001152
Double wall-mortar, pressure water and adhesion 3	0.00001152
Double wall-mortar, pressure mud and adhesion 1	0.02275002
Double wall-mortar, pressure mud and adhesion 2	0.02275002
Double wall-mortar, pressure mud and adhesion 3	0.02275002

It is noted that the probability of failure of the system is governed by the failure mechanism of hydrostatic bottom uplift, especially if one considers a sudden depletion in the level of the slurry.

However, for large diameter shafts (20 m or more) the floating mechanism can also be critical, as Archimedes' pressure increases with the square of the diameter of the shaft.

6 CONCLUSIONS

Probabilistic methods and reliability analyses can introduce a greater degree of realism in the assessment for safety engineering works, as in the case of tunnel access shafts built in the valley of Mexico. It provides a sensitivity analysis of the work in question to variations in the design parameters and allows taking into account the uncertainty that exists on them.

Rosenblueth's punctual estimation method is a very useful and easy method to use when obtaining a system reliability. It requires few calculations, and provides satisfactory results comparable to those obtained with more rigorous probabilistic methods, whenever the coefficients of variation of the independent variables do not exceed moderate limits.

Considering that the stratigraphic conditions and geometry of shafts are highly variable, it is recommended to repeat the reliability analysis presented in this paper, for shafts with different characteristics.

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