Seismic soil-structure interaction in piles and piers: case III

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ABSTRACT
A new matrix procedure is presented to determine soil-foundation contact reaction and lateral displacement on piles and piers in seismic conditions. The foundation is considered free to rotate at the head, with embedding at the point, and the base can to rotate by deformation of the surrounding material. The method takes into account the compressibility of the soil, the rigidity of the foundation and the drag forces induced by the lateral displacements of the soil by seismic action. The procedure has the advantages of not being iterative, of being easily programmed and readily extended to other boundary conditions, such as other kinds of foundations -shallow and deep-, and different subsoil conditions.

RESUMEN
Se expone un nuevo procedimiento matricial para determinar los esfuerzos de contacto y los desplazamientos horizontales en pilotes o pilas de cimentación sujetos a fuerzas sísmicas. Se considera que la pila puede girar libremente en la cabeza, con empotramiento en la base y giro en ésta por deformación del material que la rodea. El método toma en cuenta las características de compresibilidad de la masa de suelo, la rigidez de la cimentación y las fuerzas de arrastre que actúan contra la cimentación debidas al desplazamiento horizontal de la masa de suelo durante la acción sísmica. El método presenta las ventajas de no ser iterativo, de la sencillez de su programación y su posibilidad de extenderse con facilidad a otras condiciones de frontera, otros tipos de cimentaciones -superficiales o profundas- y distintas condiciones del subsuelo de cimentación.

1 INTRODUCTION

During a seismic event, soil mass is displaced in a horizontal direction δ_{si}, and due to the force of inertia that the superstructure effects upon the head of the pile foundation, the displacement of the pile, δ_i, is contrary to the seismic displacement of soil mass. Accordingly, soil mass pushes laterally the foundation's pile, producing thus important shear stresses and bending moments in the foundation's structure (figure 1).

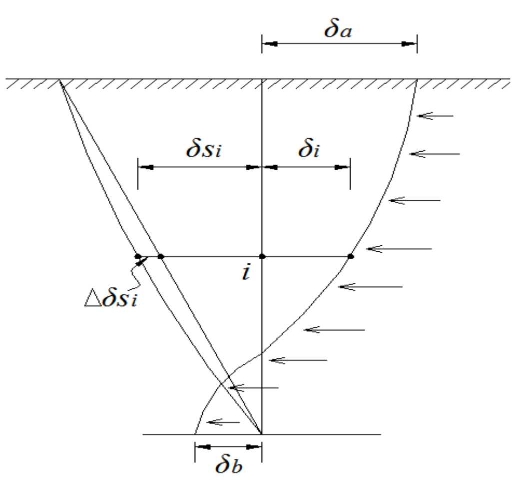


Figure 1. Lateral push forces that act against the foundation during an earthquake

This work presents a procedure for determining the contact reactions soil-pile and its lateral displacement as produced during an earthquake. Soil mass is considered as a continuous medium wherein the stress produced by a section of the foundation have influence on the same section upon which the force is applied and upon the remaining sections. Mindlin's equation (1936) is also considered applicable, obtained from elasticity theory, in order to calculate the increment of horizontal stress produced by a punctual horizontal force applied in the interior of a solid.

The seismic soil-structure interaction is obtained from the compatibility of deformations existing between the structure of foundation and soil mass. The pile foundation is considered to be embedded in its base, also, that these can rotate due to the deformation of the material where the pile foundation is fitted. The head is free to rotate.

2 SEISMIC HORIZONTAL DISPLACEMENT MATRIX EQUATION

Figure 2 shows a pile foundation submitted to lateral force P and moment M. In order to study the foundation, the pile has been divided in n+2 slices. The end slices have been termed *a* and *b*, and the middle slices numbers 1, 2, 3. . .*n*. The soil mass has been divided in sections A, B, C, D, E and F.

A unitary load is initially placed on slice *a*. The displacement from the center of each slice produced by the unitary load is then calculated. Next, a unitary load is applied to slice 1 and the displacement in the center of each slice due to the unitary load is also calculated. The procedure continues until, finally, a unitary load is applied on slice *b*.

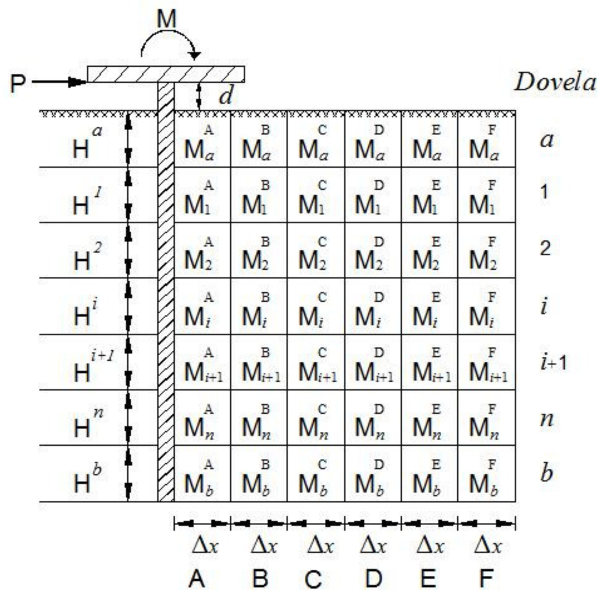


Figure 2. Pier or pile of foundation submitted to a lateral load and moment

The lateral displacement at the center of slice j produced by the unitary load acting upon slice i produces:

$$\delta_{ji} = \left[\ I_{ji}^{A}\alpha_i^{A} + I_{ji}^{B}\alpha_i^{B} + \ldots + I_{ji}^{F}\alpha_i^{F}\ \right] / a_i \qquad [1]$$

δ_{ji} is the lateral displacement in the center of slice j, produced by a unitary load acting upon slice i; I_{ji}^{N} is the influence upon the center of slice j, produced by a unitary load acting upon slice i and upon the center of section N.

$$\alpha_i^N = M_i^N \Delta x \qquad [2]$$

M_i^N is the dynamic modulus of unitary deformation of material in slice i and in section N; Δx is the width of the section of soil; a_i is the area of slice i

$$M_i^N = \frac{1}{2(1+\nu)\mu} \qquad [3]$$

ν is Poisson's relation and μ is the stiffness modulus at shear in section N.

The value of influence I_{ji}^{N} can be obtained through the integration of Mindlin's equation (1936) on a rectangular area, or else, the approximate equation proposed by Medina (2010a) can be employed:

The seismic horizontal displacement matrix equation, HEMAS, is obtained from Zeevaert (1983):

$$\begin{bmatrix} \delta_{aa} & \delta_{a1} & \delta_{a2} \ \ldots \ \delta_{an} & \delta_{ab} \\ \delta_{1a} & \delta_{11} & \delta_{12} \ \ldots \ \delta_{1n} & \delta_{1b} \\ \delta_{2a} & \delta_{21} & \delta_{22} \ \ldots \ \delta_{2n} & \delta_{2b} \\ & & \vdots & \\ \delta_{na} & \delta_{n1} & \delta_{n2} \ \ldots \ \delta_{nn} & \delta_{nb} \\ \delta_{ba} & \delta_{b1} & \delta_{b2} \ \ldots \ \delta_{bn} & \delta_{bb} \end{bmatrix} \begin{vmatrix} R_a \\ R_1 \\ R_2 \\ \vdots \\ R_n \\ R_b \end{vmatrix} = \begin{vmatrix} \delta_a \\ \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \\ \delta_b \end{vmatrix} \qquad [4]$$

R_i is the reaction of contact in slice i; δ_i is the horizontal displacement of slice i.

3 FLEXIBILITY MATRIX EQUATION

Figure 3 shows the application of the method of forces or flexibilities: McCormac and Nelson (2002) in order to obtain the reactions for a static indeterminate structure:

Δ_i = deflection at point i for condition $R_i = 0$

β_b = rotation of support b for condition $R_i = 0$

θ_b = rotation of base of pier by deformation of material where the pier is supported.

d_{ji} = deflection in j due to a unitary load applied at i (condition $R_i = 1$)

d_{jb} = Deflection at j due to an unitary moment applied at b (condition $M_b = 1$)

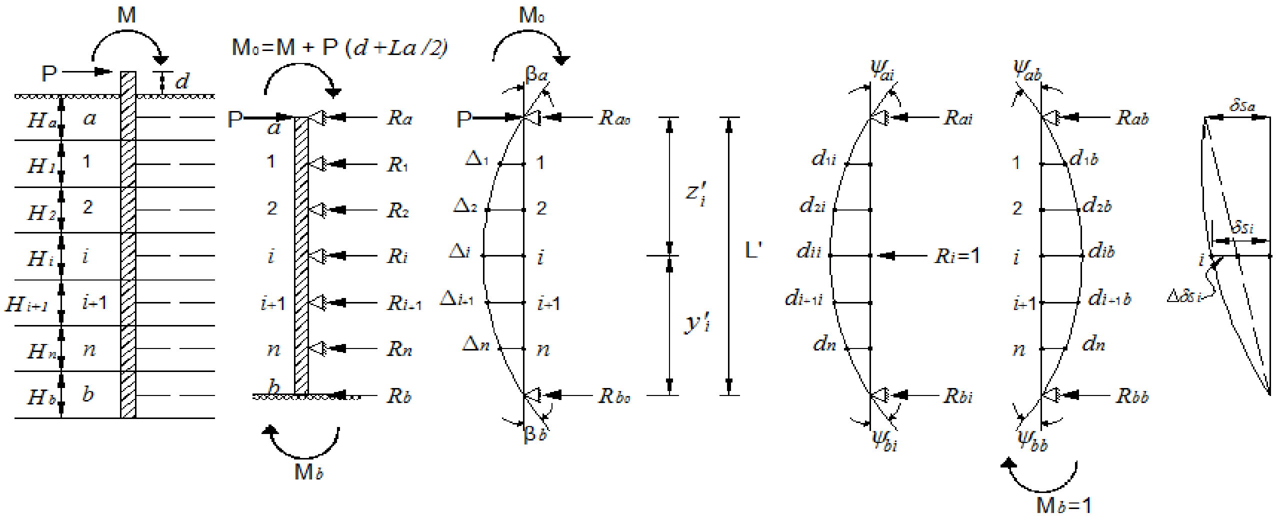
ψ_{bi} = Rotation of the support b due to a unitary load applied at i (condition $R_i = 1$)

ψ_{bb} = Rotation of the support b due to a unitary moment applied at b (condition $M_b = 1$)

$\Delta\delta_{si}$ = Relative horizontal displacement of the soil mass due to seismic action.

γ_{sb} = Rotation, at the base of the pile, caused by the seismic displacement of soil mass.

δ_i^A = Displacement at point i, by deformation of the supports *a* y *b,* considered the foundation as a rigid body.



a) b) c) d) e) f)

Figure 3. Application of the flexibilities method: a) real foundation; b) discreteness of foundation; c) condition R_i = 0; d) condition R_i = 1; e) condition M_b = 1; f) relative seismic displacement of soil mass.

$$\begin{bmatrix} d_{11} & d_{12}\dots d_{1n} & d_{1b} \\ d_{21} & d_{22}\dots d_{2n} & d_{2b} \\ & \vdots & \\ d_{n1} & d_{n2}\dots d_{nn} & d_{nb} \\ \psi_{b1} & \psi_{b2}\dots\psi_{bn} & \psi_{bb} \end{bmatrix} \begin{vmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \\ M_b \end{vmatrix} = \begin{vmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \\ \beta_b \end{vmatrix} + \begin{vmatrix} \Delta\delta_{s1} \\ \Delta\delta_{s2} \\ \vdots \\ \Delta\delta_{sn} \\ \gamma_{sb} \end{vmatrix} + \begin{vmatrix} \delta_1^A \\ \delta_2^A \\ \vdots \\ \delta_n^A \\ 0 \end{vmatrix} - \begin{vmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \\ \theta_b \end{vmatrix} \qquad [5]$$

$\Delta\delta_{si}$ and γ_{sa} are obtained from the following expressions:

$$\Delta\delta_{si} = \delta_{si} - \xi_i\,\delta_{sa} \qquad [6]$$

$$\gamma_{sb} = -\,\delta_{sa} / L' \qquad [7]$$

δ_{sa} is the horizontal seismic displacement of soil mass in slice *a* and L' is the vertical distance between slices *a* and *b*. The lateral seismic displacement of soil mass δ_{si}, can be obtained with the procedure proposed by Zeevaert (1983).

The rotation of the tip of pile, θ_b, can be estimated from elasticity theory, Frohlich (1953):

$$\theta_b = C_{\theta b}\,(M_b) \qquad [8]$$

$$C_{\theta b} = [\;3\,(1-\nu^2)\,M_D\;] \,/\, (4\,r^3) \qquad [9]$$

Where ν and M_D are Poisson`s relation and the deformation modulus of material that circulates at the tip of the pier, respectively; r is the radio of pier and M_b is the moment in *b*.

Equation 5 relates the reactions of soil foundation contact, rigidity of foundation structure, loads applied to foundation; seismic displacements of soil mass and the lateral displacements of foundation, and is called flexibility matrix equation, EMFLEX.

4 SOIL STRUCTURE INTERACTION MATRIX EQUATION

Observe that with matrix Equations HEMAS and EMFLEX there are 2n+3 equations with 2n+5 unknowns, therefore the two remaining equations are obtained from the sum of moments at supports *a* and *b* with which the system of equations can be resolved, obtaining:

$$\left[\begin{bmatrix}\delta_{11} & \delta_{12} & \dots & \delta_{1n}\\ \delta_{21} & \delta_{22} & \dots & \delta_{2n}\\ & \vdots & & \\ \delta_{n1} & \delta_{n2} & \dots & \delta_{nn}\end{bmatrix}+\begin{bmatrix}d_{11} & d_{12} & \dots & d_{1n}\\ d_{21} & d_{22} & \dots & d_{2n}\\ & \vdots & & \\ d_{n1} & d_{n2} & \dots & d_{nn}\end{bmatrix}+\begin{bmatrix}m_{11} & m_{12} & \dots & m_{1n}\\ m_{21} & m_{22} & \dots & m_{2n}\\ & \vdots & & \\ m_{n1} & m_{n2} & \dots & m_{nn}\end{bmatrix}\right]\begin{vmatrix}R_1\\ R_2\\ \vdots\\ R_n\end{vmatrix}=\begin{vmatrix}n_1\\ n_2\\ \vdots\\ n_n\end{vmatrix}+\begin{vmatrix}\Delta_1\\ \Delta_2\\ \vdots\\ \Delta_n\end{vmatrix}+\begin{vmatrix}\Delta\delta_{S1}\\ \Delta\delta_{S2}\\ \vdots\\ \Delta\delta_{Sn}\end{vmatrix} \qquad [10]$$

where:

$$\begin{bmatrix}m_{11} & m_{12} \dots & m_{1n}\\ m_{21} & m_{22} \dots & m_{2n}\\ & \vdots & \\ m_{n1} & m_{n2} \dots & m_{nn}\end{bmatrix}=\begin{bmatrix}\psi_1 & \xi_1\\ \psi_2 & \xi_2\\ \vdots & \\ \psi_n & \xi_n\end{bmatrix}\begin{bmatrix}\delta_{aa} & \delta_{ab}\\ \delta_{ba} & \delta_{bb}\end{bmatrix}\begin{bmatrix}\psi_1 & \psi_2 \dots & \psi_n\\ \xi_1 & \xi_2 \dots & \xi_n\end{bmatrix}-\begin{bmatrix}\psi_1 & \xi_1\\ \psi_2 & \xi_2\\ \vdots & \\ \psi_n & \xi_n\end{bmatrix}\begin{bmatrix}\delta_{a1} & \delta_{a2} \dots & \delta_{an}\\ \delta_{b1} & \delta_{b2} \dots & \delta_{bn}\end{bmatrix}-$$

$$-\begin{bmatrix}\delta_{1a} & \delta_{1b}\\ \delta_{2a} & \delta_{2b}\\ \vdots & \\ \delta_{na} & \delta_{nb}\end{bmatrix}\begin{bmatrix}\psi_1 & \psi_2 \dots \psi_n\\ \xi_1 & \xi_2 \dots \xi_n\end{bmatrix}-\begin{bmatrix}d_{1b}\\ d_{2b}\\ \vdots\\ d_{nb}\end{bmatrix}\left[B_1\ B_2 \dots B_n\right]-\begin{bmatrix}(\delta_{1a}-\delta_{1b)}\\ (\delta_{2a}-\delta_{2b})\\ \vdots\\ (\delta_{na}-\delta_{nb})\end{bmatrix}\left[D_1\ D_2 \dots D_n\right]+$$

$$+\begin{bmatrix}\psi_1 & \xi_1\\ \psi_2 & \xi_2\\ \vdots & \\ \psi_n & \xi_n\end{bmatrix}\begin{bmatrix}(\delta_{aa}-\delta_{ab})\\ (\delta_{ba}-\delta_{bb})\end{bmatrix}\left[D_1\ D_2 \dots D_n\right] \qquad [11]$$

$$\begin{vmatrix}n_1\\ n_2\\ \vdots\\ n_n\end{vmatrix}=\left[\begin{bmatrix}\psi_1 & \xi_1\\ \psi_2 & \xi_2\\ \vdots & \\ \psi_n & \xi_n\end{bmatrix}\begin{bmatrix}\delta_{aa} & \delta_{ab}\\ \delta_{ba} & \delta_{bb}\end{bmatrix}-\begin{bmatrix}\delta_{1a} & \delta_{1b}\\ \delta_{2a} & \delta_{2b}\\ \vdots & \\ \delta_{na} & \delta_{nb}\end{bmatrix}\right]\begin{vmatrix}R_{a0}\\ R_{b0}\end{vmatrix}-\begin{bmatrix}(\delta_{1a}-\delta_{1b})\\ (\delta_{2a}-\delta_{2b})\\ \vdots\\ (\delta_{na}-\delta_{nb})\end{bmatrix}\left|D_0\right|-$$

$$-\begin{bmatrix} d_{1b} \\ d_{2b} \\ \vdots \\ d_{nb} \end{bmatrix} \left| B_0 \right| + \begin{bmatrix} \psi_1 & \xi_1 \\ \psi_2 & \xi_2 \\ & \vdots \\ \psi_n & \xi_n \end{bmatrix} \begin{bmatrix} (\delta_{aa} - \delta_{ab}) \\ (\delta_{ba} - \delta_{bb}) \end{bmatrix} \left| D_0 \right| \qquad [12]$$

$$B_0 = \left[\frac{\beta_b + \gamma_{sb}}{\psi_{bb} + C_{\theta b}} \right] \qquad [13]$$

$$B_i = \left[\frac{\psi_{bi}}{\psi_{bb} + C_{\theta b}} \right] \qquad [14]$$

$$D_0 = B_0 / L' \qquad [15]$$

$$D_i = B_i / L' \qquad [16]$$

$$\psi_i = y'_i / L'; \quad \xi_i = z'_i / L' \qquad [17]$$

Where y'_i, z'_i are the coordinates of position of point i and L' is the distance -center to center- between slices *a* and *b* (Fig. 3).

R_{a0} and R_{b0} are the reactions at the supports *a* and *b*, respectively, for condition $R_i = 0$.

Equation 10 relates the reactions of contact soil-foundation, the rigidity of the foundation, the loads applied to the foundation, the characteristics of compressibility of the subsoil, the relative displacements of soil mass due to seismic action and the lateral displacements of the foundation; it is called soil-structure interaction matrix equation, EMISE.

Solving Equation 10, reactions R_1, R_2, … and R_n are obtained. From Equations 5 and 8 are obtained the moments M_b:

$$M_b = B_o - (B_1 R_1 + B_2 R_2 + \ldots + B_n R_n) \qquad [18]$$

By the sum of moments at the supports *a* and *b*:

$$R_a = R_{a0} - (\psi_1 R_1 + \psi_2 R_2 + \ldots + \psi_n R_n) + M_b / L' \qquad [19]$$

$$R_b = R_{b0} - (\xi_1 R_1 + \xi_2 R_2 + \ldots + \xi_n R_n) - M_b / L' \qquad [20]$$

From HEMAS, is obtained: δ_a, δ_1, δ_2, …, δ_n and δ_b.

5 EXAMPLE

5.1 Problem set up

Pier foundation with EI = 2.600E+06 kN-m^2. E is Young's modulus of reinforced concrete and I is the moment of inertia of the pier.

P = 560 kN; M = 40 kN-m; d = 1.00 m; r = 0.60 m.

At the base of the pier:

$C_{\theta b}$ = 4.4259E-05 rad/kN-m;

Acceleration of ground surface, 1 m/s^2.

Poisson's relation of subsoil, ν = 0.5.

Table 1 shows the foundation subsoil's characteristics.

5.2 Solution

The pier foundation was divided into eight slices: *a*, 1, 2, 3, 4, 5, 6 and *b*.

Δx = 1.80 m was used and six sections in soil mass: A, B, C, D, E and F.

The values of influence were obtained by Medina's (2010a) proposed equation.

The seismic displacements of soil mass were obtained with the procedure proposed by Zeevaert (1983), considering the first mode of vibration of soil mass with a frequency of 6.791 cycles by second (see Table 1).

The problem was solved by means of a computer program using a spreadsheet. The results were the following:

Table 1. Characteristics and seismic displacement of soil mass, δ_{Si}

Slice	Thickness (m)	unit weight (kN/m^3)	μ_l (kN/m^2)	M^N (m^2/kN)	z'$_i$ (m)	δ_{Si} (m)
a	1.50	14.220	8 826	3.7767E-05	0.00	-1.4158E-03
1	2.00	13.729	6 374	5.2293E-05	1.75	1.8340E-03
2	3.50	12.945	3 727	8.9449E-05	4.50	2.0267E-03
3	2.00	13.729	6 374	5.2293E-05	7.25	1.3031E-03
4	2.00	13.729	6 374	5.2293E-05	9.25	9.7500E-04
5	2.00	13.729	6 374	5.2293E-05	11.25	2.6713E-04
6	2.00	14.220	8 826	3.7767E-05	13.25	-1.4710E-04
b	2.00	15.691	19 613	1.6995E-05	1.75	-1.4158E-03

Matriz de influencias correspondiente a la dovela *a*, $\left[I^{N}_{ja}\right]$:

zi (m)		A	B	C	D	E	F
0.75	*a*	2.8984E-01	6.2789E-02	2.5329E-02	1.3438E-02	8.2730E-03	5.5895E-03
2.50	1	1.4365E-02	2.6407E-02	1.6916E-02	1.0677E-02	7.1415E-03	5.0497E-03
5.25	2	1.7726E-04	2.4134E-03	4.2944E-03	4.5182E-03	3.9966E-03	3.3362E-03
8.00	3	1.7317E-05	3.5198E-04	1.0058E-03	1.5459E-03	1.7938E-03	1.8110E-03
10.00	4	5.2526E-06	1.1871E-04	3.9885E-04	7.2423E-04	9.7169E-04	1.1032E-03
12.00	5	2.0102E-06	4.8058E-05	1.7742E-04	3.5911E-04	5.3542E-04	6.6760E-04
14.00	6	8.9938E-07	2.2230E-05	8.7034E-05	1.8943E-04	3.0480E-04	4.0868E-04
16.00	*b*	4.5022E-07	1.1368E-05	4.6266E-05	1.0589E-04	1.8012E-04	2.5541E-04

Matriz de desplazamientos unitarios. $\left[\delta_{ji}\right]$ (m/kN):

a	1	2	3	4	5	6	b
1.5305E-05	3.2217E-06	7.4993E-07	2.4812E-07	1.2617E-07	6.7913E-08	3.8418E-08	2.2722E-08
4.2125E-06	1.7551E-05	2.2904E-06	5.9260E-07	2.8306E-07	1.4522E-07	7.8829E-08	4.4954E-08
1.6759E-06	3.5182E-06	2.1342E-05	3.4156E-06	1.3215E-06	6.0592E-07	3.0216E-07	1.6015E-07
3.4125E-07	5.9260E-07	2.2284E-06	1.7350E-05	3.2351E-06	1.0640E-06	4.6483E-07	2.2509E-07
1.7371E-07	2.8306E-07	8.0946E-07	3.2351E-06	1.7346E-05	3.2328E-06	1.0626E-06	4.6393E-07
9.3585E-08	1.4522E-07	3.6645E-07	1.0640E-06	3.2328E-06	1.7345E-05	3.2319E-06	1.0620E-06
3.8261E-08	5.6932E-08	1.3117E-07	3.3571E-07	7.6745E-07	2.3341E-06	1.2527E-05	2.3339E-06
1.0189E-08	1.4610E-08	3.1155E-08	7.3154E-08	1.5078E-07	3.4516E-07	1.0502E-06	5.6369E-06

Soil structure interaction matrix equation (EMISE):

(m/kN)						(kN)		(m)
2.7686E-05	1.7302E-05	1.5166E-05	1.2608E-05	9.1146E-06	4.9302E-06	R_1		1.0568E-02
1.8369E-05	4.7046E-05	3.0749E-05	2.5271E-05	1.8440E-05	1.0120E-05	R_2		1.3064E-02
1.5540E-05	2.9967E-05	5.0064E-05	3.3314E-05	2.4136E-05	1.3471E-05	R_3	=	1.1789E-02
1.2868E-05	2.5027E-05	3.3248E-05	4.6518E-05	2.6494E-05	1.4494E-05	R_4		9.5534E-03
9.2240E-06	1.8254E-05	2.3884E-05	2.6277E-05	3.6878E-05	1.4964E-05	R_5		6.2813E-03
4.8674E-06	9.7588E-06	1.2843E-05	1.3678E-05	1.3736E-05	1.9782E-05	R_6		2.9102E-03

Solving the matrix above:

R_1 = 259.734 kN; R_2 = 121.979 kN; R_3 = 76.921 kN; R_4 = 32.003 kN; R_5 = -10.772 kN; R_6 = -41.559 kN

From Equations (18, 19, 20, and 8):

M_b = -38.214 kN-m;

R_a = 263.798 kN; R_b = -142.104 kN;

θ_b = -1.691E-03 rad.

From matrix equation of displacements, HEMAS:

δ_a = 0.498 cm; δ_1 = 0.599 cm; δ_2 = 0.422 cm;

δ_3 = 0.189 cm; δ_4 = 0.088 cm; δ_5 = -0.018 cm;

δ_6 = -0.079 cm; δ_b = -0.083 cm.

Figures 4, 5, 6 and 7 are shown in the lateral displacement, lateral contact pressure, shear force and bending moment diagrams of the pier, respectively.

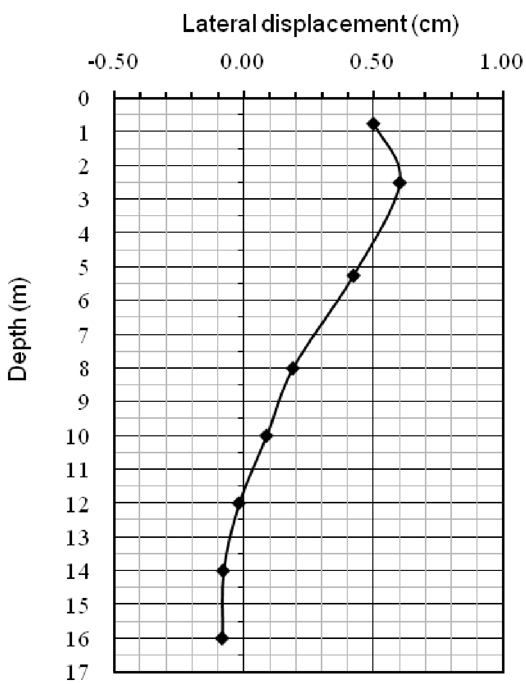


Figure 4. Lateral displacement diagram

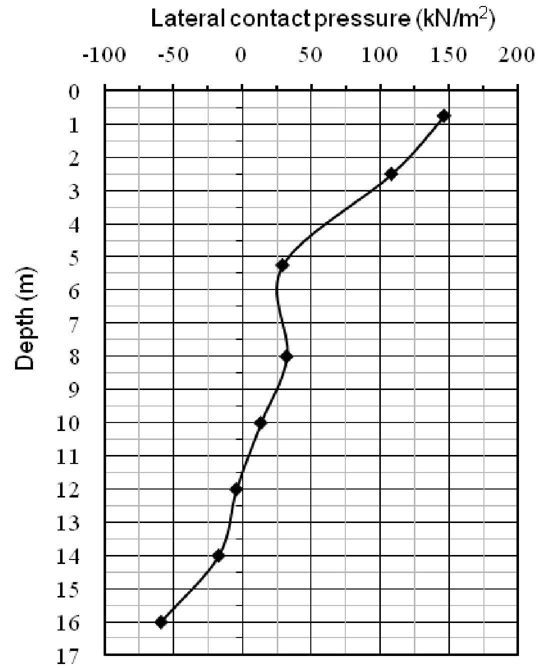


Figure 5. Lateral contact pressure diagram

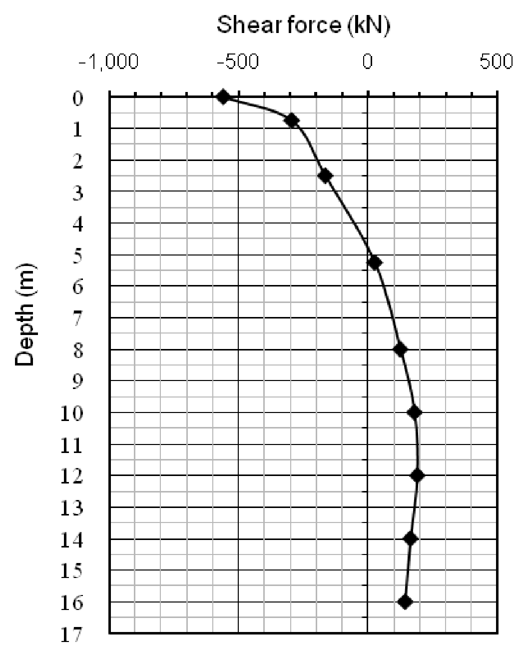


Figure 6. Shear force diagram

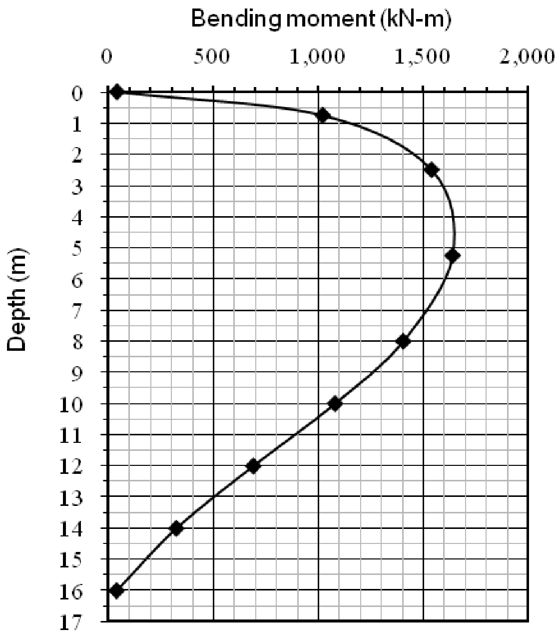


Figure 7. Bending moment diagram

6 CONCLUSIONS

A procedure for obtaining the contact reactions and the horizontal displacement for piles or pier foundations submitted to seismic forces has been presented. The method has the following advantages:

a) It considers soil mass as a continuous medium where the stress produced by a section of the foundation has influence on the same slice in which the load is applied as well as in the reminder of the considered sections.
b) It takes into account the compression characteristics of soil mass.
c) It considers the lateral push forces that are applied by soil mass against the foundation during an earthquake.
d) The increments of the horizontal stress induced by the foundation on soil mass are estimated, and the pore pressure can be estimated.
e) It takes into account the rigidity of the pile or pier foundation.
f) The setting up of the problem has been presented in a matrix form; therefore, the soil structure interaction matrix equation is constructed in a simpler manner.
g) The method can be easily programmed on a spreadsheet and can be easily resolved.
h) The method can be extended to different boundary conditions, other foundation types and subsoil conditions, Medina (2011a, 2011b, 2010b, 2005a, 2005b, 2005c).

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