Multivariate distribution models for design spectral accelerations based on uniform hazard spectra

Shun-Hao Ni, De-Yi Zhang, Wei-Chau Xie, & Mahesh D. Pandey
Department of Civil and Environmental Engineering – University of Waterloo, Waterloo, Ontario, Canada

ABSTRACT
The site-specific design spectra, Uniform Hazard Spectra (UHS), have been adopted as input design ground motions in several seismic design codes. However, the UHS, which are derived from the conventional Probabilistic Seismic Hazard Analysis (PSHA), can only provide the probability of exceeding a specified value of spectral acceleration at individual vibration period. To perform reliability-based seismic design of structures, the analytical multivariate distribution of design spectral accelerations at multiple periods for a given site is desired. In this paper, a multivariate distribution model of design spectral accelerations at multiple periods are derived based on the marginal distribution model, knowledge of correlations among the spectral accelerations, and the existing uniform hazard spectra.

RÉSUMÉ
Les spectres de conception spécifique au site, les Spectres Uniforme de Risque Sismique (SURS), ont été adoptés comme motions du sol dans plusieurs codes de conception parasismique. Toutefois, SURS, qui sont dérivées de la Méthode d'Analyse Probabiliste du Danger Sismique (MAPDS), ne peuvent fournir que la probabilité de dépasser une certaine valeur de l'accélération spectrale à la période individuelle de vibration. Pour effectuer une conception parasismique basée sur la fiabilité des structures, la distribution analytique multi variée des accélérations spectrale de conception à des périodes multiples pour un site donné, est désire. Dans cet article, un modèle de distribution multi variée des accélérations spectrale de conception à de multiples périodes est dérivées basée sur le modèle de distribution marginale, la connaissance des corrélations entre les accélérations spectrales, et les spectres du risque uniforme existantes.

1 INTRODUCTION

Uniform Hazard Spectra (UHS) have been used as input design spectral accelerations (acceleration seismic design spectra) in a number of seismic design codes (e.g., ASCE, 2005, NBCC, 2010). The UHS, however, are derived from the conventional Probabilistic Seismic Hazard Analysis (PSHA) (McGuire, 2004), which computes the probability of exceeding a threshold value of spectral acceleration at individual vibration period separately. As results from a suite of marginal hazard analyses for spectral accelerations at different periods, the UHS are thus unable to provide any knowledge about the simultaneous occurrence of spectral accelerations at multiple vibration periods (Baker and Cornell, 2006). The concept of a single “design earthquake” is then lost on an UHS.

To obtain knowledge about the simultaneous occurrence of spectral accelerations at multiple periods, a Vector-valued Probabilistic Seismic Hazard Analysis (VPSHA) was performed by Bazzurro and Cornell (2002). This analysis is a direct extension of the conventional PSHA. The only additional information required is the joint distribution of spectral accelerations for a given magnitude and distance. However, the results of both the conventional and the vector-valued PSHA can only provide numerical spectral accelerations associated with a small number of corresponding levels of probability of exceedance, rather than analytical multivariate distribution functions of spectral accelerations at multiple periods.

To perform reliability-based seismic design of structures, an analytical multivariate distribution function of design spectral accelerations at multiple vibration periods for a given site is desired. For seismic design and analysis of structures, the original perception of a single “design earthquake” can also be achieved if the multivariate distribution is used. The probability of exceedance obtained from the multivariate distribution is for the simultaneous occurrence of the design spectral accelerations.

In this paper, the conventional and vector-valued PSHA are first described in Sections 2 and 3, respectively. The basic concept of spectral correlation is briefly introduced in Section 4. In Section 5, the marginal distribution model for spectral acceleration at individual period is verified empirically based on the results of PSHA. The multivariate distribution model of design spectral accelerations at multiple periods are then derived based on the marginal distribution model, knowledge of correlations among the spectral accelerations, and the UHS. The multivariate distribution model is validated using the results of the VPSHA.

2 PROBABILISTIC SEISMIC HAZARD ANALYSIS

Probabilistic Seismic Hazard Analysis (PSHA) has been widely used in earthquake engineering for decades. It combines all possible and relevant earthquake scenarios and the levels of probability of ground motion parameters in the analysis. The framework of the PSHA was established by Cornell (1968) 40 years ago. The current well-known PSHA method is recognized as the Cornell-McGuire PSHA (McGuire, 2004).

For the seismic hazard evaluation of a site of interest having n potential seismic sources, the annual mean rate of exceedance of ground motion parameter, such as spectral acceleration at a specified period S_a, is given by:

$$\lambda_{s^*} = \sum_{i=1}^{n} \nu_i \left\{ \int_r \int_m P\left\{ S_a > s^* \left| m, r \right. \right\} f_{M,R}(m,r)\, dm dr \right\}_i, \quad [1]$$

where ν_i is the annual mean rate of occurrence of earthquakes on seismic source i above a lower bound of earthquake magnitude and s^* is the threshold value of spectral acceleration S_a. $f_{M,R}(m,r)$ is the joint probability density function of magnitude M and source-site distance R. In some cases, the magnitude and distance are assumed to be statistically independent. $f_M(m)$ can usually be assumed to be an exponential distribution, which is truncated at lower- and upper bounds, namely, $m_{\min}$ and $m_{\max}$. $f_R(r)$ can be derived based on the assumption that a seismic focus is equally likely to occur anywhere on the seismic source given the occurrence of an earthquake having certain magnitude.

$P(\cdot)$ in equation 1 is the complementary standard normal cumulative distribution function:

$$P\left\{ S_a > s^* \left| m, r \right. \right\} = 1 - \Phi_{S_a}\left(\frac{\ln s^* - \mu_{\ln S_a|m,r}}{\sigma_{\ln S_a|m,r}} \right), \quad [2]$$

where $\mu_{\ln Sa|m,r}$ and $\sigma_{\ln Sa|m,r}$ are the conditional mean and standard deviation of the natural logarithm of S_a provided in any customary attenuation relation. Given a pair of magnitude m and distance r, the natural logarithm of S_a has been widely verified to be well represented by the normal distribution marginally (Jayaram and Baker, 2008).

The temporal uncertainties of the occurrence of earthquakes are defined using the Poisson process and the probability of exceeding s^* in a time period t is given by

$$P\left\{ S_a^t > s^* \right\} = 1 - \exp\left(-\lambda_{s^*} t \right) \quad [3]$$

In most structural engineering practices, the time period t is taken as one year or 50 years. The use of the Poisson process can also be extended to describe the temporal uncertainties of the occurrence of earthquakes associated with multiple ground motion parameters, such as spectral accelerations at multiple vibration periods as will be discussed in Section 3.

For a given annual probability of exceeding s^*, a plot of the threshold s^* for a range of vibration periods of interest at the site gives the Uniform Hazard Spectrum (UHS) as shown in Figure 1. The spectral accelerations on a UHS are separately derived by the PSHA. The plot of the probability of exceedance against the spectral acceleration at a given period is called seismic hazard curve.

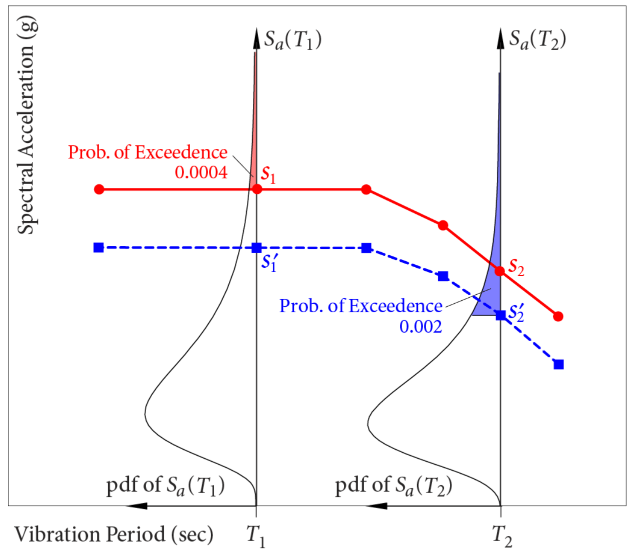

Figure 1. Concept of uniform hazard spectra

3 VECTOR-VALUED PROBABILISTIC SEISMIC HAZARD ANALYSIS

The conventional PSHA for a specific site provides the annual probability of exceeding a threshold value of a single ground motion parameter, such as spectral acceleration at a given period. In many practical applications, however, the joint knowledge of a vector of ground motion intensity measures can improve the accuracy in the prediction of structural response induced by the earthquake.

To demonstrate the procedure of Vector-valued Probabilistic Seismic Hazard Analysis (VPSHA), first proposed by Bazzurro and Cornell (2002), two spectral accelerations at different vibration periods are considered as follows.

In the two-dimensional case, the joint annual mean rate of exceedance of spectral accelerations at periods T_1 and T_2, i.e., $S_a(T_1)$ and $S_a(T_2)$, is given by

$$\lambda_{s_1,s_2} = \sum_{i=1}^{n} \nu_i \left\{ \begin{array}{l} \int_r \int_m P\left\{ S_a(T_1) > s_1, S_a(T_2) > s_2 \left| m, r \right. \right\} \\ \cdot f_{M,R}(m,r)\, dm dr \end{array} \right\}_i. \quad [4]$$

The joint complementary cumulative distribution function of $S_a(T_1)$ and $S_a(T_2)$ conditional on a pair of magnitude m and distance r for seismic source i, in equation 4, can be obtained by integrating the corresponding probability density function, i.e.,

$$P\left\{S_a(T_1) > s_1, S_a(T_2) > s_2 \left| m, r\right.\right\} = \int_{s_2}^{\infty}\int_{s_1}^{\infty} f_{S_a(T_1),S_a(T_2)}(s_1, s_2 \left| m, r\right.) ds_1 ds_2. \quad [5]$$

Given a pair of m and r, a vector of the natural logarithm of spectral accelerations at multiple periods has been empirically tested to follow multivariate normal distribution (Jayaram and Baker, 2008).

The joint probability density function conditional on m and r in equation 5 can be rewritten in the conditional form:

$$f_{S_a(T_1),S_a(T_2)}(s_1, s_2 \left| m, r\right.) = f_{S_a(T_1)}(s_1 \left| m, r\right.) \cdot f_{S_a(T_2)|S_a(T_1)}(s_2 \left| s_1, m, r\right.). \quad [6]$$

Based on the verified assumption of multivariate lognormal distribution for spectral accelerations at multiple periods given m and r, the marginal distribution of $S_a(T_1)$ is given by

$$f_{S_a(T_1)}\left(s_1 \left| m, r\right.\right) = \frac{1}{s_1 \sigma_{\ln S_a(T_1)|m,r}} \Phi_{S_a(T_1)}\left(\frac{\ln s_1 - \mu_{\ln S_a(T_1)|m,r}}{\sigma_{\ln S_a(T_1)|m,r}}\right), \quad [7]$$

and the distribution of $S_a(T_2)$ conditional on $S_a(T_1)=s_1$ can be expressed as

$$f_{S_a(T_2)|S_a(T_1)}\left(s_2 \left| s_1, m, r\right.\right) = \frac{1}{s_2 \sigma_{\ln S_a(T_2)|s_1,m,r}} \cdot \Phi_{S_a(T_2)}\left(\frac{\ln s_2 - \mu_{\ln S_a(T_2)|s_1,m,r}}{\sigma_{\ln S_a(T_2)|s_1,m,r}}\right). \quad [8]$$

The mean and standard deviation of the conditional distribution in equation 8 are given by

$$\mu_{\ln S_a(T_2)|s_1,m,r} = \mu_{\ln S_a(T_2)|m,r} + \rho_{\ln S_a(T_1),\ln S_a(T_2)} \frac{\sigma_{\ln S_a(T_2)|m,r}}{\sigma_{\ln S_a(T_1)|m,r}}\left(\ln s_1 - \mu_{\ln S_a(T_1)|m,r}\right),$$
$$\sigma_{\ln S_a(T_2)|s_1,m,r} = \sigma_{\ln S_a(T_2)|m,r}\sqrt{1-\rho^2_{\ln S_a(T_1),\ln S_a(T_2)}}. \quad [9]$$

In equations 7 and 9, the means and standard deviations for $\ln S_a(T_1)$ and $\ln S_a(T_2)$ conditional on m and r can be obtained from the attenuation relations of $S_a(T_1)$ and $S_a(T_2)$, respectively. The correlation coefficient between $\ln S_a(T_1)$ and $\ln S_a(T_2)$ has been empirically derived (Baker and Jayaram, 2008) as will be discussed in Section 4.

Theoretically, two-dimensional case can be easily extended to a vector of ground motion parameters and the additional requirement is to replace the correlation coefficient with the full matrix of correlation coefficient. Equation 3 can then be applied for describing the temporal uncertainties when the joint annual mean rate of exceedance of spectral accelerations at multiple periods has been obtained in equation 4.

4 CORRELATION COEFFICIENTS OF SPECTRAL ACCELERATIONS

The correlation coefficient between the natural logarithm of spectral accelerations at any two periods, given a scenario earthquake (m and r), has been empirically obtained. Based on the correlation coefficient for any two periods, a symmetric positive semi-definite matrix of the correlation coefficient can be determined from all the combinations of spectral accelerations at different periods.

The most recent matrix of correlation coefficient for a scenario earthquake has been defined based on regression analysis of a large strong motion database by Baker and Jayaram (2008). In the following numerical example, this matrix of spectral correlation is adopted for the determination of multivariate distribution model.

It is noted that, due to the limited ground motion records, the correlation coefficient between spectral accelerations at any two periods from different pairs of m and r is still unavailable. This limitation may prevent the application of the multivariate distribution model on structures having long fundamental period as will be seen later. This problem can be resolved with the increase of recorded ground motions.

5 DISTRIBUTION MODELS OF SPECTRAL ACCELERATIONS

In Sections 2 and 3, the conventional PSHA and vector-valued PSHA were described. Having obtained the marginal and joint distributions of spectral accelerations conditional on a pair of m and r, the probabilities of exceedance can be determined by integrating all possible magnitudes and distances for the site of interest based on equations 1, 3 and 4.

The resulting probabilities of exceedance associated with the corresponding threshold values of spectral accelerations, however, can only be plotted numerically. The analytical marginal and joint distribution functions, for the numerical results generated by PSHA and VPSHA, are still unknown. To perform reliability-based seismic design of structures, the analytical multivariate distribution function of design spectral accelerations at multiple periods for a given site is desired.

Moreover, except for the UHS associated with a small number of corresponding levels of probability of exceedance for a given site, the full information required for performing a PSHA or VPSHA is usually unavailable for structural analysts. A reliable analytical multivariate distribution function of design spectral accelerations at multiple periods, which can be obtained based on as little information as possible, such as only the existing UHS, is preferable.

5.1 Marginal Distribution Model

According to the seismic hazard curve from the PSHA for a site of interest (i.e., UHS in this paper), given a level of probability of exceedance, a spectral acceleration value at a specified period can be obtained. For this spectral acceleration value associated with the corresponding probability of exceedance, a scenario earthquake (or so-called "beta earthquake"), represented by magnitude m_β, distance r_β, and ε_β, can be determined by applying the Probabilistic Seismic Hazard Deaggregation (PSHD) (McGuire, 1995). ε_β is defined as the number of logarithmic standard deviations by which the logarithmic spectral acceleration deviates from the mean.

The "beta earthquake" derived for a given probability level contributes the most seismic hazard to the spectral acceleration at the selected period among all possible earthquakes considered in the PSHA. The "beta earthquake" can thus approximately represent the seismic hazard to the spectral acceleration at the given period at the site of interest by the derived m_β, r_β, and ε_β, instead of all possible earthquake threats. This approximation has been used for reducing the UHS by Baker and Cornell (2006).

It is also assumed that the "beta earthquakes" (represented by magnitudes m_β and distances r_β) obtained using the PSHD for the same hazard curve (i.e., spectral accelerations at the same period) but different probability levels are identical. This assumption is made because this is not an unusual case for the numerical results of the PSHD. Only the ε_β value changes with the change of probability of exceedance to characterize the variation of spectral acceleration. Further investigation is needed to validate this assumption.

Therefore, the spectral accelerations on different levels of probability of exceedance at the same period are generally induced by the same "beta earthquake". Since the spectral acceleration at a given period conditional on a scenario earthquake follows lognormal distribution as discussed in Section 2 and illustrated in Figure 1, the spectral acceleration at a given period can thus be assumed to be lognormally distributed marginally conditional on the "beta earthquake", i.e., the seismic hazard curve for spectral acceleration at a given period for a site empirically matches the complementary lognormal cumulative distribution function.

Since all the information of variation on the spectral acceleration, including magnitude, distance, activity rate of seismic source, and any other ground motion parameters, has been included in the hazard curve from the PSHA, the mean and standard deviation of the marginal lognormal distribution for the spectral acceleration can be simply determined by using the regression analysis on the hazard curve.

It should be mentioned that although the assumption of lognormal distribution is based on the concept of the PSHD, the numerical results of the PSHD (i.e., "beta earthquake" represented by m_β, r_β, ε_β) is not actually needed. The only information, on which the marginal lognormal distribution depends, is at least two spectral acceleration values at a given period associated with two corresponding levels of probability of exceedance as shown in Figure 1.

5.2 Multivariate Distribution Model

Having obtained the marginal lognormal distributions (mean and standard deviation values) for the spectral accelerations at given periods separately as presented in Subsection 5.1, the multivariate lognormal distribution function for those spectral accelerations can be directly determined, if the corresponding matrix of correlation coefficient is available, by using the Nataf joint distribution model (Liu and Der Kiureghian, 1986).

There is also a more compelling reason. According to the concept of the UHS, the long period portion of the UHS represents large far-field earthquakes, whereas the short period portion of the UHS represents small near-field earthquakes (Atkinson and Beresnev, 1998). In practical applications, the response spectra of one large far-field and one small near-field ground motions are frequently used to cover the long period and short period segments of the UHS, respectively (McGuire, 1995, Atkinson and Beresnev, 1998). The spectral accelerations at short periods can thus be safely assumed to be induced by a small near-field beta earthquake. Since the spectral accelerations at multiple periods have been tested to follow multivariate lognormal distribution conditional on a given scenario earthquake, the assumption of the multivariate lognormal distribution for spectral accelerations within short period range is thus theoretically reasonable.

Given a scenario earthquake, the matrix of correlation coefficient between spectral accelerations at any combinations of different periods is available as discussed in Section 4. The multivariate lognormal distribution function for the spectral accelerations at short periods can then be easily determined based on the means and standard deviations of the marginal distributions derived in Subsection 5.1 and the empirical matrix of correlation coefficient.

In practice, the proposed distribution model and the analysis procedure are suitable only for structures having short fundamental periods (usually less than 0.3 sec). This multivariate distribution model could benefit the nuclear energy industry, since nuclear structures (exiting plants and new designs) are very stiff with fundamental periods in the range of 0.06-0.3 sec (EPRI, 2007).

As mentioned in Section 4, the procedure described above cannot be readily used for long-period structures.

The oscillatory modes of a long-period structure may cover the long and short period regions, controlled by large far-field and small near-field beta earthquakes, respectively. The correlation coefficients between spectral accelerations at two periods from different pairs of *m* and *r* are thus needed. This type of spectral correlation will become available in future with the increase of recorded ground motions.

It should be emphasized that the design spectral accelerations (threshold values for different periods), obtained from the multivariate distribution model for a given annual probability of exceedance, occur simultaneously. Hence, the combination of these design spectral accelerations gives a single "design earthquake" conceptually.

5.3 Numerical example

In this section, the procedure for obtaining the marginal and joint distribution functions for the design spectral accelerations (i.e., approximate results) is demonstrated through a PSHA example and then numerically verified by the results of the VPSHA (i.e., accurate results) for the same hazard environment.

As illustrated in Figure 2, a simple numerical PSHA example is performed to obtain the seismic hazard curves for spectral accelerations at short periods of 0.06 and 0.3 sec, respectively, using equations 1-3. The attenuation relation, proposed by Abrahamson and Silva (1997), is used for obtaining the parameters in equation 2.

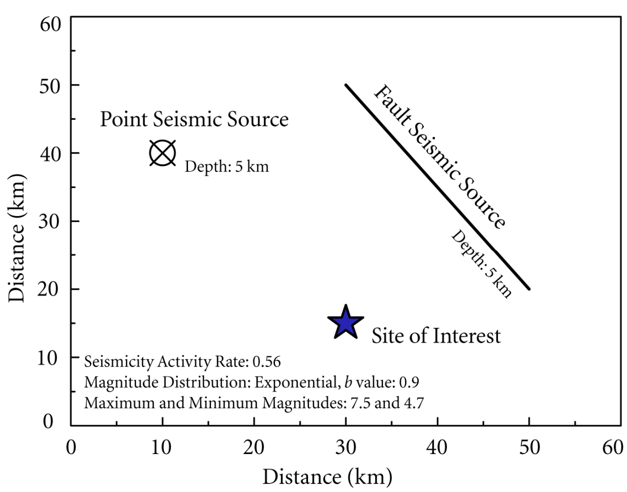

Figure 2. Seismicity of the site for PSHA and VPSHA

The resulting seismic hazard curves for spectral accelerations at 0.06 and 0.3 sec are shown in Figures 3 and 4, respectively, in square symbols. To determine the parameters of the logarithm of spectral acceleration for the marginal lognormal distribution function, two resulting hazard curve points at frequently used probability of exceedance of 0.002 and 0.0004 are adopted as shown in Figures 3 and 4 in shaded square symbols. Most of the resulting marginal complementary lognormal distribution curves go through the unused hazard curve points. It is noted that more hazard curve points can be used for estimating parameters of the distribution functions by using the regression analysis if applicable.

Having obtained the marginal lognormal distributions for spectral accelerations at 0.06 and 0.3 sec, based on the proposed procedure, the joint lognormal distribution for these two spectral accelerations can then be determined by using the matrix of correlation coefficient described in Section 4. The contours of the joint annual probability of exceedance for spectral accelerations at 0.06 and 0.3 sec, derived from the proposed procedure, are illustrated in Figure 5.

To verify the joint lognormal distribution model, a VPSHA, based on the same seismic hazard information as the PSHA, is performed as shown in Figure 6. Compared to the VPSHA (incorporating all possible earthquake threats), the relative errors of the spectral accelerations of the joint lognormal distribution (approximating all possible earthquake threats by a single beta earthquake) are less than 5 percent for a given joint probability of exceedance. For example, as shown in Figures 5 and 6, for the marginal probability of exceedance of 0.0004, spectral accelerations at 0.3 sec from the lognormal model and the PSHA are 1.47 g and 1.45 g, respectively. Having determined the spectral accelerations at 0.3 sec, to achieve the joint probability of exceedance of 0.0002, spectral accelerations at 0.06 sec from the lognormal model and the VPSHA are 0.51 g and 0.53 g, respectively. The relative errors for the marginal and joint probabilities of exceedance are only 1.38% and 3.77%, respectively. Theoretically, the bivariate lognormal distribution model demonstrated in the numerical example can be easily extended to multivariate lognormal distribution with more than two random variables.

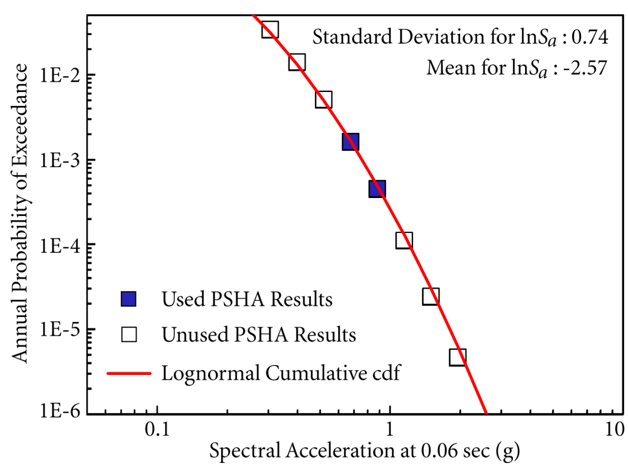

Figure 3. Seismic hazard curve for spectral acceleration at 0.06 sec from PSHA

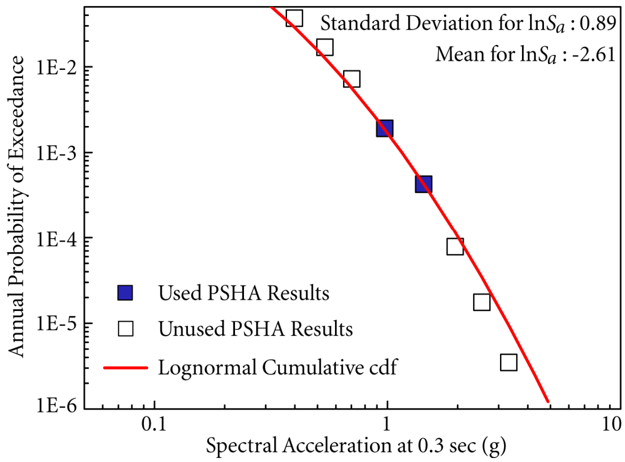

Figure 4. Seismic hazard curve for spectral acceleration at 0.3 sec from PSHA

As a result, the seismic hazard curves derived from the PSHA can be well represented by the complementary marginal lognormal distribution. It is also satisfactory to represent the results of the VPSHA by multivariate lognormal distribution. The parameters can be easily determined from the seismic hazard curves using regression analysis.

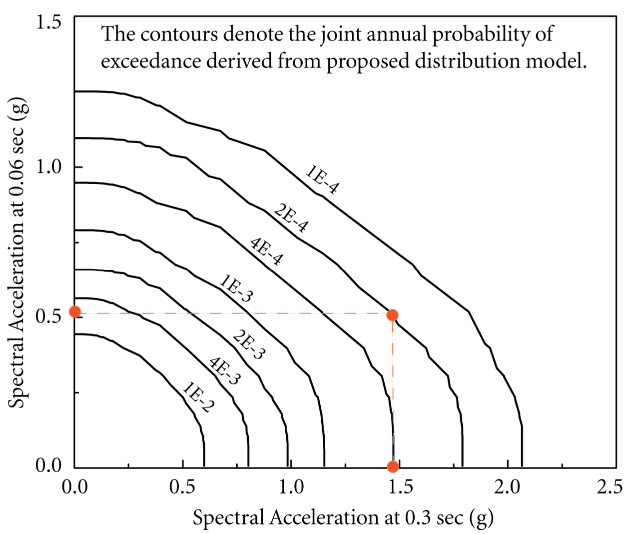

Figure 5. Contours of joint lognormal distribution

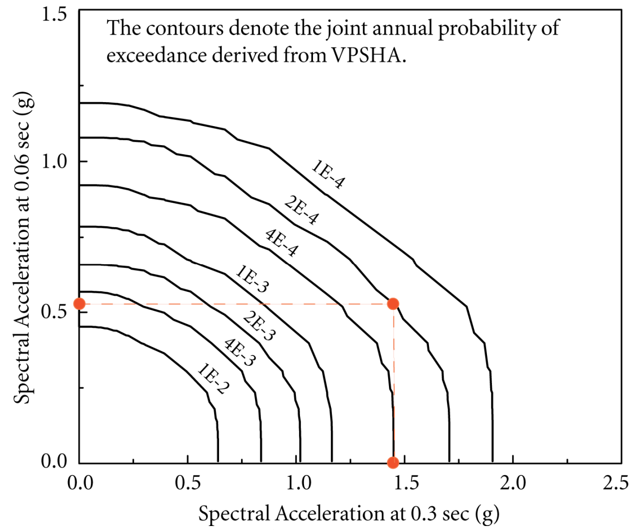

Figure 6. Contours of VPSHA

6 CONCLUSION

For the proper use of the uniform hazard spectra in reliability-based seismic design, a multivariate (lognormal) distribution model is proposed, which can appropriately represent the variation of spectral accelerations at multiple periods for a site of interest. The procedure of analysis contains the following main steps:

1. Select at least two spectral accelerations associated with the corresponding annual probabilities of exceedance at each vibration period of interest for the UHS at the site.
2. Estimate the parameters of the marginal lognormal distribution for spectral accelerations at all interested periods by solving the equations (for the cases with two points) or using the regression analysis (for the cases with more than two points).
3. Establish the multivariate lognormal distribution of the spectral accelerations at the periods of interest based on the parameters, obtained in Step 2, and the empirical matrix of correlation coefficient of the spectral accelerations.

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