State parameter based model for unsaturated soils

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ABSTRACT
This paper presents a state parameter based model for partially saturated soils based on Generalized Plasticity. The proposed model is based on two pairs of work conjugated variables and a suitable hardening law taking into account the bonding – debonding effect of suction and the degree of saturation. A generalized state parameter for unsaturated state is proposed to reproduce monotonic and cyclic behaviour of soils with a single set of constitutive parameters. The developed constitutive model has a hydro-mechanical formulation including a modified water retention curve relating the saturation degree and the normalized suction. The model includes two important features of the hydraulic behaviour of soils: void ratio dependency and hydraulic hysteresis. Comparison between model simulations and series of experimental data available, both cohesive and granular, are given to illustrate the accuracy of the enhanced generalized plasticity equation.

RESUMEN
Este trabajo presenta un modelo constitutivo para suelos parcialmente saturados basado en la Plasticidad Generalizada. El modelo propuesto está formulado con dos pares de variables conjugadas de tensión y deformación y un ley de endurecimiento que tiene en cuenta el efecto de cementación debido a la succión. Se propone una generalización del concepto de parámetro de estado para reproducir el comportamiento monótono y ciclico con un juego de constantes del modelo. El modelo tiene una formulación hidro-mecánica acoplada a través de la ecuación de rentención de agua en el suelo (WRC) relacionando el grado de saturación y la succión normalizada. El modelo incluye dos aspectos importantes del comportamiento hidráulico de los suelos: la dependencia con la relación de vacíos y la histéresis hidráulica. Con el objeto de ilustrar la capacidad de la ecuación constitutiva de Plasticidad Generalizada propuesta, se presentan comparaciones entre las simulaciones del modelo y una serie de datos experimentales disponibles, tanto para suelos cohesivos como granulares.

1 INTRODUCTION

In many practical problems concerning monotonic or cyclic behaviour of soils, the material is unsaturated. As examples we can mention road or railroad geostructures, natural slopes and embankments. Modelling of unsaturated soils is still a young research branch, which is presenting important difficulties. First of all, there is the problem of choosing a suitable stress measure. There are two main alternatives: the bitensorial formulation, which has been used in the Barcelona model of Alonso and co-workers (1990), or the Bishop effective stress, which has attracted the attention of researchers since the work of Houlsby (1997). Based on Houlsby work, a new generation of models for unsaturated soils based both on the effective stress and suction was produced (Jommi, 2000, Gallipoli et al., 2003a, Tamagnini & Pastor, 2004, among others).

The aim of this paper is to present experimental evidence that allowed the extension of the Generalized Plasticity model based on the state parameter (Pastor et al. 2009) to reproduce the main characteristics of partially saturated soils under monotonic and cyclic loadings. The model is formulated in two pairs of work conjugated variables. The stress variables are the effective stress tensor defined in equation **Error! Reference source not found.**~~(6)~~ and the matrix suction *s*, and the strain variables are the soil skeleton strain and degree of saturation.

The idea of a single parameter that incorporates several aspects of soil behaviour has been studied mainly in saturated granular soils to include the double dependence on the density and confining pressure. The state parameter, as it is known today, has several definitions depending on different combinations of current state of the soil and its critical state (Been & Jefferies, 1985; Ishihara, 1993). The state parameter most widely used today was proposed by Been and Jefferies (1985), and is defined as:

$$\psi_s = e - e_c = e - e_{atm} + \lambda \cdot \left(\frac{p'}{p'_{atm}} \right)^{\varsigma} \qquad [1]$$

where e_{atm} is the void ratio at a confining pressure of 100 kPa, λ is the slope of critical state line , ζ_c varies between 0.60 to 0.80 and p'_{atm} is the atmospheric pressure.

ψ_s combines the influence of void ratio and the confining pressure into a single parameter. Granular soils with equal value of state parameter show a relatively similar behaviour. Positive values of ψ_s are related to a contractive behaviour, while negative values of ψ_s indicate dilative behaviour.

The extension of the state parameter for partially saturated soils includes a generalization of the critical state for different suctions as a function of a bonding parameter.

2 STATE PARAMETER FOR UNSATURATED SOILS

An important aspect of the model is the definition of state parameter defined in section 1. In the case of partially saturated soils, the critical state line at the *e* - *p'* depends on suction (see Figure 1). Therefore, to define the state parameter in partially saturated soils is essential to find the variation of the unsaturated CSL's. Recently, Gallipoli et al (2003a) introduced a bonding parameter ξ defined by:

$$\xi = f(s)\cdot(1-S_r) \qquad [2]$$

where $f(s)=\Delta\sigma/\Delta\sigma_0$ represent the capillary force due to suction increments (Fisher, 1926).

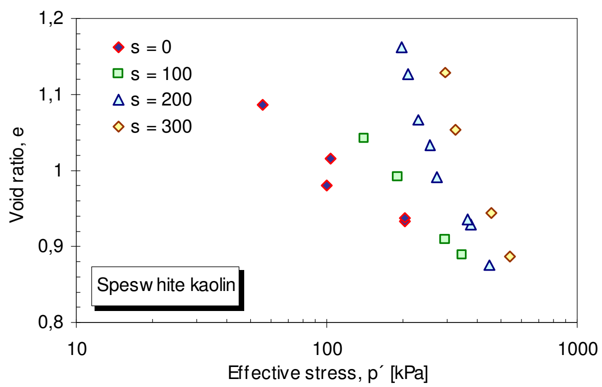


Figure 1. Critical state points for different suction (Experimental data from Sivakumar, 1993).

Here, we will use the following alternative relation linking the values of the critical effective stress *p´* at saturation and at a given suction for a fixed void ratio:

$$\frac{p_{cs}^{\prime unsat}}{p_{cs}^{\prime sat}} = 1 + g(\xi) \qquad [3]$$

where

$$g(\xi) = a\cdot\left[exp(b\cdot\xi)-1\right] \qquad [4]$$

and ξ is bonding parameter defined in eq. [2]. The function *g*(ξ) depends on the degree of saturation and on suction and equals zero at saturation. The parameters *a* and *b* are calibrated from experimental data.

Combining equation [3]~~[3]~~ and **Error! Reference source not found.**~~[4]~~ with a suitable definition of a CSL for saturated states, we will obtain a generalization of the critical state line to unsaturated states:

$$e_c = e_{atm} - \lambda\cdot\left(\frac{p'}{p_a}\right)^{\zeta_c}\cdot\left(1+g(\xi)\right)^{-\zeta_c} \qquad [5]$$

where e_{atm} , λ and ζ_c are model parameters, *p´* is the mean effective stress given by [6] and *g*(ξ) is the bonding function expressed by [4].

To show the effectiveness of the proposed approach, we will consider two cases for which experimental data is available: Speswhite kaolin (Sivakumar, 1993) and the decomposed granitic soil (Chiu, 2001).

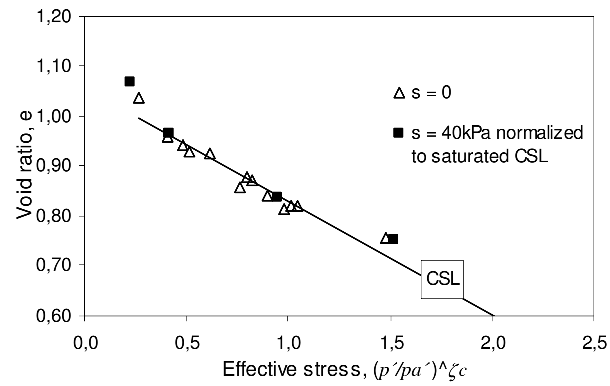


Figure 2. Normalization of CSLs (Experimental data from Chiu, 2001]).

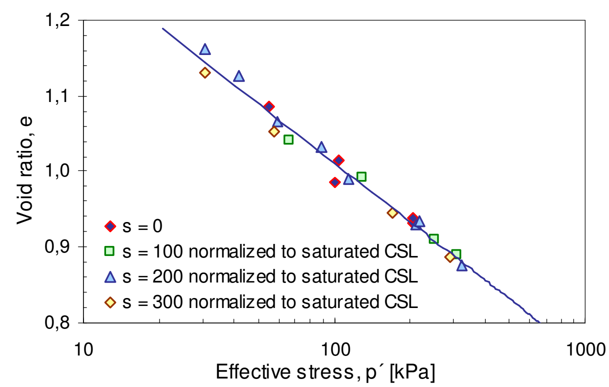


Figure 3. Normalization of CSLs (Experimental data from Sivakumar, 1993]).

In both cases we plotted the critical state conditions observed in the laboratory tests on the effective stress vs. void ratio planes (Figures 2 and 3), normalized by equation [3].

3 STRESS AND STRAIN VARIABLES

The model is based on two pairs of work conjugated variables (Houlsby, 1997). The stress variables are the effective stress tensor and the matrix suction *s*; the strain variables are the soil skeleton strain and degree of saturation. The effective stress tensor is the generalization of Terzaghi's effective stress for unsaturated soils proposed by Bishop (1959):

$$\sigma''_{ij} = \sigma_{ij} - u_a\cdot\delta_{ij} + \chi\cdot(u_a-u_w)\cdot\delta_{ij}$$

[6]

where σ_{ij} is total stress tensor, u_a is pore air pressure, u_w is pore water pressure, χ is a scalar function and δ_{ij} is Kronecker delta. It was found that χ depends on the degree of saturation, the type of the soil and the hysteresis effects due to water content or stress changes. The expression (u_a - u_w) is called matrix suction (s). In the definition of the effective stress we have introduced a modification which takes into account the existence of a residual degree of saturation S_{r0}:

$$\chi = S_{re} = \frac{S_r - S_{r0}}{1 - S_{r0}} \quad [7]$$

We provide two examples which illustrate the effectiveness of the proposed approach is order to improve the capability of the model performance.

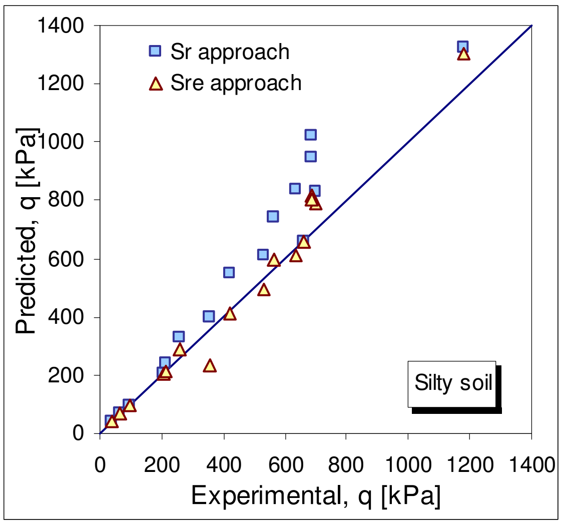


Figure 4. Comparison between deviator stress predicted and experimental for silty soil described by Maâtouk et al (1995)

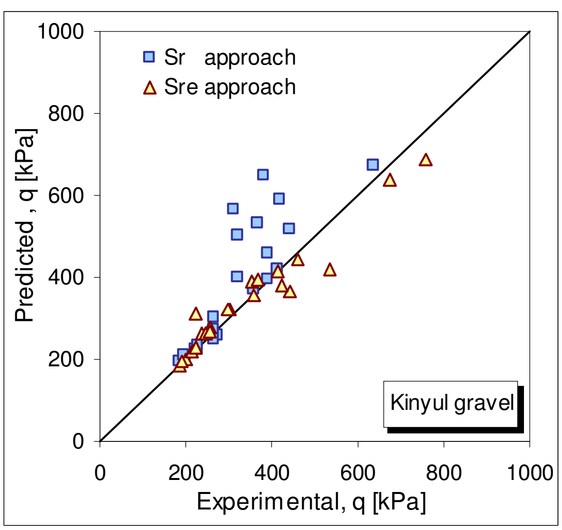


Figure 5. Comparison between deviator stress predicted and experimental for kinyul gravel (Toll, 1990).

4 HYDRUALIC EQUATION

In order to model the hydro-mechanical behaviour of partially saturated soils the proposed model incorporates a water retention curve (WRC) which takes into account both the hydraulic hysteresis during a drying – wetting cycle and its dependency on past history. A modified version of the water retention curve of Fredlund & Xing (1994) is adopted:

$$Sr = Sr_0 + (1 - Sr_0) \cdot \left\{ \ln \left[\exp(1) + \left(\frac{s^*}{a_w \cdot p_0} \right)^n \right] \right\}^{-m} \quad [8]$$

where s^* is the normalized suction proposed by Gallipolli et al (2003) to account for void ratio dependency

$$s^* = e^{\Omega} \cdot s \quad [9]$$

where Ω, a_w, $n_{w/d}$ and $m_{w/d}$ are model parameters, e is the void ratio and s the matrix suction. Figure 6 displays the results of a series of tests conducted on Kiunyu Gravel (Toll, 1990) at different void ratio normalized by equation [8] (symbols) and the calibration of the modified Fredlund & Xing equation (line).

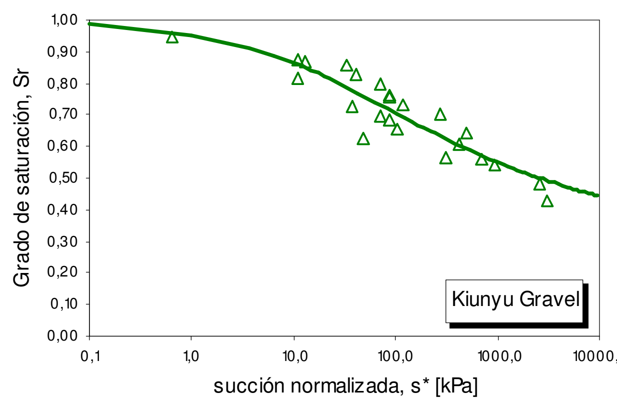


Figure 6. S_r – s^* curve for experimental data at critical state of Kiunyu Gravel. (Experimental data from Toll, 1990).

The main wetting and drying curves are obtained by assuming different values for a_w, $n_{w/d}$ and $m_{w/d}$. We assume non-linear scanning curves (Figure 7).

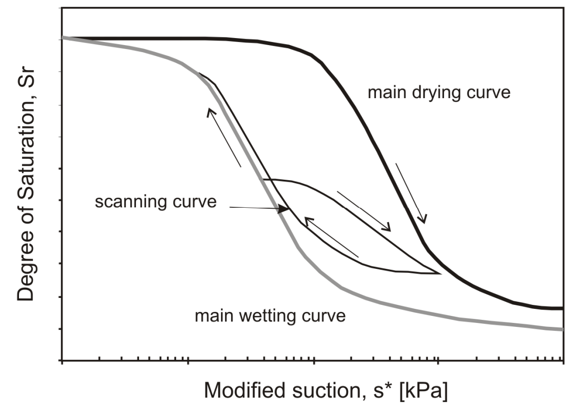


Figure 7. Hydraulic relationship

5 GENERALIZED PLASTICITY FRAMEWORK

Since the basic idea of Generalized Plasticity Theory (GPT) was introduced (Zienkiewicz & Mroz, 1984; Pastor et al. (1990), several improvements have been proposed to describe many important features of soil behaviour (Manzanal et al 2010a). Based on PZ model for sand and clays under different loading conditions, the proposed constitutive model is combining versatile and hierarchical formulation of Generalized Plasticity with the Critical State and the concept of state and bonging parameter described in previous sections. GPT provides a framework within which accurate models can be developed to describe softening and liquefaction under monotonic and cyclic loading.

Based on the original ideal of Tamagnini & Pastor (2004), the total strain rate is defined as a sum of the elastic component, the plastic component coupled with the stress tensor and the plastic component coupled with suction. Therefore, the constitutive equation is:

$$d\varepsilon = \left(\mathbf{D}^{e}\right)^{-1} : d\sigma'' + \frac{1}{H_{L/U}} \cdot \mathbf{n}_{gL/U} \otimes \mathbf{n} : d\sigma'' + \frac{1}{H_b} \cdot \mathbf{n}_{gL/U} \cdot ds \qquad [10]$$

In order to reproduce the elastoplastic behaviour of a material according to the Generalized Plasticity Theory, the following items must be known: the elastic behaviour tensor $\mathbf{D}^e$, the tensor discriminating loading and unloading situations $\mathbf{n}$, the tensor of plastic flow direction in loading and unloading $\mathbf{n}_g$; the plastic modulus in loading and unloading $H_{L/U}$; and the plastic modulus in wetting and drying paths H_b. Details about the formulation can be found in Manzanal et al (2010b).

6 SIMULATIONS

In order to show the predictive capabilities of the model for cohesive and frictional unsaturated soils, we provide here some validation cases concerning some well documented experiments (Sivakumar, 1993; Sharma 1998, Russell & Khalili, 2006).

In order to show the influence of the wetting- drying cycle on the mechanical soil behaviour, we have chosen an experiment performed by Sharma (1998). The test consists in a constant suction isotropic compression loading/unloading cycle (a-b-c) at s = 200kPa, followed by wetting – drying cycle (c-d-e) at p_{net} = 10kPa and a second constant suction isotropic reloading and unloading cycle (e-f-g). Figure 8 provides the model predictions and experimental data on compacted bentonite- kaolin sample in (i) net confining pressure vs void ratio and (ii) degree of saturation vs net stress.

Concerning deviator conditions, we have chosen the experiments performed in cohesive soils by Sivakumar (1993) and in sands by Russell & Khalili (2006). The first example is a series of constant volume triaxial tests on Speswhite Kaolin, denoted as 2A, 3A, 4A and 5A, with net confining pressures of 200, 100, 150 and 300 kPa, and at initial suction of 200 kPa. Figure 9 provides (i) net confining pressure vs deviatoric stress, (ii) axial strain vs deviatoric stress, and (iii) axial strain vs pore pressure change.

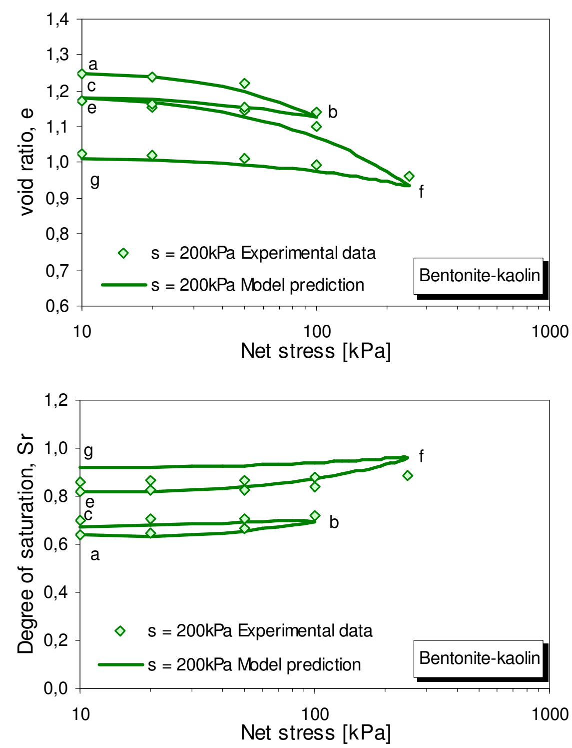


Figure 8. Comparison between model predictions and experimental data of isotropic loading/unloading tests with a wetting-drying cycle on a bentonite - kaolin sample (Experimental data from Sharma, 1998).

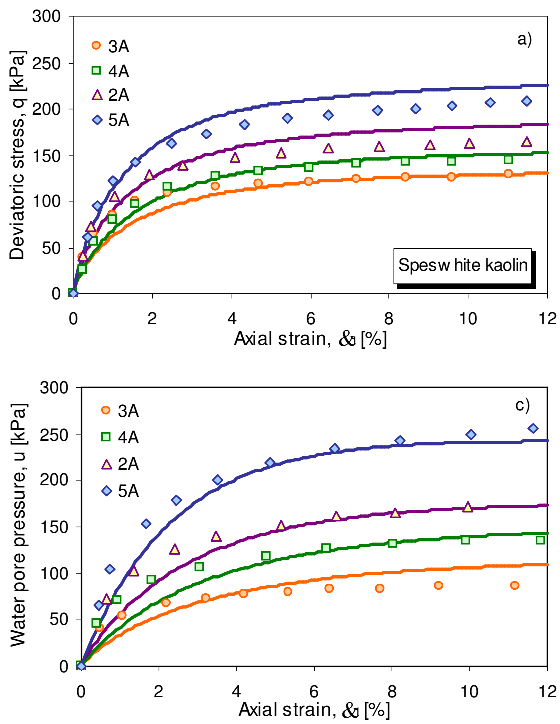
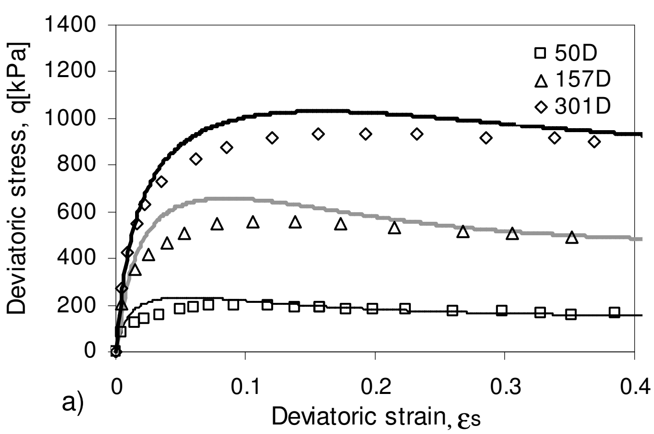


Figure 9. Comparison between model predictions and experimental data of undrained triaxial compression tests at constant suction on Speswhite kaolin (Experimental data from Sivakumar, 1993)

Concerning the granular soil, we have chosen the tests on Kurnell sand, described by Russell & Khalili (2006). Figure 10 shows the predictions of the model during the fully saturated triaxial tests under the confining pressures of 50kPa, 157kPa and 100kPa.



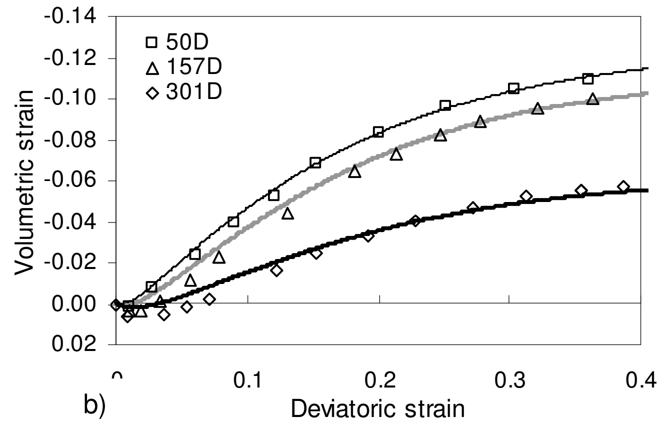


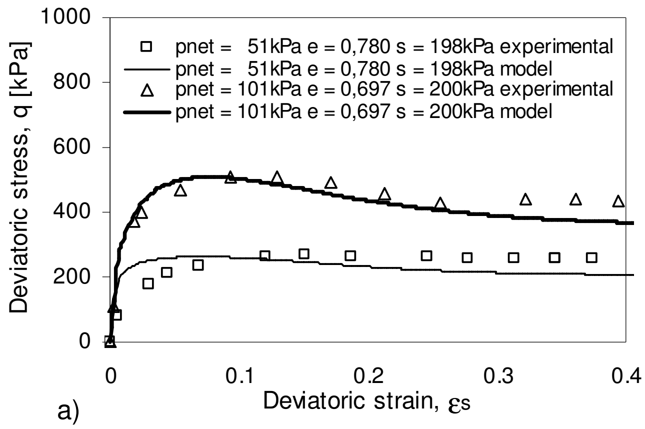
Figure 10. Comparison between model simulations and fully saturated drained triaxial compression test results. (experimental data from Russell & Khalili, 2006). a) deviatoric stress vs deviatoric strain and b) volumetric strain vs deviatoric strain

Figure 11 shows the experimental results and the model predictions for a drained triaxial on Kurnell sand, under two initial net confining pressures of 50 and 100 kPa at constant suction *s* = 200kPa.

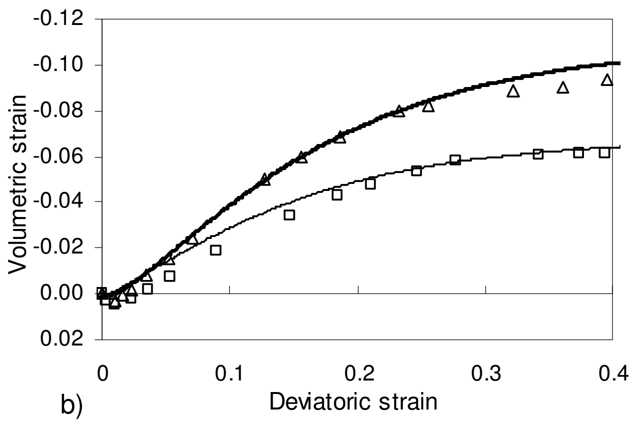
7 CONCLUSIONS

The concept of state parameter, which takes into account both the effect of density and confining pressure in saturated granular soils, has been extended to unsaturated soils analysis as a function of bonding parameter. The proposed relationships were implemented to Generalized Plasticity Model to reproduce the main features of unsaturated soil behaviour. The main aspects of the formulation are: (i) it uses a modified definition of the effective stress to unsaturated soils, (ii) it introduces the normalization of the critical state lines for different suction values, and (iii) it introduces the influence of the hydraulic path and the void ratio on the hydro-mechanical soil behaviour.

The model is capable of reproducing stress-strain behaviour of unsaturated soils for different densities, confining pressures and suctions, by using the same materials constants. The Generalized Plasticity Theory gives a suitable framework to reproduce not only the monotonic stress path, but also the cyclic behaviour.



a)



b)

Figure 11. Comparison between model simulations and drained triaxial compression test results at constant suction (experimental data from Russell & Khalili, 2006) a) deviatoric stress vs deviatoric strain and b) volumetric strain vs deviatoric strain.

8 ACKNOWLEDGEMENTS

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